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CONCEPTUAL DESIGN OF LAMINATED PIEZOELECTRIC ENERGY HARVESTING DEVICES USING TOPOLOGY OPTIMIZATION SUBJECTED TO GLOBAL STRESS CONSTRAINTS

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Abstract. *Energy harvesters convert wasted energy into electrical energy. The available energy sources are solar, mechanical, temperature gradient-based, among others. In this work, the conceptual design of laminated piezoelectric energy harvesting devices capable of harvesting vibrational energy have been studied using the topology optimization method (TOM). The developed TOM code combines the finite element method and an optimization algorithm to determine the layout of the piezoelectric material that maximizes the effective electric power (measured at a coupled electric resistor) generated by a harmonic excitation. A global stress constraint, which accounts for distinct failure criteria for different types of materials (isotropic and piezoelectric) is enforced to prevent device failure. Regarding the TOM formulation, the Piezoelectric Material with Penalization and Polarization (PEMAP-P) material model, which allows for piezoelectric material distribution, is used. Regarding the finite element formulation, a piezoelectric shell element is employed. Numerical examples show that it is possible to obtain a non-intuitive optimized design that satisfies all design constraints, which would be difficult to accomplish by using trial and error based design procedures or even by considering the practical expertise of a designer.*

Keywords: *topology optimization, stress constraints, energy harvesting, electric power maximization*

1. INTRODUCTION

Energy harvesters are smart systems that convert wasted energy into electrical energy. The available energy sources can be solar, mechanical, temperature gradient-based, among others sources. In this work we devote our attention to piezoelectric harvesters. This particular type can be incorporated into vibrating structures so that they convert mechanical vibrations into usable electrical energy. We also focus our efforts on small vibration frequencies, e.g., those related to some vibration modes of a bridge. In order to roughly tune the resonance frequency of the harvester to the vibration (excitation) frequency, the piezoelectric transducer is fixed to a metallic substrate.

Piezoelectric-based energy generation is directly related to the strain distribution in the harvester, which in turn is related to its geometry. Therefore, one must consider finding the optimal location, amount, size and shape of the piezoelectric patches attached to the substrate. The topology optimization method (TOM) have been successfully applied to carry these tasks out by maximizing the power generation (Rupp, C.J. et al., 2009) or the electro-mechanical coupling coefficient (Kiyono, C.Y., Silva, E.C.N. and Reddy, J.N., 2012; Zheng, B., Chang, C.J. and Gea, H.C., 2009). TOM is a computational technique that finds the optimal material distribution inside a fixed region, such that, a prescribed objective is maximized or minimized subjected to design constraints.

TOM applied to problems under dynamic assumptions generally gives rise to structures with prohibitive stress values. Because the piezoelectric transducer and the metallic substrate might be bonded together and because a piezoceramic is fragile, some optimization results may become impractical. Wein, F., Kaltenbacher, M. and Stingl, M. (2013) first brought the importance of having a stress constraint in the design of piezoelectric energy harvesters using TOM. In addition to using a 2D model and optimizing the metallic substrate based on the Solid Isotropic Material Penalization (SIMP) method, their approach uses a globalized and normalized stress constraint function for which the admissible stress value is slightly below the stress function value of a fully covered design.

In this work, the conceptual design of laminated piezoelectric energy harvesting devices capable of harvesting vibrational energy have been studied using the topology optimization method (TOM). The developed TOM code combines the finite element method and an optimization algorithm to determine the layout of the piezoelectric material that maximizes the effective electric power (measured at a coupled electric resistor) generated by a harmonic excitation. The resistor emulates an electric load. A global stress constraint, which accounts for distinct failure criteria for different types of materials (isotropic and piezoelectric), is enforced. Additionally, the Piezoelectric Material with Penalization and Polarization (PEMAP-P) material model (Kogl, M. and Silva, E.C.N., 2005), which allows for piezoelectric material distribution and polarization direction determination, is used (only material distribution is optimized to facilitate fabrication, though). Regarding the finite element formulation, a piezoelectric shell element is employed and a fully coupled harvesting system is modeled.

This paper is organized as follows. In Section 2, the finite element formulation applied to the particular harvesting device addressed in this work is briefly introduced. In Section 3, topology optimization is discussed, the optimization problem is proposed and implementation details are provided. Numerical results are discussed in Section 4, and concluding remarks are provided in Section 5.

2. FINITE ELEMENT FORMULATION

2.1 Finite element equations

In this work, energy harvesting plate structures are modeled by using the finite element method. An appropriate element for this case is the 8-node multilayer piezocomposite shell element, which accounts for thin or thick walls and different material layers (Ahmad, S., Irons, B.M. and Zienkiewicz, O.C., 1970; Kogl, M. and Bucalem, M. L., 2005a; Kogl, M. and Bucalem, M. L., 2005b; Reddy, J.N., 2004; Reddy, J.N., 2015). We assume a constant thickness. Further detailing on the piezoelectric shell element can be found in the literature (Balamurugan, V. and Narayanan, S., 2008; Kiyono, C.Y., Silva, E.C.N. and Reddy, J.N., 2012).

Using Hamilton's variational principle and assuming small deformations (linear theory), the finite element equilibrium equations that describe the harmonic behavior are written as (Rupp, C.J. et al., 2009):

$$\begin{bmatrix} \mathbf{K_{uu}} - \omega^2\mathbf{M} + i\omega\mathbf{C} & \mathbf{K_{u\phi}} \\ \mathbf{K}_{\mathbf{u}\phi}^T & \mathbf{K}_{\phi\phi} + i\omega\mathbf{K}_R \end{bmatrix} \begin{Bmatrix} \mathbf{U} \\ \mathbf{\Phi} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{Q} \end{Bmatrix} \text{ or } \tilde{\mathbf{K}}\tilde{\mathbf{U}} = \tilde{\mathbf{F}} \tag{1}$$

where $\omega = 2\pi f$, f is the harmonic excitation frequency, and $\mathbf{U}$, $\mathbf{\Phi}$, $\mathbf{F}$, and $\mathbf{Q}$ are the mechanical displacements, electric potentials, mechanical force, and electric charge global vectors, respectively. It must be noticed that $\mathbf{U}$ and $\mathbf{\Phi}$ are vectors with complex numbers. $\mathbf{K_{uu}}$, $\mathbf{K_{u\phi}}$, $\mathbf{K_{\phi\phi}}$, $\mathbf{K}_R$, $\mathbf{M}$ and $\mathbf{C}$ are the stiffness, piezoelectric, dielectric, electric resistor, mass, and damping global matrices, respectively. These matrices are adequately built by using the respective local matrices (Kiyono, C.Y., Silva, E.C.N. and Reddy, J.N., 2012). The electric resistor matrix $\mathbf{K}_R$ has only one value. It is equal to $-1/(\omega^2 R)$ at the diagonal corresponding to the electrode degree of freedom (Rupp, C.J. et al., 2009).

This work uses the layer-wise theory to model a multilayered structure. This theory assumes that the displacement field must be continuous at the interface between two adjacent layers, i.e., the layers are perfectly coupled. Thus, the equilibrium equation system has to be modified such as described in the papers by Kiyono, C.Y., Silva, E.C.N. and Reddy, J.N. (2012), and Kogl, M. and Bucalem, M.L. (2005a).

Mesh is generated using ANSYS in this work.

3. TOPOLOGY OPTIMIZATION METHOD

The objective is to maximize the electric power generated by a bilayered plate structure, which is excited by a harmonic base vibration. This configuration (considering a generic geometry) can be seen in Fig. 1.

The electric power is calculated by using the electric potential difference between the electrodes of the piezoelectric layer ($\Delta\phi$), the resistor value coupled to the electrodes, and the following equation (Rupp, C.J. et al., 2009)

$$\wp = \frac{|\Delta\phi|^2}{2R} \tag{2}$$

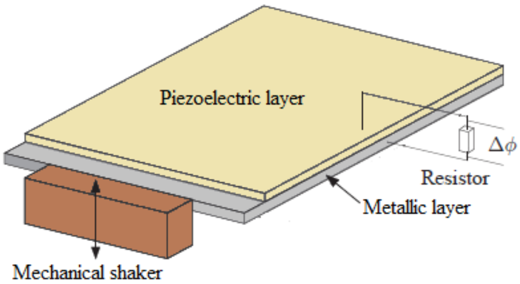

Figure 1 Schematic of a bilayered plate energy harvester excited by a harmonic excitation.

where $\Delta\phi$ is computed considering (i) Eq. (1) modified by the constraint equations regarding the layer-wise theory, and (ii) the prescription of displacements described by a sinusoidal function.

We apply topology optimization techniques to determine the piezoelectric material distribution. The resistor value, on the other hand, is assumed to be given for a given application, and polarization direction is not optimized to facilitate fabrication, as already mentioned. The piezoelectric material distribution is parameterized by the PEMAP-P material model (Kogl, M. and Silva, E.C.N., 2005) in this work.

The heaviside projection technique with β-continuation proposed by Guest, J.K., Prevost, J.H. and Belytschko, T. (2004) has been used to avoid checkerboarding, to eliminate mesh dependency, to reduce intermediate values of the design or optimization parameters at the borders of the optimized topology, and to control the minimum length scale.

3.1 Stress constrained topology optimization

In stress-related optimization problems, there are three major challenges that need to be overcome (Bendsøe, M.P. and Sigmund, O., 2003): the "singularity" phenomenon (Rozvany, G.I.N., 2001), the non-linear dependence on the design, and the local nature of the constraint. The first two issues can be overcome by using an appropriate material model. Several authors have proposed different stress relaxation approaches (Bruggi, M. and Venini, P., 2008; Cheng, G.D. and Guo, X., 1997; Duysinx, P. and Bendsøe, M.P., 1998). Bruggi, M. and Venini, P. (2008) have successfully proposed the qp-approach, which is basically the SIMP method (Bendsøe, M.P. and Sigmund, O., 2003) using different penalization coefficients for stiffness and stress calculations. The third issue mentioned has been properly addressed by Le, C. et al. (2010), who proposed global calculations of the stress constraints. In this work, we employ the qp-approach and the procedure reported by Le, C. et al. (2010).

Before discussing the globalized stress constraint, we will discuss failure criteria related to distinct material types (piezoelectric and metallic) to properly evaluate failure at each material layer. At both layers, the von Mises criterion is used. However, the upper limit is based upon the material yield stress at the metallic layer, and on the material depolarization stress at the piezoelectric layer. The failure criteria can be described by the following inequalities

$$\sigma_{vm}^{met} < \bar{\sigma}_y \quad ; \quad \sigma_{vm}^{piezo} < \bar{\sigma}_d \tag{3}$$

where σ_{vm} is the von Mises stress, and $\bar{\sigma}_y$ and $\bar{\sigma}_d$ are the admissible yield and depolarization stresses, respectively. To avoid scale problems, we normalize the constraints by dividing both sides by the corresponding von Mises stress, which gives rise to the terms λ^{met} and λ^{piezo}. In future work, we intend to model a bonding layer as well (made of a cured silver epoxy paste, for instance), thus proposing a failure criterion for it. It must be noted that this layer does not exist in the proposed model.

The normalized stress constraints need to be calculated at every Gauss point g of the reduced numerical integration to give better precision (Cook, R.D., Malkus, D.S., Plesha, M.E. and Witt, R.J., 2007). To calculate the constraints sensitivities, we have to solve the adjoint problem at every Gauss point, making the procedure computationally inefficient. One way to circumvent this issue is to use the minimum value of λ instead of λ itself (for simplicity, we write λ; however, it can be either λ^{met} or λ^{piezo}). In fact, it must be noted that only its minimum needs to be higher than 1. Because the minimum function is not differentiable, one can use the p-norm formulation (Duysinx, P. and Bendsøe, M.P., 1998; Le, C. et al., 2010) with negative exponent, given by

$$\Lambda = \left(\sum_{g=1}^{n_g} \lambda_g^{-p_n} \right)^{-\frac{1}{p_n}} \tag{4}$$

where p_n is the p-norm penalization coefficient, n_g is the number of Gauss points and λ_g is λ computed at a particular Gauss point. Λ tends to the smallest value of λ_g as p_n tends to infinite. In practice, a range between about 8 and 12 is adopted for p_n to mitigate oscillatory behavior of the optimization procedure (Le, C. et al., 2010).

Depending on the values of λ_g, Λ can be slightly smaller than the smallest λ_g, thus jeopardizing the efficiency of the stress constraint (although safety is favored). This typically happens if there is a large number of variables λ_g with approximately the same magnitude. A solution for a similar problem has been proposed by Le, C. et al. (2010) and Holmberg, E., Torstenfelt, B. and Klarbring, A. (2013). They introduced multiplier factors into their globalized stress constraints. The multiplier proposed by Le, C. et al. (2010) is calculated by using the ratio between the maximum stress value and the p-norm value of the previous iteration. Although this formulation is not differentiable because this ratio is changed discontinuously, they observed that as the optimization converges, the effects of the non-differentiability are reduced and do not cause convergence issues. To prevent this non-differentiability, Holmberg, E., Torstenfelt, B. and Klarbring, A. (2013) proposed another multiplier based on the number of stress evaluation points in the domain. Although this formulation is differentiable, it does not give a good approximation of the maximum stress value when stress concentration occurs. Therefore, we use the approach proposed by Le, C. et al. (2010), adapting the formulation to normalized stress constraints:

$$\Lambda = c\left(\sum_{g=1}^{n_g} \lambda_g^{-p_n}\right)^{-\frac{1}{p_n}} \tag{5}$$

where c is given by

$$c = \frac{\lambda_{min}^{I-1}}{\Lambda^{I-1}} \tag{6}$$

and I is the current iteration.

Finally, to deal with complex variables, we propose to use the absolute value of Λ, which indicates the amplitude of the stress constraint. Thus, we write for each material type:

$$|\Lambda| \geq 1 \tag{7}$$

3.2 Optimization Problem

In this work, the generated electric power (Eq. (2)) is maximized with respect to the material distribution, subjected to globalized stress constraints. The parameters that define the material distribution are the design variables d of each finite element, which are constrained between 0 and 1 (Kogl, M. and Silva, E.C.N., 2005). Summarizing:

$$\begin{aligned} &\underset{\mathbf{d}}{\text{Maximize}}: && \wp \\ &\text{Subjected to}: && |\Lambda_{\Xi}| \geq 1 \\ & && 0 \leq \mathbf{d} \leq 1 \end{aligned} \tag{8}$$

The term Λ_{Ξ} is Λ computed using all Gauss points, i.e., those points regarding both the metallic and piezoelectric layers. Eq. (5) and Eq. (6) must be updated to take these changes into account.

The optimization routine used in this work is the GCMMA method (Svanberg, K., 1987), which is a gradient-based method that updates the design variables iteratively. Thus, the sensitivity analysis of the objective function and stress constraints with respect to the design variables must be performed. The adjoint method is used to efficiently compute sensitivities in this work, as already suggested. All implementations have been done in Matlab.

4. NUMERICAL RESULTS

In this section, we present results obtained with the proposed approach. The design of the laminated energy harvesting devices with stress constraints has been carried out, whenever interesting, by considering boundary conditions and other parameters we expect to reproduce in future experiments. For instance, the material properties of the piezoelectric material and the size of each piezoelectric patch that are provided by a piezoelectric transducer manufacturer have been used. The vibration amplitude, on the other hand, has been changed in the experiments.

The number and position of the piezoelectric patch models coupled to the metallic plate model for the initial guess are shown in Fig. 2. This guess is referred to as fully covered design (FCD). All results present positive polarization. Positive polarization direction follows the positive direction of z axis. It is important to note that (i) the patch models delimit the design domain, i.e., the place where optimization is carried out, and that (ii) the material distribution of the metallic layer is kept fixed during the optimization.

Regarding the PEMAP-P material model, there are some parameters to be set. The authors are well aware that there may be an optimal set of values (Kim, J.E., Kim, D.S., Ma, P.S. and Kim, Y.Y., 2010), and that different combinations lead to different local optima, however, selecting the best set is beyond the scope of this research. Thus, the parameters are empirically chosen for all examples. Many combinations were tried, and the chosen combination provided the best performance together with better convergence of all aspects (generated electric power, stress constraint and topology).

The harvester is clamped at the metallic layer at the hashed area shown in Fig. 2. The thicknesses of the piezoelectric layer and the metallic layer are equal to 0.5 mm and 1 mm, respectively. The plate is subjected to harmonic excitation of 230 Hz at the clamped location.

A mass proportional damping is used, where $\alpha_D = 0.01$ and $\beta_D = 0$. For the circuit resistor, we consider R = 100 kΩ. The materials used for the piezoelectric and metallic layers are PZT-5A and aluminum, respectively.

In the figures that show the optimized material distributions, black and white mean solid and (approximately) void elements, respectively. Grayscale means elements with intermediate properties (those must be eliminated for proper fabrication).

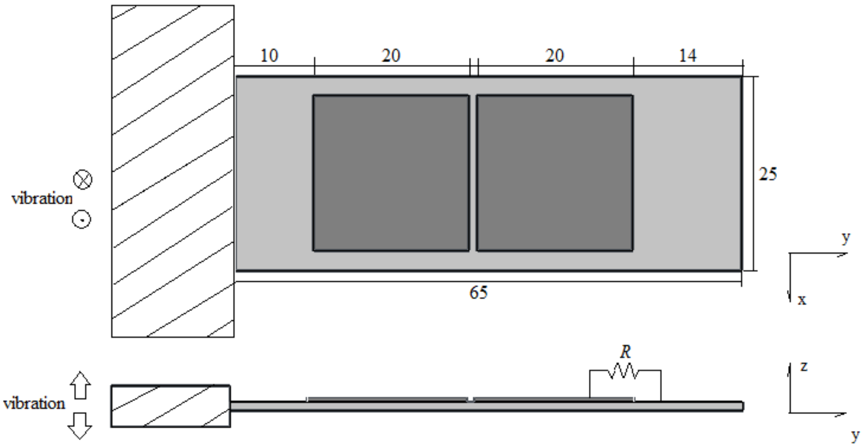

Figure 2 Initial guess: fully covered design (FCD). The patch models delimit the design domain, i.e., the place where optimization is carried out. Measurements are in mm.

Regarding the first results, a displacement amplitude equal to 1 mm peak-to-peak, normal to the plane of the plate (z axis direction), is prescribed. This amplitude value is employed because it approaches some values reported in the literature related to topology optimization methodology for designing piezoelectric energy harvesters.

By applying the optimization formulation to the piezoelectric material distribution, we obtain the non-intuitive results shown in Fig. 3. As it can be seen, the (piezoelectric) material volume is reduced. It is also interesting to note that the optimization algorithm avoids placing material near the clamped area (left side), which is usually the region with higher stresses. Regarding the generated electric power amplitude $\wp$, it is equal to 25 mW.

The resulting stress distribution is shown in Fig. 4. The maximum stresses are equal to σ_{vm}^{met} = 113 MPa and σ_{vm}^{piezo} = 48 MPa, which are under the stress limits of the respective layers (350 MPa and 50 MPa). The maximum stress in the piezoelectric layer of the initial guess (FCD), on the other hand, is equal to 94 MPa, substantially higher than the stress limit. Thus, the initial geometry mechanically excited by a 1mm (peak-to-peak) displacement amplitude is impractical (although the electric power generated, i.e., 92mW, is higher than the optimized value).

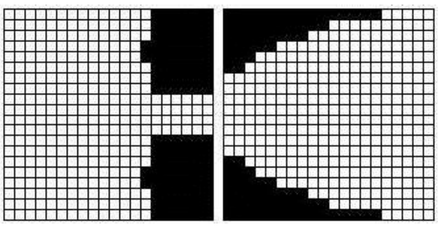

Figure 3 Optimized material distributions considering a prescribed displacement amplitude of 1 mm; black and white mean solid and (approximately) void elements, respectively. The clamped area is at the left side.

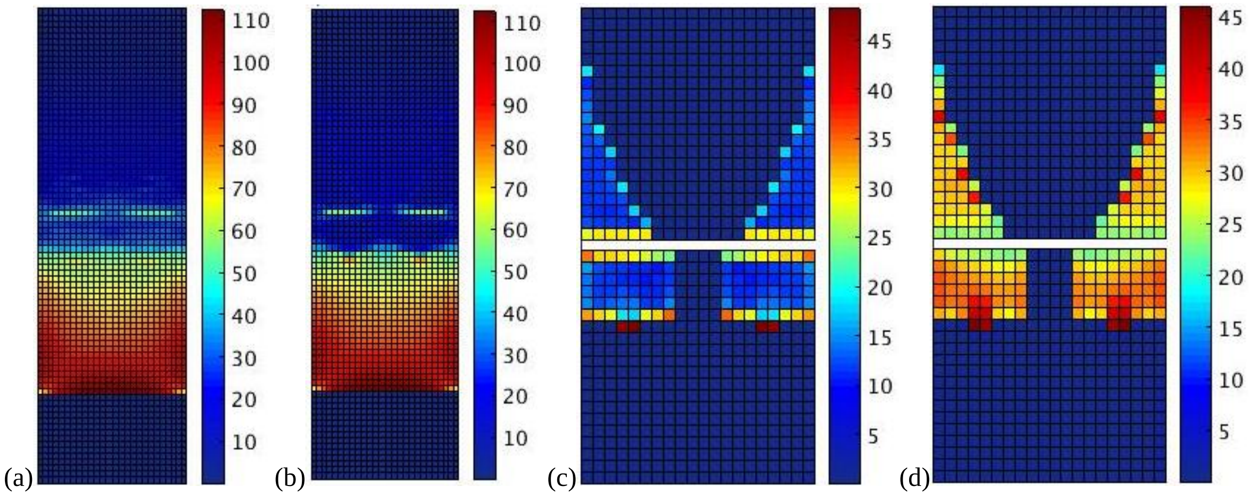

Figure 4 Stress results for the first example: (a) bottom of metallic layer; (b) top of metallic layer; (c) bottom of piezoelectric layer; (b) top of piezoelectric layer. Stresses are in MPa.

By reducing the input displacement amplitude to 9.6 µm peak-to-peak (which corresponds to an acceleration of 2 g peak-to-peak), on the other hand, a value we could reproduce in experiments, we obtain the topologies shown in Fig. 5. In this case, the stress constraints are not active in the final design, as illustrated in Fig. 6. In fact, stress values are substantially smaller than those from the previous example, which means that the stress is not an issue in this case. Regarding the initial design (FCD), it is feasible; however, its electric power (0.009 mW) is substantially smaller than the optimized value (0.25 mW). It is worth mentioning that, at this order of magnitude, the optimized signal might power a digital watch or a temperature sensor. By connecting the harvester to a dedicated system containing a suitable capacitor, on the other hand, alternative tasks requiring either higher power levels or autonomy can be carried out.

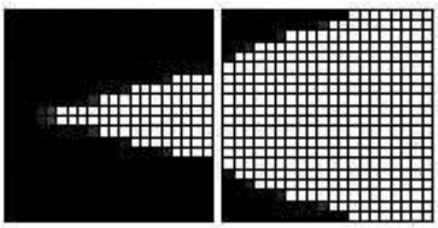

Figure 5 Optimized material distributions for smaller amplitudes; black and white mean solid and (approximately) void elements, respectively. We can see more material in the left square (and less material in the right square) than the previous result. The clamped area is at the left side.

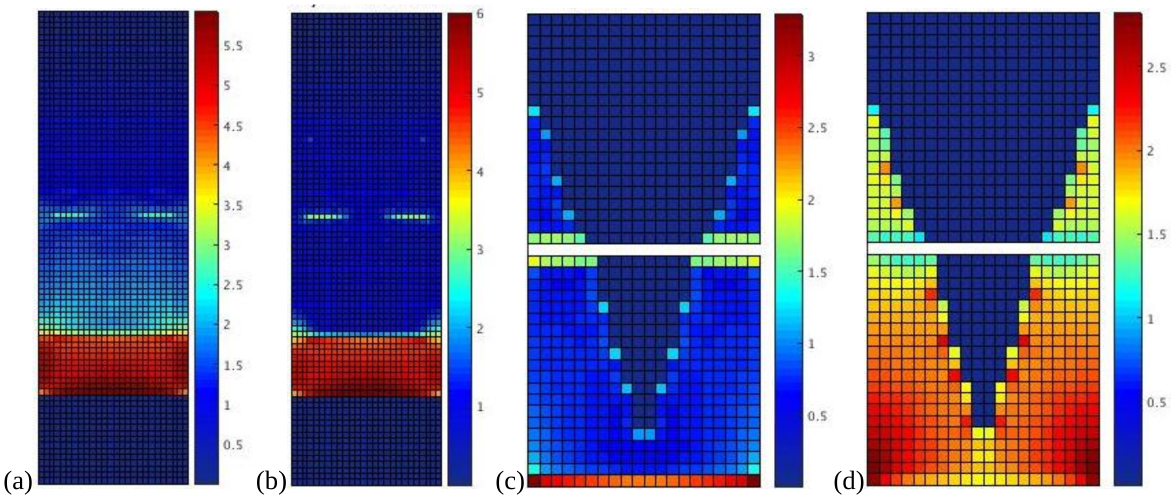

Figure 6 Stress results for smaller amplitudes: (a) bottom of metallic layer; (b) top of metallic layer; (c) bottom of piezoelectric layer; (b) top of piezoelectric layer. Stresses are in MPa.

In all results, it can be noticed that there are very few elements with intermediate material at the borders of the optimized topology. Thus, those designs are eligible for manufacturing and characterization.

5. CONCLUDING REMARKS

In this work, the conceptual design of laminated piezoelectric energy harvesting devices capable of harvesting vibrational energy has been investigated. The TOM has been employed as the design tool. The layout of the piezoelectric transducers that maximizes the effective electric power generated by a harmonic excitation, subjected to a global stress constraint, is estimated by the optimization code. Piezoelectric shell elements model the piezoelectric transducers.

The numerical results show that it is possible to obtain a non-intuitive layout satisfying all design criteria. In addition, initial guesses have been compared to the optimized results, showing improvements. Regarding the stress constraints, our results suggest that they might be employed depending on the application. For relatively high excitation amplitudes, they successfully prevented the optimization algorithm from generating geometries with excessive stress values, thus preventing mechanical failure.

In future work we intend to fabricate and run experiments with designed harvesters. The experiments will provide us with information to set damping parameters. We also intend to improve our finite element model by modeling the conductive adhesive that will be used to bond the piezoelectric transducer to the metallic substrate, as already suggested. Then, by proposing a suitable additional stress constraint, we will be able to make our optimization results even more realistic.

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