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ON THE REALIZATION OF GHM VISCOELASTIC MODEL TO THE TREATMENT OF THE AEROELASTIC STABILITY OF PLATES

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Abstract. *This paper investigates the influence of viscoelastic damping in the aeroelastic stability boundary of flutter in aeronautical panels. Flutter in aeronautical panels is a kind of self-excited oscillation, which can occurs during supersonic flights. In the point of flutter occurrence, when the critical dynamic pressure is reached, the vibrations of the panel become unstable and increase significantly in time. As a result, it can leads to a fatigue failure of the considered structure. Thus, it becomes important to investigate the possibilities of reducing the effects of the flutter in order to increase the reliability of the structures during service. In the present contribution, the Piston Theory is adopted to model the aerodynamic loading in supersonic condition. Aiming the reduction of the aeroelastic instabilities effects and the improvement of the structural dynamic performance, viscoelastic materials applied as constraining damping layers is an interesting strategy to be investigated, since very little effort has been carried-out on this subject. To do so, the Golla-Hughes-McTavish (GHM) parametric model is developed through the identification of the model parameters from the experimental data. Within this context, the finite element model of a three-layer sandwich plate element is presented. After exposing theoretical foundations of the methodology, the description of a numerical study on the flutter analysis of a rectangular plate treated by passive constraining layer damping is addressed.*

Keywords: *Passive control, viscoelastic materials, finite elements, aeroelasticity, flutter.*

1. INTRODUCTION

The aeronautical panel flutter is a major problem in the analysis and design of aerospace vehicles. Jordan (1956) was the first in the identification of the phenomenon, which is a kind of self-excited oscillation normally occurred during supersonic flights in which the vibrations of the panel become unstable and increase in time when the critical dynamic pressure is reached. However, the vibrations amplitudes in this operation region are restrained due to the in-plane tensile stress induced by the geometric nonlinearities. As a result, this phenomenon can affect significantly the fatigue life of engineering structures (Hodges & Pirce, 2002). Although the presence of the geometrical nonlinearities usually takes place in this kind of phenomenon, linear theories are capable to determine the values of critical dynamic pressure, frequency and modal shape of vibrations (Dowell, 1975). In this context, the Piston Theory is adopted in order to model the aerodynamic loading in supersonic conditions regardless the effects of the aerodynamic damping, resulting in the quasi-steady supersonic aerodynamic theory (Kuo, 2011). This approach is the so-named Ackeret's Model.

Kuo (2011) described the effects of variable fiber spacing in composites plates flutter concluding that this parameter has direct influence at the sequence of natural modes changing the flutter coalescence. Song and Li (2012) and Almeida et al (2012) have proposed an active control of the aeroelastic flutter by using piezoelectric materials. Kouchakzadeh et al. (2010) have evaluated the effects of the in-plane forces, the fiber orientations and the aerodynamic damping on the aeroelastic flutter in laminated composites plates. Zhao and Cao (2013) investigated the influence of the stiffness of laminated composite panels in the supersonic flow by placing stiffeners on the surface of the plate. Pegado (2003) studied the aeroelastic instability of panels in supersonic flows employing structural and aerodynamics nonlinearities using the perturbation method. Sohn and Kim (2009), and Prakash and Ganapathi (2006) reported the analysis of supersonic flutter characteristics of functionally graded panels under thermal and aerodynamics loads. More recently, many researchers have been investigated the flutter in panels. However, few studies have addressed the problem of applying the viscoelastic materials to reduce the effects of flutter phenomenon, which motivates the study reported here.

With the purpose of improving the structural dynamic performance of the aerodynamic panels, the passive control strategies by applying viscoelastic materials can be used in the configuration called passive constrained damping layers, as developed by Ioannides and Groothenhuis (1979). Thus, the finite element model of a three-layer sandwich plate is required.

It is widely known that the dynamic behavior of viscoelastic materials is strongly dependent on the excitation frequency and temperature. In the present study, the Golla-Hughes-McTavish (GHM) parametric model (McTavish & Hughes, 1993) is used as a model of viscoelastic material. This methodology requires the realization of a curve fitting to set the exact parameters to represent the behavior of an experimental model. The insertion of such parameters may implies in heavy computational costs due to the augmentation of the matrix dimensions. In terms of the finite element procedures of engineering systems incorporating viscoelastic materials, de Lima et al (2010) presented a viscoelastic reduction method to simulate complex engineering structures treated with viscoelastic materials. Friswell and Lam (1997) implemented a numerical strategy of viscoelastically-damped structures by using the Golla-Hughes-McTavish model in which a new formulation of the series used into the model is proposed. Martin (2011) proposed a new material modulus function feasible to be applied within the framework of the GHM method, in order to improve the curve fitting error generated.

Once the parametric model is incorporated into the sandwich structure, the aerodynamic forces can be obtained by the computation of the work done by the non-conservative aerodynamic loadings. After the matrix assembling, based on the standard finite element procedures, the governing equations of motion of the system can be generated. However, as the viscoelastic damping is a viscous structural one and it is not proportional to the mass and stiff matrix, it is necessary to rewrite the equations into the space state in order to uncouple the equations and solve it.

The results obtained show that the viscoelastic damping increases the rate of occurrence of flutter, stabilizing the plate structure. Finally, it is demonstrated the influence of the temperature and the thicknesses of the layers on the performance of the viscoelastic treatment.

2. FE FLUTTER MODELING IN SANDWICH PLATES WITH VISCOELASTIC LAYERS

In this section, the flutter behavior of a moderately thin three-layer sandwich plate commonly employed in aerospace panels is investigated by using the finite element method. The FE formulation is based on the original developments made by Khatua and Cheung (1973) and implemented by de Lima et al. (2006). Figure 1 depicts a rectangular element formed by an elastic base-plate (1), a viscoelastic core (2) and an elastic constraining layer (3). This element contains four nodes and seven DOFs per node, representing the in-plane displacements in the middle plane of the base-plate in directions x and y (denoted by u_1 and v_1, respectively), the in-plane displacements of the middle plane of the constraining layer in directions x and y (denoted by u_3 and v_3, respectively), the transverse displacements, w, and the cross-section rotations about x and y, denoted by θ_x and θ_y, respectively.

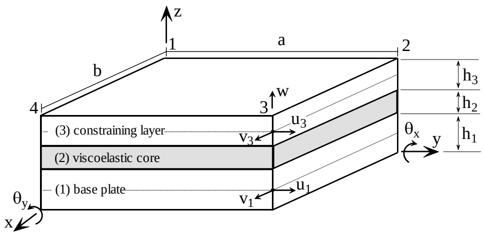


Figure 1 – Schematic representation of three-layer sandwich plate element.

In the development of the theory, the following assumptions are adopted: (i) all the materials involved are homogeneous, isotropic and linear elastic; (ii) normal strains in direction *z* are null for the three layers; (iii) the elastic layers (1) and (3) are modeled according to Kirchhoff's plate theory; (iv) for the viscoelastic core, Mindlin's theory is adopted (transverse shear is included); (v) the cross-section rotations, θ_x and θ_y, are the same for the elastic layers; (vi) the transverse displacement, w, is the same for all the layers. These assumptions have been considered by many authors as being adequate for the modeling of thin panels, as is the case of the structures addressed herein (Austin, 1999). Moreover, previous studies carried-out by the authors demonstrated satisfactory correlation between model predictions and experimental results (de Lima, Stoppa, & Rade, 2003).

The discretization of the displacement fields within the element is made by using bilinear interpolation functions for the displacements in the middle plane of the elastic layers in directions x and y, and a cubic interpolation function for the transverse displacement by the relation, $\boldsymbol{u}(x,y,t) = \boldsymbol{N}(x,y)\,\boldsymbol{u}_{(e)}(t)$, where $\boldsymbol{N}(x,y)$ is the matrix formed by the shape functions, and $\boldsymbol{u}_{(e)}(t)=\left[u_1^i\ v_1^i\ u_3^i\ v_3^i\ w^i\ \theta_x^i\ \theta_y^i\right]^T$ with $i=1\,to\,4$ designates the vector containing the nodal variables as functions of time. The strain-displacement relations, $\boldsymbol{\varepsilon}(x,y,z,t)=\boldsymbol{B}(x,y,z)\boldsymbol{u}_{(e)}(t)$, where the strains for the elastic layers, $\boldsymbol{\varepsilon}_k=\left[\varepsilon_x^{(k)}\ \ \varepsilon_y^{(k)}\ \ \gamma_{xy}^{(k)}\right]^T$ $(k=1{,}3)$, and for the viscoelastic core, $\boldsymbol{\varepsilon}_2=\left[\varepsilon_x^{(2)}\ \ \varepsilon_y^{(2)}\ \ \gamma_{xy}^{(2)}\ \ \gamma_{xz}^{(2)}\ \ \gamma_{yz}^{(2)}\right]^T$, are generated. Thus, based on the stress-states assumed for each layer and the corresponding stress-strain relations, following standard analytical developments based on variational approaches, the strain and kinetic energies of the three-layer sandwich plate FE are formulated and the elementary mass and stiffness matrices are obtained as:

$$\begin{gathered}\boldsymbol{M}^{(e)}=\sum_{k=1}^{3}\rho_k h_k\int_{x=0}^{b}\int_{y=0}^{a}\boldsymbol{N}^T(x,y)\boldsymbol{N}(x,y)\,dydx\\ \boldsymbol{K}_e^{(e)}=\sum_{k=1,3}\int_{z=0}^{h_k}\int_{x=0}^{b}\int_{y=0}^{a}\boldsymbol{B}_k^T(x,y,z)\boldsymbol{C}_k\boldsymbol{B}_k(x,y,z)\,dydxdz,\boldsymbol{K}_2^{(e*)}=\int_{z=0}^{h^{(v)}}\int_{x=0}^{b}\int_{y=0}^{a}\boldsymbol{B}_2^T(x,y,z)\boldsymbol{C}_2^*\boldsymbol{B}_2(x,y,z)\,dydxdz\end{gathered}\tag{1}$$

The matrixes $\boldsymbol{C}_k$ $(k=1{,}3)$ are the isotropic elastic material properties, and $\boldsymbol{C}_2^*=\boldsymbol{C}_v^*$ contains the frequency- and temperature-dependent material properties for the viscoelastic core. Matrix $\boldsymbol{B}(x,y,z)$ is formed by differential operators appearing in the strain-displacement relations for each layer, as detailed in de Lima et al. (2006). Also, b and a designate, respectively, the dimensions of the plate element in directions *x* and *y*, and h_k and ρ_k are the thickness and the mass density of the k-th layer, respectively. From the elementary matrices and neglecting other forms of damping, the elementary equations of motion are given as:

$$\boldsymbol{M}^{(e)}\ddot{\boldsymbol{u}}_{(e)}(t)+\boldsymbol{D}^{(e)}\dot{\boldsymbol{u}}_{(e)}(t)+\left[\boldsymbol{K}_e^{(e)}+\boldsymbol{K}_v^{(e*)}\right]\boldsymbol{u}_{(e)}(t)=\boldsymbol{f}^{(e)}(t)\tag{2}$$

where the stiffness matrices, $\boldsymbol{K}_e^{(e)}=\boldsymbol{K}_1^{(e)}+\boldsymbol{K}_3^{(e)}$ and $\boldsymbol{K}_v^{(e*)}=\boldsymbol{K}_2^{(e*)}$ give, respectively, the contributions of the purely elastic and viscoelastic parts of the damped structure, $\boldsymbol{D}^{(e)}$ is the viscous damping (symmetric, non-negative-definite) and $\boldsymbol{M}^{(e)}=\boldsymbol{M}_1^{(e)}+\boldsymbol{M}_2^{(e)}+\boldsymbol{M}_3^{(e)}$ includes the contribution of the base-plate, the viscoelastic core and the constraining layer to the inertia matrix. $\boldsymbol{u}_{(e)}(t)$ and $\boldsymbol{f}^{(e)}(t)$ are, respectively, the vectors of the generalized displacements and external loads.

The global equations of motion in the frequency domain of a viscoelastic damped system containing N DOFs, for which one of the *moduli* appears factored-out of the viscoelastic stiffness matrix, can be expressed as:

$$\left[\boldsymbol{K}_e+G(\omega,T)\bar{\boldsymbol{K}}_v+i\omega\boldsymbol{D}-\omega^2\boldsymbol{M}\right]\boldsymbol{Q}(\omega,T)=\boldsymbol{F}(\omega)\tag{3}$$

where $\boldsymbol{M}\in R^{N\times N}$ is the mass matrix, $\boldsymbol{K}_e\in R^{N\times N}$ is the stiffness matrix corresponding to the purely elastic parts, and $\bar{\boldsymbol{K}}_v\in R^{N\times N}$ is the frequency and temperature-independent part of the viscoelastic stiffness matrix. $\boldsymbol{Q}(\omega,T)\in R^N$ and $\boldsymbol{F}(\omega)\in R^N$ are, respectively, the vectors of the amplitudes of the harmonic generalized displacements and external loads.

2.1 Introduction of the aerodynamic instability into the viscoelastic system

In this section, the three-layer sandwich plate element presented previously is extended for the flutter analysis of viscoelastic systems in order to investigate the effects of the viscoelastic materials on the flutter of plates in the frequency domain. Thus, it is necessary to compute the work done by the non-conservative forces due to the aerodynamic loadings. Also, it is assumed a supersonic unidirectional airflow along the x axis over a surface of the sandwich plate, as depicted in Figure 2.

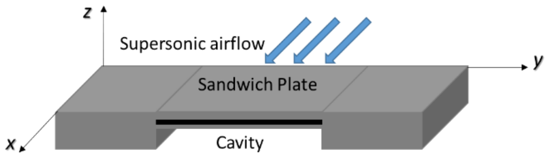


Figure 2 – Three layer sandwich plate under supersonic airflow.

In the present study, the linear form of the piston theory is used. Furthermore, the effects of the aerodynamic damping is negligible in such a way that the quasi-steady aerodynamic model of Ackeret (Almeida, Donadon, & de Almeida, 2012), described by Eq. (4), is retained.

$$p - p_{\infty} = \frac{2\lambda}{M}\left(\frac{\partial \boldsymbol{w}}{\partial \boldsymbol{x}}\right) \tag{4}$$

To introduce the aeroelastic instability, the aerodynamic matrix is calculated by computing the work done by the non-conservative forces:

$$W_a = -\frac{\rho_\infty V_\infty^2}{\sqrt{M_\infty^2 - 1}} \int_S \frac{d\boldsymbol{w}}{d\boldsymbol{x}} \boldsymbol{w}\, dS = -\lambda \boldsymbol{A} \tag{5}$$

where $\boldsymbol{A} = \int_S \frac{d\boldsymbol{w}}{d\boldsymbol{x}} \boldsymbol{w}\, dS \in R^{N\times N}$ is the aerodynamic matrix (real and non-symmetric) and $\lambda = \rho_\infty V_\infty^2 \big/ \sqrt{M_\infty^2 - 1}$ is the aerodynamic factor, having ρ_∞ as the air density, V_∞ as the air speed, and M_∞ designates Mach number. Thus, from Eq. (2) it is possible to incorporate the aerodynamic effects, through the addition of the aerodynamic stiffness matrix. After assuming a harmonic response, the equation of motion is described as:

$$\left[\boldsymbol{K}_e + G(\omega,T)\bar{\boldsymbol{K}}_v + \lambda\boldsymbol{A} + i\omega\boldsymbol{D} - \omega^2\boldsymbol{M}\right]\boldsymbol{Q}(\omega,T) = 0 \tag{6}$$

2.2 Introduction of the GHM viscoelastic model into FE models

According to the studies of McTavish and Hughes (1993), the viscoelastic modulus function is expressed as follows:

$$G(s) = G_0\left(1 + \sum_{i=1}^{N_G} \alpha_i \frac{s^2 + 2\zeta_i\omega_i s}{s^2 + 2\zeta_i\omega_i s + \omega_i^2}\right) \tag{7}$$

where G_0 is called low frequency modulus or static modulus or yet, relaxed modulus, s represents the Laplace variable. The Eq. (7) can be interpreted as a series of N_G mini-oscillators terms represented by three positive parameters $(\alpha_i, \zeta_i, \omega_i)$. The dynamic or higher frequency modulus takes the following form:

$$G_\infty = G_0\left(1 + \sum_{i=1}^{N_G} \alpha_i\right) \tag{8}$$

In order to develop the GHM method it is necessary to apply the Laplace Transformation to Eq. (6) and replace $G(\omega,T)$ by the modulus function given by Eq. (7). After assuming a steady-state harmonic response $\boldsymbol{Q}(t) = \boldsymbol{Q}e^{i\omega t}$ the equation of motion is written.

$$\left[s^2\boldsymbol{M} + s\boldsymbol{D} + \boldsymbol{K}_e + \lambda\boldsymbol{A} + G_0\bar{\boldsymbol{K}}_v\left(1 + \sum_{i=1}^{N_G} \alpha_i \frac{s^2 + 2\zeta_i\omega_i s}{s^2 + 2\zeta_i\omega_i s + \omega_i^2}\right)\right]\boldsymbol{Q} = \boldsymbol{0} \tag{9}$$

A series of dissipation coordinates $\boldsymbol{Q}_i^G\left(i=1,\dots,N_G\right)$ related with the physical coordinates are defined according to the following expression:

$$\boldsymbol{Q}_{\mathrm{i}}^{\mathrm{G}}=\frac{\omega_{\mathrm{i}}^{2}}{\mathrm{s}^{2}+2\zeta_{\mathrm{i}}\omega_{\mathrm{i}}\mathrm{s}+\omega_{\mathrm{i}}^{2}}\boldsymbol{Q} \tag{10}$$

Upon introduction of Eq. (10) into (9), after some mathematical manipulations and back transformation to time domain, the following augmented coupled system of equations is obtained:

$$\boldsymbol{M}_G\ddot{\boldsymbol{q}}_G\left(t\right)+\boldsymbol{D}_G\dot{\boldsymbol{q}}_G\left(t\right)+\boldsymbol{K}_G\boldsymbol{q}_G\left(t\right)=0 \tag{11}$$

where:

$$\boldsymbol{M}_G=\begin{bmatrix}\boldsymbol{M} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \boldsymbol{0} & \frac{\alpha_1}{\omega_1^2}\boldsymbol{K}_v^0 & \boldsymbol{0} & \vdots\\ \vdots & \boldsymbol{0} & \ddots & \boldsymbol{0}\\ \boldsymbol{0} & \cdots & \boldsymbol{0} & \frac{\alpha_{\mathrm{N_G}}}{\omega_{\mathrm{N_G}}^2}\boldsymbol{K}_v^0\end{bmatrix},\boldsymbol{D}_G=\begin{bmatrix}\boldsymbol{D} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \boldsymbol{0} & \frac{2\zeta_1\alpha_1}{\omega_1}\boldsymbol{K}_v^0 & \boldsymbol{0} & \vdots\\ \vdots & 0 & \ddots & \boldsymbol{0}\\ \boldsymbol{0} & \cdots & \boldsymbol{0} & \frac{2\zeta_{\mathrm{N_G}}\alpha_{\mathrm{N_G}}}{\omega_{\mathrm{N_G}}}\boldsymbol{K}_v^0\end{bmatrix},\boldsymbol{K}_G=\begin{bmatrix}\boldsymbol{K}_e+\boldsymbol{K}_v^\infty+\lambda\boldsymbol{A} & -\alpha_1\boldsymbol{K}_v^0 & \cdots & -\alpha_{\mathrm{N_G}}\boldsymbol{K}_v^0\\ -\alpha_{\mathrm{N_G}}\boldsymbol{K}_v^{0T} & \alpha_1\boldsymbol{K}_v^0 & \boldsymbol{0} & \vdots\\ \vdots & \boldsymbol{0} & \ddots & \boldsymbol{0}\\ -\alpha_{\mathrm{N_G}}\boldsymbol{K}_v^{0T} & \cdots & \boldsymbol{0} & \alpha_{\mathrm{N_G}}\boldsymbol{K}_v^0\end{bmatrix},$$

$$\boldsymbol{q}_G=\left[\boldsymbol{q},\boldsymbol{q}_1^{\mathrm{G}},\dots,\boldsymbol{q}_{\mathrm{N_G}}^{\mathrm{G}}\right]^{\mathrm{T}}.$$

The matrix above respects the following rule: $\boldsymbol{M}_G,\boldsymbol{D}_G,\boldsymbol{K}_G\in\mathrm{R}^{\mathrm{T_G x T_G}}$ with $\mathrm{T_G}=\mathrm{N}\left(1+\mathrm{N_G}\right)$. $\boldsymbol{K}_v^0=\mathrm{G}_0\,\bar{\boldsymbol{K}}_v$ is the static low frequency stiffness matrix and $\boldsymbol{K}_v^\infty=\mathrm{G}_\infty\,\bar{\boldsymbol{K}}_v$ is the dynamic or high frequency stiffness matrix.

It is observed from Eq. (11) that the of the dissipative coordinates (internal variables) in the GHM model, leads to an augmented coupled system of equations of motion, in which the total number of d.o.f largely exceeds the initial number of structural degrees of freedom. In order to avoid problems of singularity, an adequate transformation of the augmented system of second-order equations into a state-space first-order form is carried through obtaining the state matrix, $\boldsymbol{A}$:

$$\boldsymbol{A}=\begin{bmatrix}\boldsymbol{Z} & \boldsymbol{I}\\ -\boldsymbol{M}_G^{-1}\boldsymbol{K}_G & -\boldsymbol{M}_G^{-1}\boldsymbol{D}_G\end{bmatrix}$$

where the matrix $\boldsymbol{Z}\in\mathrm{R}^{\mathrm{T_G x T_G}}$ is a matrix composed of zeros and $\boldsymbol{I}\in\mathrm{R}^{\mathrm{T_G x T_G}}$ is an identity matrix, with T_G respecting the same rule as previously.

According to the reference (McTavish & Hughes, 1993), a positive-definite inertia matrix can be obtained for the GHM model by performing the spectral decomposition of the stiffness matrix related to the viscoelastic substructure (denoted by $\bar{\boldsymbol{K}}_v$). The null eigenvalues and the corresponding eigenvectors are eliminated and, as result, fewer dissipative coordinates and a positive-definite viscoelastic matrix are obtained.

2.3 Curve fitting

A curve fitting process is required in order to adjust the exact values of the N_{par}^G parameters (G_0,α_i,ζ_i and ω_i). As shown in Eq. (9), the GHM model makes use of $N_{par}^G=1+3N^G$ parameters, where N^G is the quantity of mini-oscillators involved in the series. For the problem dealt herein, thirteen parameters ($N^G=4$) are required to well fit the curves.

The optimization is carried-out taking as reference four curves of the ISD112 3M® viscoelastic material : 20, 30, 40 and 60°C. To facilitate the visualization of the results the Figure 3 only shows the curves fitted for the temperatures of 30°C and 60°C. The red pointed lines represent the optimized fitted curves and the solid and dashed lines represent the properties obtained from the ISD112 3M®. To facilitate the visualization, only the temperatures of 30 and 60 °C are plotted.

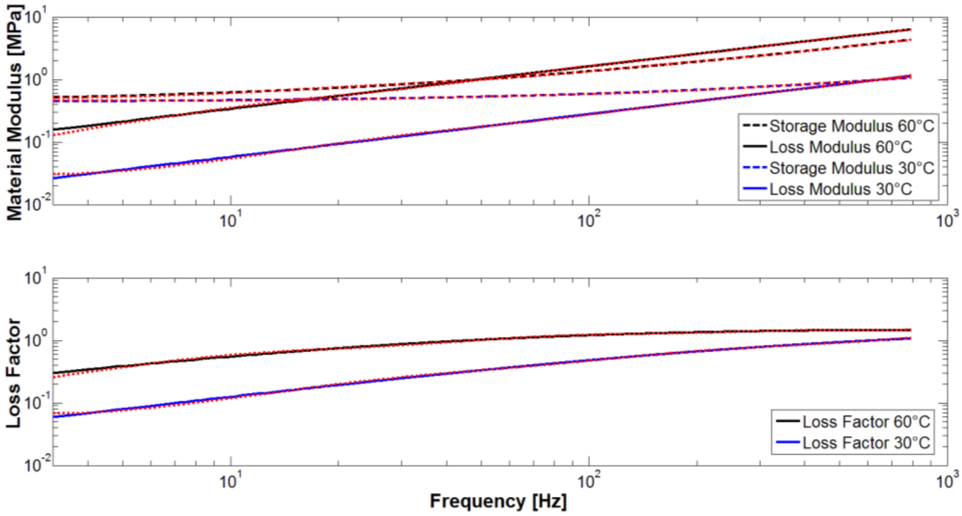


Figure 3 - GHM curve fitting for temperatures of 30°C and 60°C.

The Table 1 presents the values obtained for the optimized parameters of the GHM model.

Table 1: Values of the GHM parameters obtained from the curve fitting optimization

	Temperature of 30°C				Temperature of 60°C			
	Gr=0.63159				Gr=0.28291			
Mini oscillator	1	2	3	4	1	2	3	4
αi	8.7236	22.289	2.0276	298.4	9.9654	1.4118	7.6494	20.942
ζi	0.65348	0.12758	6.6081	3257.1	0.59262	0.65573	10.196	3807.9
ωi [Hz]	2.3441	1.6652	10247	98510	0.38184	6.2515	18686	42542

3. THE K METHOD

In order to determine the critical pressure, which defines the point of flutter occurrence, in the present contribution the *k*-method (Hodges & Pirce, 2002) is implemented by considering the viscoelastic damping. It can be done by introducing a fictitious damping with no physical meaning into the eigenvalue problem as follows:

$$\left[\frac{(1+ig)}{\Omega^2}\boldsymbol{K}_G+\omega_j\boldsymbol{D}_G-\omega_j^2\boldsymbol{M}_G\right]\boldsymbol{Q}_G(\omega,T)=\boldsymbol{0} \tag{12}$$

After transforming the Eq. (12) to the space state form, its solutions obtained are complex eigenvalues in which the complex part represents the structural damping. However, the physical structural damping caused by the viscoelastic material must be separated from the non-physical damping introduced by the k-method. This process can be carried out by solving two eigenvalue problems considering two different stiffness matrix, $\boldsymbol{K}_G$, where the coefficient $\boldsymbol{K}_G(1,1)$ changes in order to consider the aerodynamic influence. To do so, two eigenvalue problems are solved: (a) $\boldsymbol{K}_G(1,1)=\left[\boldsymbol{K}_e+\boldsymbol{K}_v^{\infty}+\lambda\boldsymbol{A}\right]$, resulting in g_1, from Eq. (12); (b) $\boldsymbol{K}_G(1,1)=\left[\boldsymbol{K}_e+\boldsymbol{K}_v^{\infty}\right]$, resulting in g_2, from Eq. (12). The procedure (a) results in a damping coefficient containing both structural and non-physical damping and the process (b) enables to obtain a damping coefficient related to the viscoelastic damping only. Thus, subtracting g_2 from g_1 the result is purely the non-physical damping required to predict the speed of flutter. Flutter will occurs for $g=g_1-g_2=0$.

4. RESULTS AND DISCUSSIONS

To illustrate the main features and capabilities of the modeling procedure, a numerical example composed by a three-layer sandwich plate with a viscoelastic core under supersonic flow is performed. The geometrical dimensions in *x* and y directions are, respectively, 0.39m×0.33m, and the mesh is composed by 6×6 three-layer sandwich plate elements. As mechanical boundary conditions it is considered that all the edges of the plate simply supported, which means $w = 0$.

The Young modulus and Poisson ratio of the base-plate and constraining layer are, respectively, $70GPa$ and 0.34 with the following thicknesses values $1.5^{-3}m$ and $0.5^{-3}m$. The thickness of the viscoelastic layer is $2.54^{-5}m$ and its Poisson ratio is 0.49 with its modulus defined by Eq. (4). The frequencies are computed in a dimensionless form by performing the relation: $\varpi_j = \omega_j a^2 \big/ h\sqrt{E/\rho}$.

The flutter speed is evaluated by observing the first coalescence (1st and 2nd modes of vibration) of the non-dimensional natural frequencies as a function of airflow speed. First, an aluminum plate of dimensions $0.39m$ vs $0.33m$ and $1.5mm$ of thickness is analyzed as reference, obtaining the following value of critical airflow speed: 2062 km/h.

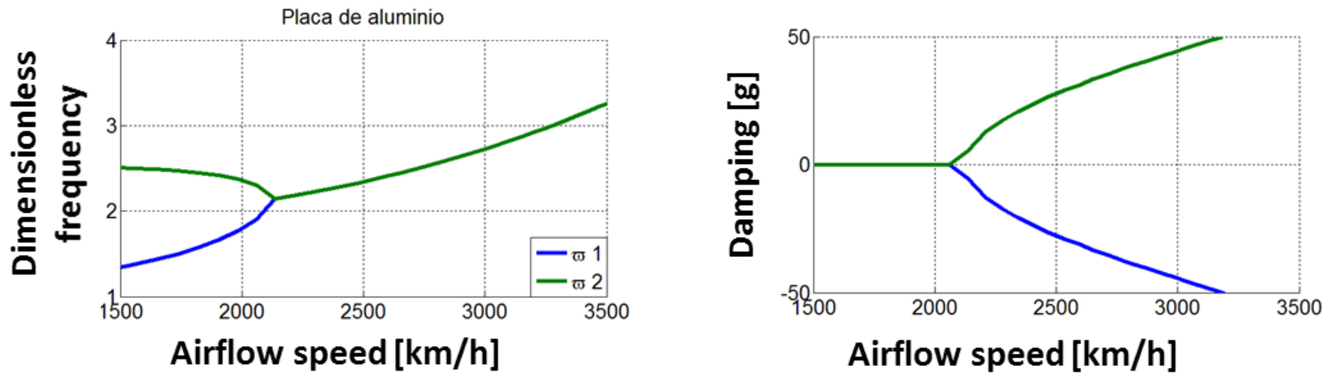


Figure 4 – Coalescence of a simply supported aluminum plate

Finally, the sandwich plate is evaluated taking into account four different values of temperature for the viscoelastic material: $T = 20°C$, $T = 30°C$, $T = 40°C$ and T=60°C.

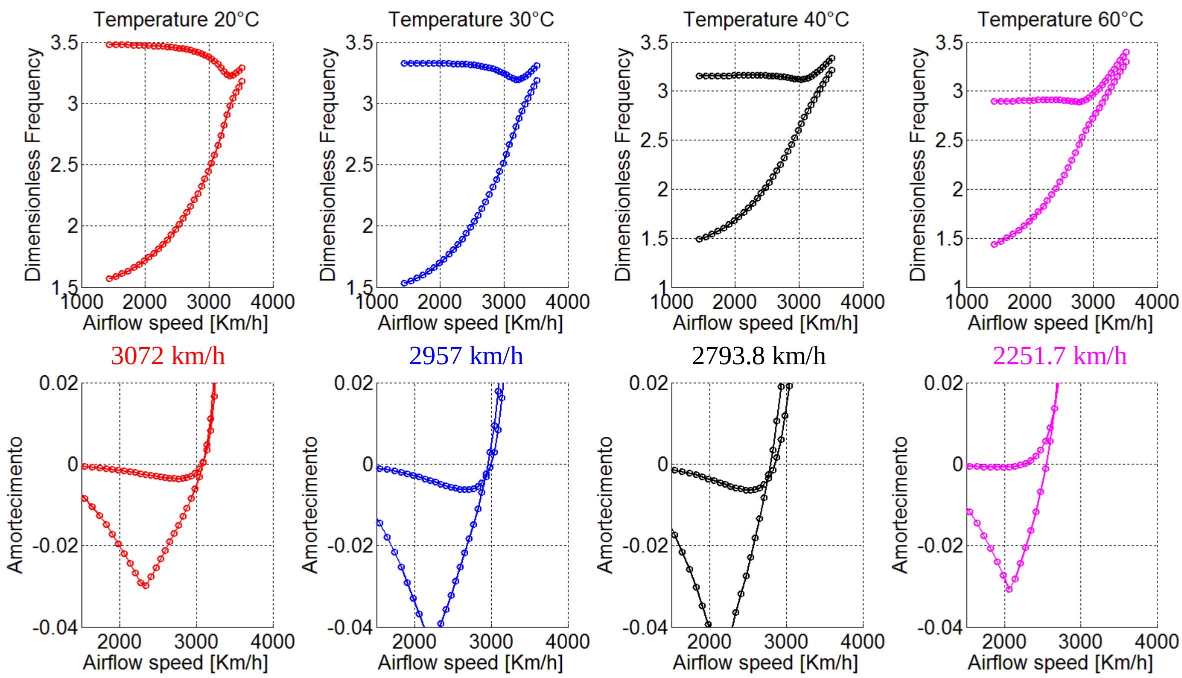


Figure 4 – V-g diagram of the sandwich plate for the following temperatures of viscoelastic material: 20, 30, 40 and 60°C.

The comparison between the base-plate without viscoelastic treatment and the sandwich plate enables to conclude about the advantage in using the surface viscoelastic treatment to reduce the undesirable vibrations due to aeroelastic instabilities and to increase the point in which the flutter occurs. Also, the numerical results obtained show a different behavior between the base-plate where no damping is considered, and the viscoelastically damped system. For this later, it is more difficult to predict the occurrence of the first coalescence due to the fact when studying damped structures, the eigenvalues obtained by the resolution of Eq. (8) are complex for any value of airflow speed given and approach to each other, but do not coalesce. In this case, flutter occurs when an imaginary part of an eigenvalues is equal to zero.

Furthermore, the increasing temperature of the viscoelastic material considerably degrades the dynamic performance of the plate.

In order to remark the dynamic nonlinear behavior of the viscoelastically treated plate the evaluation of the viscoelastic layer thickness is addressed.

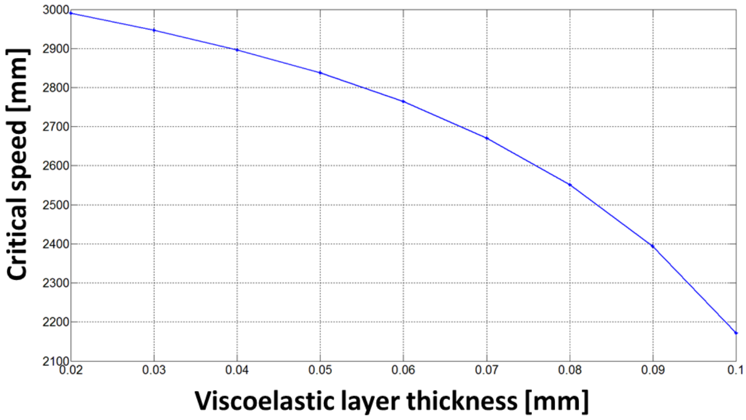
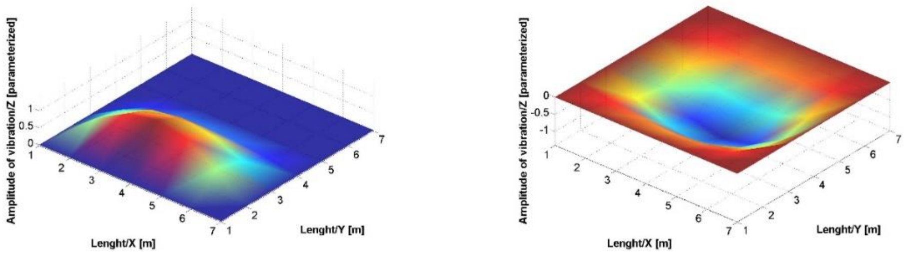


Figure 5 – Dimensionless Critical Pressure vs Temperature of Viscoelastic Material

The mode shapes of vibration were evaluated to verify the dynamic behavior of the plate in the points exactely before and after the occurrence of the flutter phenomenon as depicted in Fig. 7.



(a) Point before flutter (b) Point after flutter

Figure 7 – Representation of the second mode shape of vibration before and after the occurrence of the flutter.

It can be seen that the first mode takes the shape of the second one after the flutter occurrence, indicating the good prediction of the coalescence.

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