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NUMERICAL ANALYSIS OF PERFORATED STEEL PLATES SUBJECTED TO ELASTIC AND ELASTO-PLASTIC BUCKLING BY MEANS THE CONSTRUCTAL DESIGN METHOD

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Abstract. *Steel plates are structural elements widely employed in several mechanical engineering applications. It is well known that if an axial compressive load is imposed to these plates an undesired instability phenomenon can occur: buckling. At a certain load magnitude a limit stress is reached and the plate suffers lateral displacements (out of plane) indicating the buckling occurrence. In plates an elastic buckling or an elasto-plastic buckling can occur, depending on dimensional, constructive or operational aspects. Therefore, in the present work, the Constructal Design method was adopted to investigate the influence of the type and shape of perforations in the plate buckling. To do so, using the ANSYS® software, which is based on the finite element method (FEM), computational models were developed to tackle with the elastic (linear) and elasto-plastic (nonlinear) plate buckling. Square and rectangular thin steel plates, simply supported along its four edges, with a centered perforation, were analyzed, being the objective function to maximize the buckling limit stress, avoiding the plate buckling occurrence. The square and rectangular plates have H/L (ratio between width and length of the plate) of 1.0 and 0.5, respectively. A value of 0.15 for the cutout volume fraction (ratio between the cutout volume and the total plate volume) was considered for different types of cutout: elliptical, rectangular, hexagonal and diamond. The cutout shape variations were promoted by the H_0/L_0 degree of freedom (which relates the characteristic dimensions of the cutout). The results showed that the cutout shape variation has a fundamental influence in the plate buckling behavior, determining if the buckling is elastic or elasto-plastic, allowing to define a buckling stress limit curve for each perforation type. In addition, it was observed that the Constructal Design method conducts to the definition of optimal geometries and dimensions of the cutouts.*

Keywords: *Buckling, Constructal Design, ANSYS®, Finite Element Method, Numerical Simulation.*

1. INTRODUCTION

In several practical applications it is necessary to provide cutouts in plate structures to allow access for services or inspection and even aesthetics purposes, as well as to reduce the structure self-weight. The presence of a hole in a plate panel changes the stress distribution within the member, alters its elastic buckling and post-buckling characteristics and generally reduces its ultimate load carrying capacity. The performance of a plate containing an opening is influenced by the nature of the applied stress (e.g. compressive, tensile, shear, etc.), besides the type, size and location of the hole (Narayanan, 1984).

There are several works dedicated to the study of the elastic (linear) buckling (e.g.: El-Sawy and Nazmy, 2001; El-Sawy and Martini, 2007; Rocha *et al*., 2012; Rocha *et al*., 2013; Isoldi *et al*., 2013) as well as to the analysis of the elasto-plastic buckling (e.g.: Paik *et al*., 2001; El-Sawy *et al*., 2004; Kumar *et al*., 2007; Helbig *et al*., 2013; Helbig *et al*., 2014). Considering the publications above one can note the fundamental importance of the study and understanding of the mechanical behavior of perforated steel plates subjected to buckling in structural engineering, especially if the goal is to improve the performance of these structural elements. Therefore, the main objective of this work is numerically investigate the influence of the hole type and shape in the behavior of buckling perforated steel plates, in order to improve its mechanical behavior. The Constructal Design method is used in order to guarantee an adequate and consistent comparison among the studied cases. The objective function is to maximize the compressive stress, avoiding the plate buckling occurrence. To do so, it was considered a hole volume fraction (ratio between the hole and the volume of the total volume of the plate) of 0.15, for different cutout types: elliptical, rectangular, diamond, longitudinal hexagonal and transversal hexagonal. The shape of these perforations can vary by means the ratio H_0/L_0, which relates the characteristics dimensions of each hole. Besides, two ratios between the plate width (H) and the plate length (L) were studied: H/L = 0.5 and H/L = 1.0, with a plate thickness (t) of 10 mm and keeping constant the total volume of the plate. Besides, in all studied cases, the plate is simply supported along its four edges and has a centered perforation. A constraint which limits a minimal distance of 100 mm from the plate edges to the hole edges is also employed.

2. BUCKLING AND POSTBUCKLING OF PLATES

Buckling is an instability phenomenon that can occur if a slender and thin-walled plate (plane or curved) is subjected to axial compression. At a certain critical load, the plate will suddenly buckle in the out-of-plane transverse direction. However, plate buckling has a post-critical load-carrying capacity that enables for additional loading after elastic buckling has occurred. A plate is in that sense inner statically indeterminate, which makes the collapse of the plate not coming when elastic buckling occurs, but instead later, at a higher loading level reached in the elasto-plastic buckling. This is taken into consideration in the ultimate limit state design of plates because the elastic buckling does not restrict the load carrying capacity to the critical buckling stress, instead the maximum capacity consists of the two parts: the buckling load added to the additional post-critical load (Åkesson, 2007). In other words, when a load level called critical (P_{cr}) is achieved the plate is subjected to the elastic buckling, however the ultimate loading capacity (P_u) of plates is not restricted to the occurrence of elastic buckling.

This capacity to carry additional load after elastic buckling is due to the formation of a membrane that stabilizes the buckle through a transverse tension band. When the central part of the plate buckles, it loses the major part of its stiffness, and then the load is forced to be "linked'' around this weakened zone into the stiffer parts on either side. Additionally, due to this redistribution a transverse membrane in tension is formed and anchored (Åkesson, 2007).

The relative magnitude of the post-buckling strength to the buckling load depends on various parameters such as dimensional properties, boundary conditions, types of loading, and the ratio of buckling stress to yield stress (Yoo and Lee, 2011). There is an analytical solution for the problem of the elastic buckling of a simply supported plate (without perforations) of length L, width H, thickness t, and subjected to a distributed uniaxial compressive load, which is given by (Vinson, 2005):

$$P_{cr} = k\frac{\pi^2 D}{H^2} \tag{1}$$

where P_{cr} is the critical load per unit length, π is the mathematical constant, k is the function of aspect radio H/L, m is the wavelength parameter, defined as:

$$k = \left(m\frac{H}{L} + \frac{1}{m}\frac{L}{H} \right)^2 \tag{2}$$

and D is the plate bending stiffness, determined as:

$$D = \frac{Et^3}{12\left(1 - v^2\right)} \tag{3}$$

being E the Young's Modulus and v the Poisson's ratio of the plate material. The optimum value of m that gives the lowest σ_{cr} depends on the aspect ratio H/L. For a plate with a large aspect ratio, k = 4.0 serves as a good approximation (Yamaguchi, 1999).

The stress at which elastic buckling occurs, σ_{cr}, is defined by the average stress that is equal to the uniformly applied compressive load, P_{cr}, divided by the thickness of the plate, t. This stress is called elastic buckling stress, defined by:

$$\sigma_{cr} = k\frac{\pi^2 E}{12\left(1 - v^2\right)}\left(\frac{t}{H}\right)^2 \tag{4}$$

3. COMPUTATIONAL MODELS

There are many practical engineering problems for which exact solutions cannot be obtained. This inability to obtain an exact solution may be attributed to either the complex nature of governing differential equations or the difficulties that arise from dealing with the boundary and initial conditions. To deal with such problems, one may resort to numerical methods (Moaveni, 1999). In this context, the ANSYS® software, which is based on the Finite Element Method (FEM), was used to solve elastic and elasto-plastic plate buckling problems.

In the present work the 8-Node Structural Shell finite element called SHELL93 was used. This element is particularly well suited to model curved shells. The element has six degrees of freedom at each node: translations in the nodal *x*, *y* and *z* directions and rotations about the nodal *x*, *y* and *z* axes. It was employed in its quadrilateral form and the deformation shapes are quadratic in both in-plane directions. The element has plasticity, stress stiffening, large deflection, and large strain capabilities (Ansys®, 2005).

3.1. Elastic Buckling

Eigenvalue linear buckling analysis is generally used to estimate the critical buckling load of ideal structures (Vinson, 2005). This numerical procedure is used for calculating the theoretical critical buckling load of a linear elastic structure. Since it assumes the structure exhibits linearly elastic behavior, the predicted critical buckling loads are overestimated. So, if the component is expected to exhibit structural instability, the search for the load that causes structural bifurcation is referred to as a critical buckling load analysis. Because the critical buckling load is not known a priori, the finite element equilibrium equations for this type of analysis involve the solution of homogeneous algebraic equations whose lowest eigenvalue corresponds to the critical buckling load, and the associated eigenvector represents the primary buckling mode (Madenci and Guven, 2006).

The strain formulation used in the analysis includes both the linear and nonlinear terms. Thus, the total stiffness matrix, $[K]$, is obtained by summing the conventional stiffness matrix for small deformation, $[K_E]$, with another matrix, $[K_G]$, which is the so-called geometrical stiffness matrix (Przemieniecki, 1985). The matrix $[K_G]$ depends not only on the geometry but also on the initial internal forces (stresses) existing at the start of the loading step, $\{P_0\}$. Therefore the total stiffness matrix of the plate with load level $\{P_0\}$ can be written as:

$$[K]=[K_E]+[K_G]. \tag{5}$$

When the load reaches the level of $\{P\} = \lambda\{P_0\}$, where λ is a scalar, the stiffness matrix can be defined as:

$$[K]=[K_E]+\lambda[K_G]. \tag{6}$$

Now, the governing equilibrium equations for the plate behavior can be written as:

$$[[K_E]+\lambda[K_G]]\{U\}=\lambda\{P_0\}. \tag{7}$$

where $\{U\}$ is the total displacement vector that may therefore be determined from:

$$\{U\}=[[K_E]+\lambda[K_G]]^{-1}\lambda\{P_0\}. \tag{8}$$

At buckling, the plate exhibits a large increase in its displacements with no increase in the load. From the mathematical definition of the inverse matrix as the adjoint matrix divided by the determinant of the coefficients it is possible to note that the displacements $\{U\}$ tend to infinity when:

$$det[[K_E]+\lambda[K_G]]=0\,. \tag{9}$$

Equation (9) represents an eigenvalue problem, which when solved provides the lowest eigenvalue, λ_1 that corresponds to the critical load level $\{P_{cr}\} = \lambda_1\{P_0\}$ at which buckling occurs. In addition, the associated scaled displacement vector $\{U\}$ defines the mode shape at buckling. In the finite element program ANSYS®, the eigenvalue problem is solved by using the Lanczos numerical method (Ansys®, 2005).

3.2. Elasto-Plastic Buckling

Nonlinear or collapse buckling analysis is a more accurate approach since this finite element analysis has capability of analyzing the structures in a more realistic way, i.e., considering its imperfections. Hence this approach is highly recommended for design or evaluation of actual structures (Budiansky, 1968). To do so, the plate material was assumed to be linear elasto-perfectly plastic (with no strain hardening), which is the most critical case for the steel material. An initial imperfect geometry that follows the buckling mode of an elastic eigenvalue pre-analysis is assumed. The adopted maximum value of the imperfection is H/2000 in accordance with El-Sawy *et al.* (2004), being H the plate width.

To find out the plate ultimate load, a reference load given by $P_y = \sigma_y.t$, where σ_y is the material yielding strength, was applied in little increments to the plate edge parallel to the y axis. For each load increment the standard Newton-Raphson method was applied to determine the displacements that correspond to the equilibrium configuration of the plate through the equations:

$$\{P\}_{i+1} = \{P\}_i + \{\Delta P\}, \tag{10}$$

$$\{\psi\} = \{P\}_{i+1} - \{F_{NL}\}, \tag{11}$$

$$[K_t]\{\Delta U\} = \{\psi\}, \tag{12}$$

$$\{U\}_{i+1} = \{U\}_i + \{\Delta U\}, \tag{13}$$

where $[K_t]$ is updated tangent stiffness matrix, $\{\Delta U\}$ is the displacements increment vector necessary to reach the equilibrium configuration, $\{F_{NL}\}$is the nonlinear internal nodal forces vector and $\{\psi\}$ is the out-of-balance load vector. The vectors $\{U\}_i$ and $\{U\}_{i+1}$ correspond to the displacements, while the vectors $\{P\}_i$ and $\{P\}_{i+1}$ correspond to the applied external loads, at two successive equilibrium configurations of the structure.

If at a certain load stage the convergence could not be achieved; that is, a finite displacement increment cannot be determined so that the out-of-balance load vector $\{\psi\}$ is annulled; it means that the failure load of the structure has been reached. This occurs because no matter as large as the displacements and strains can be, the stresses and internal forces cannot increase as it would be required to balance the external loads. The material has reached the exhaustion of its strength capacity.

4. CONSTRUCTAL DESIGN METHOD

It is well known that a major engineering goal is to improve the system configuration aiming to improve its performance. In this sense the Constructal Design method can be used to guide the designer toward flow architectures that have better and better global performance for the specified flow access conditions (fluid flow, heat flow, flow of stresses). So, the Constructal Design method is about the optimal distribution of imperfections. This natural tendency of flowing with better and better configuration is the essence of the Constructal Law (Bejan and Lorente, 2008).

So, in order to apply this philosophy the Constructal Design method needs one or more degrees of freedom (DOF) and constraints to achieve an objective function. To do so, considering the perforated plates of Fig. 1, the DOF H_0/L_0 is free to vary respecting the vertical limit of $H - H_0$ of 200 mm and horizontal limit of $L - L_0$ of 200 mm; while the DOF H/L assumes values of 0.5 (H = 1000 mm and L = 2000 mm) and 1.0 ($H = L$ = 1414.2 mm), keeping the plate volume.

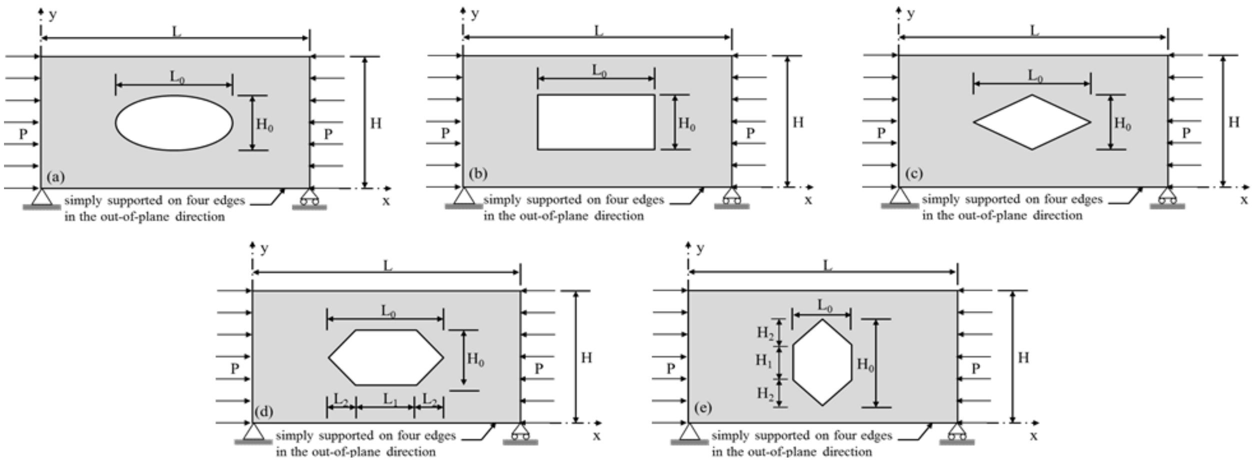

Figure 1. Plate with a centered cutout of the type: (a) elliptical, (b) rectangular, (c) diamond, (d) longitudinal hexagonal, and (e) transversal hexagonal.

To allow an adequate and consistent comparison among the different hole types, a constraint called hole volume fraction, given by the ratio between the hole volume (V_0) and total plate volume (V) (without perforation), is also taken into account with a value of 0.15, being defined for the elliptical, rectangular, diamond, longitudinal hexagonal and transversal hexagonal cutouts, respectively, by:

$$\phi = \frac{V_0}{V} = \frac{(\pi H_0 L_0 t)/4}{HLt} = \frac{\pi H_0 L_0}{4HL} \tag{14}$$

$$\phi = \frac{V_0}{V} = \frac{H_0 L_0 t}{HLt} = \frac{H_0 L_0}{HL} \tag{15}$$

$$\phi = \frac{V_0}{V} = \frac{(H_0 L_0 t)/2}{HLt} = \frac{H_0 L_0}{2HL} \tag{16}$$

$$\phi = \frac{V_0}{V} = \frac{H_0(L_1 + L_2)t}{HLt} = \frac{H_0(L_1 + L_2)}{HL} = \frac{3}{4}\frac{L_0 H_0}{HL} \tag{17}$$

$$\phi = \frac{V_0}{V} = \frac{L_0(H_1 + H_2)t}{HLt} = \frac{L_0(H_1 + H_2)}{HL} = \frac{3}{4}\frac{L_0 H_0}{HL} \tag{18}$$

where π is the mathematical constant; H_0 and L_0 the characteristic dimensions of hole in y and x directions (see Fig. 1), respectively; H, L and t are the width, length and thickness of the plate, respectively.

The objective function is to maximize the buckling limit stress and hence to define the optimal hole geometry (H_0/L_0) for each perforated plate. In order to normalize the obtained results, the critical stress (elastic buckling) and ultimate stress (elasto-plastic buckling), a Normalized Limit Stress (*NLS*) was adopted, being defined by:

$$NLS = \frac{\sigma_{cr}}{\sigma_y} \tag{18}$$

or

$$NLS = \frac{\sigma_u}{\sigma_y} \tag{19}$$

where σ_y is the yielding strength of the plate material, being 250 MPa, for steel A-36, the value adopted in this work.

5. RESULTS AND DISCUSSION

Initially the computational model used to solve the elastic plate buckling problem was verified. To do so, a simply supported steel plate (A-36) with H = 1000 mm, L = 2000 mm and t = 10 mm, without perforations and discretized with a mesh generated with quadrilateral SHELL93 elements having a maximum size of 20 mm. The critical load defined by the analytical solution, Eq. (1), was 759.20 kN/m while the value obtained by the numerical solution was 753.74 kN/m, showing a difference of only -0.72% and thus verifying the computational model.

After that, for the validation of the elasto-plastic buckling computational model the experimental result presented by El-Sawy *et al.* (2001) was used. A simply supported steel plate with H = 1000 mm, L = 1000 mm, t = 20 mm, E = 210 GPa, v = 0.3 and σ_y = 350 MPa having a centered circular hole ($H_0 = L_0$ = 300 mm) were considered. Its discretization was done by quadrilateral SHELL93 elements with maximum size of 20 mm. The experimental result obtained by El Sawy *et al.* (2001) for the rupture stress of the plate was σ_u = 213.50 MPa while the value obtained numerically was σ_u = 217.00 MPa. Therefore, the computational model was validated with an error of 1.61%.

Following the verification and validation of the computational models and considering a fixed value for the hole volume fraction of 0.15, the influence of the type and shape of the cutouts were numerically investigated. To do so, for each hole type the DOF H_0/L_0 was varied. Hence it was possible to determine the *NLS* for elastic (linear) buckling and elasto-plastic (nonlinear) buckling for all studied cases. If for each hole type these results were plotted together, as presented in Fig. 2 for the plate with H/L = 0.5 and a centered elliptical perforation, one can observe the formation of a limit curve that avoids the buckling occurrence. This curve is composed by the left part obtained from the linear analysis and the right part obtained from the nonlinear analysis, usually being the intersection point between linear and nonlinear behaviors the maximum *NLS* value.

It is important to highlight that for lower values of H_0/L_0 (see Fig. 2) there is a wider transversal dimension between the cutout and the longitudinal edges of the plate which facilitates the flow of stresses, do not causing meaningful stress concentrations, occurring the elastic buckling. However, for the higher values of H_0/L_0 this transversal dimension is reduced, making difficult the flow of stress, generating stress concentrations, and occurring the plasticizing of the plate material. So, in these cases, the elasto-plastic buckling behavior is reached before than the elastic buckling behavior. In other words, the plate buckling behavior can be explained by means the flow of stresses in accordance with the Constructal Theory.

Therefore, this analysis procedure was repeated for the other hole types for the plate with H/L = 0.5, as well as for all studied hole types for the plate with H/L = 1.0. These limit curves are showed in Figs. 3(a) and 3(b), respectively.

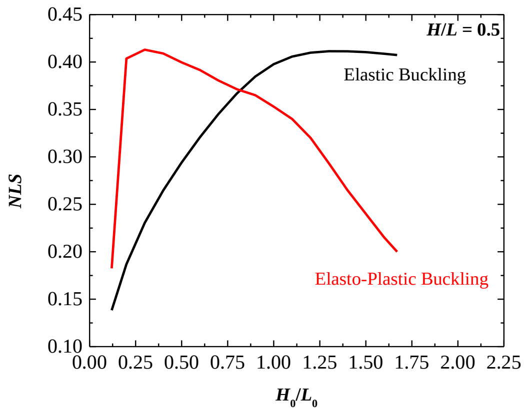

Figure 2. Elastic buckling and elasto-plastic buckling curves of plate with H/L = 0.5 and centered elliptical hole.

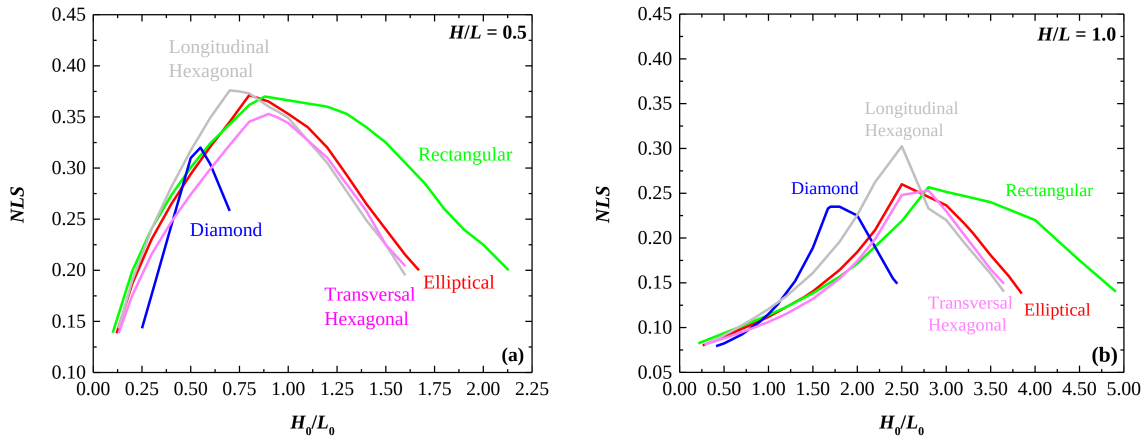

Figure 3. Limit curves to avoid buckling for perforated plates with: (a) H/L = 0.5 and (b) H/L = 1.0.

In Fig. 3(a) and 3(b) it is possible to determine the transition point from elastic to elasto-plastic buckling behavior for each hole type. In addition, these points are also the H_0/L_0 ratios which define the best shape for each hole type, i.e. the H_0/L_0 ratio that allows to reach the maximized *NLS*. Table 1 presents these H_0/L_0 values with its correspondent *NLS*, as well as the worst values for the *NLS* and its correspondent H_0/L_0 value considering the elasto-plastic behavior.

Type	H/L = 0.5					H/L = 1.0				
	$(H_0/L_0)_{opt}$	$(NLS)_{max}$	H_0/L_0	*NLS*	Difference %	$(H_0/L_0)_{opt}$	$(NLS)_{max}$	H_0/L_0	*NLS*	Difference %
Elliptical	0.82	0.37	1.67	0.20	85.00	2.50	0.26	3.85	0.14	85.71
Rectangular	0.88	0.37	2.13	0.20	85.00	2.88	0.26	4.91	0.14	85.71
Diamond	0.51	0.32	0.80	0.20	60.00	1.67	0.23	2.45	0.15	53.33
Longitudinal Hexagonal	0.71	0.38	1.60	0.20	90.00	2.25	0.27	3.65	0.14	92.86
Transversal Hexagonal	0.86	0.36	1.60	0.20	80.00	2.52	0.25	3.65	0.15	66.67

Table 1. Comparison between best and worst shape for each hole type for the plate with H/L = 0.5, and H/L = 1.0.

From Tab. 1 one can note that significant improvements in *NLS* can be obtained by the shape variation of each hole type, being this behavior in accordance with the Constructal Law. Other interesting observation can be made when the von Mises stress distribution is presented in Figs. 4 and 5, for the cases presented in Tab. 1. Figure 4 corresponds to the plates with H/L = 0.5 and Fig. 5 to the plates with H/L = 1.0.

It is possible to note in Figs. 4 and 5 that the optimal cutout shapes promote a better distribution of the maximum stress, i.e., there are more regions submitted to the maximum stress, being this behavior in agreement with the Constructal principle of the optimal distribution of imperfections.

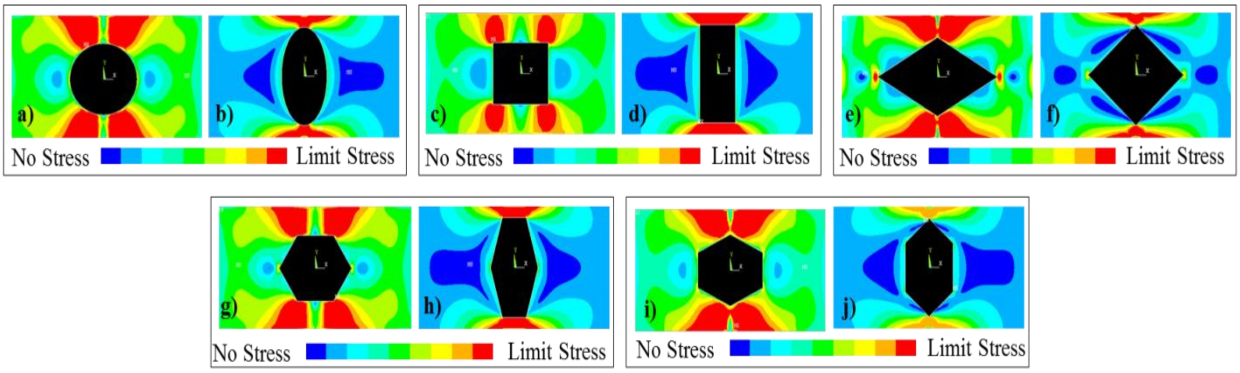

Figure 4. Stress distribution in rectangular perforated plates: (a, c, e, g, i) optimal shape and (b, d, f, h, j) worst shape.

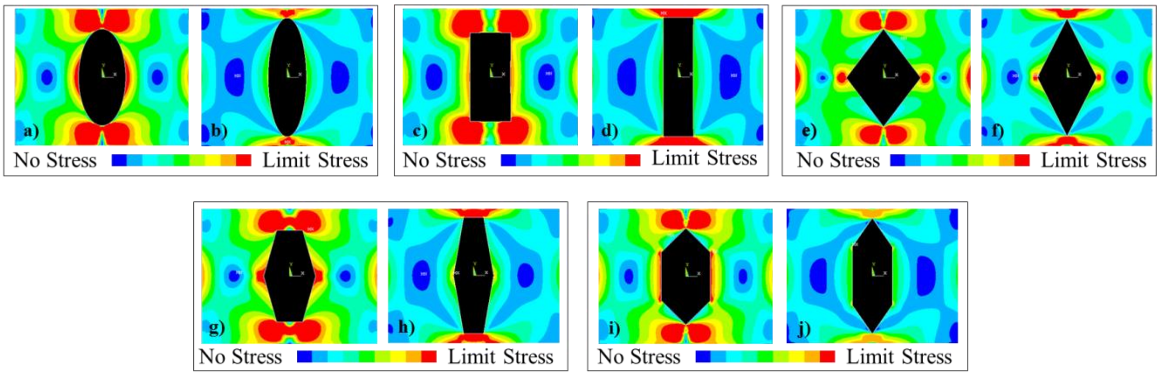

Figure 5. Stress distribution in square perforated plates: (a, c, e, g, i) optimal shape and (b, d, f, h, j) worst shape.

Moreover, if a comparison for the maximized *NLS* among the different hole types is carried out (see Fig. 3 and Tab. 1), it is possible to observe to the plate with H/L = 0.5 that the longitudinal hexagonal cutout is 2.63%, 2.63%, 15.75%, and 5.26% superior than the elliptical, rectangular, diamond, and transversal hexagonal perforations, respectively; while for the H/L = 1.0 the longitudinal hexagonal hole is 3.70%, 3.70%, 14.81%, and 7.41% more efficient than the elliptical, rectangular, diamond, and transversal hexagonal cutouts, respectively.

However, still referring to Fig. 3, it is also observed that in some specific regions of the H_0/L_0 range the longitudinal hexagonal hole do not conducts to the superior performance, despite having reached the highest *NLS* level. For the H/L = 0.5 plates the rectangular perforation presents a better mechanical behavior at the intervals $0.10 \leq H_0/L_0 \leq 0.30$ and $0.86 \leq H_0/L_0 \leq 2.13$. In the case of H/L = 1.0 plates in the interval $1.14 \leq H_0/L_0 \leq 2.00$ the diamond cutout is more efficient and at the interval $2.73 \leq H_0/L_0 \leq 4.91$the rectangular perforation has a superior performance.

6. CONCLUSIONS

The present work shows that the hole type and shape have a fundamental importance for the definition of the buckling behavior in perforated plates. The adequate hole type choice allow to obtain a superior performance, i.e., removing the same material amount of the plate it is possible to improve its performance only by the correct hole type choice. In addition, for all studied hole types, its shape variation promoted by means the DOF H_0/L_0 is responsible to define if the buckling will be elastic (linear) or elasto-plastic (nonlinear). For lower values of H_0/L_0 an elastic buckling occur while an elasto-plastic buckling happens for higher values for the ratio H_0/L_0. Hence, there is an intersection point between the curves that define the elastic and elasto-plastic plate buckling. This point determines the transitions between these mechanical behaviors, also being where the level maximum of the stress is reached among all studied DOF H_0/L_0 for each hole type.

Another important observation is that there is no optimal global geometry. Depending of the H_0/L_0 value, a specific hole type presents the best performance. This is an important aspect if the cutout at the plate need to be done with a pre-defined geometry.

In general it can be said that the plates with H/L = 0.5 and H/L = 1.0 have a similar buckling behavior. However, the H/L = 0.5 perforated plates can support a more elevated stress level when compared with the H/L = 1.0 perforated plates. As the total volume of the plate material is the same in both cases, the recommendation is, if possible, to use plates with $H/L = 0.5$.

Finally, the Constructal Design method proved to be able to analyze the geometric configuration influence at the plate buckling problem, allowing to define for each studied hole type a limit stress curve as function of the hole shape variation which avoids the buckling phenomenon occurrence.

7. ACKNOWLEDGEMENTS

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