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NUMERICAL STUDY OF THE HELMHOLTZ RESONATORS BEHAVIOR USING IMPEDANCE FUNCTIONS

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Abstract. *Helmholtz Resonator (HR) consists of a device dedicated to attenuating the acoustic pressure inside cavities, rooms and other volumes. It is not rare to find this type of device in cases in which absorption material cannot be applied or in cases that the acoustic energy must be attenuated in a specific range of frequencies (narrow band). The last option is suitable to decrease the acoustic pressure in the natural frequencies (resonances) of a cavity. The proposal of the paper is to analyze the effect of HR behavior in the impedance functions of a cavity. Using a cylindrical tube as the reference cavity, a sort of HR was designed and attached (one at a time) to one of its ends. The impedance functions were obtained from harmonic analysis of a Finite Element model, which was developed considering the pressure response due to an excitation at the opposite end. The numerical acoustic impedance functions of each configuration (each HR) were compared to the function of the reference tube alone (without HR). The effect of the HR behavior could be detected not only in resonance peaks (as expected) but in the whole considered frequency range. Even if the HR tuned frequency is located between two resonances (anti-node regions), the effect could be verified clearly. In this way, it was concluded that the HR behavior can be noticed in the tube; even if the HR is tuned to a frequency which pressure responses meet the lowest levels inside it. This conclusion establishes the basis of the next study stage: to construct a HR parametric model using the acoustic impedance functions of the cavity where the HR is installed.*

Keywords: *Helmholtz Resonator, Finite Element Analysis, Acoustic Impedance.*

1. INTRODUCTION

The Helmholtz Resonators (HR) have been used as sound pressure absorbers with reactive and resistive features. The attenuation occurs mostly at low frequencies and the attenuated bandwidth is considered narrow. The HR is suitable in cases which absorption material cannot be applied or in cases that the acoustic energy must be attenuated in a specific range of frequencies. In order to enhance its bandwidth and its sound absorption capacity many researchers have been exploring it. Tang, *et al*., 2012 showed that the coupled resonator can enhance the sound capacity and widen the working bandwidth of the resonators. Similarly, it was studied by Sang and Yang (2005) that combining many resonators can broaden their bandwidth; Tang (2003) also showed that the use of tapered necks could provide significant improvement on the sound absorption capacity of a resonator.

Fahy (2001) stated that the HR is most effective when its resonance frequency is tuned to the fundamental frequency of the tube of interest. Regarding the latest statement, the present study analyzes numerically the behavior of the impedance function of the cavity when a Helmholtz resonator is attached to the cavity, considering both cases: when the HR resonance frequency coincides and when does not coincide with the cavity natural frequency. This reference cavity is a cylindrical tube, which presents a Helmholtz Resonator attached to one of its end and at the other end is applied an excitation.

2. THEORY REVIEW

2.1 Helmholtz Resonator

The Helmholtz resonator is often used to reduce noise in a narrow frequency band (Sang and Yang, 2005). It uses passive technique, which attenuates sound by reactive and resistive principles (Guimarães, 2013). The resonant frequency is determined by its geometric dimensions (neck and cavity).

The considered arrangement of a HR consists of a small cavity attached to a tube (at which the sound will be attenuated) by a neck, as Fig. 1. The HR dimensions must be smaller than the minimum wavelength of interest in order to consider it as an element with lumped parameters connected to a geometric discontinuity. The condition of attachment is that the oscillatory volume flow in the neck is equal to that imposed on the fluid in the cavity, thus the air elastic property of the neck fluid is neglected, being treated as an incompressible volume (Fahy, 2001).

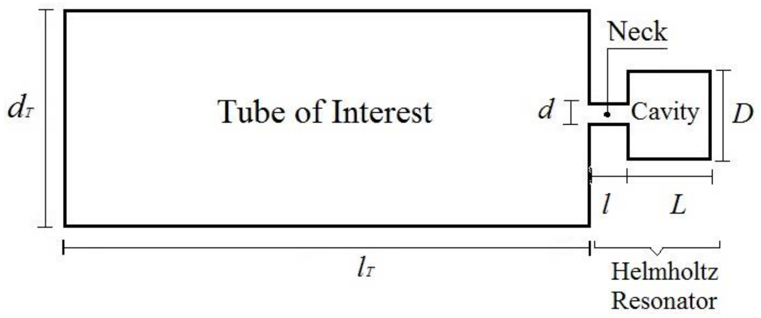


Figure 1. Illustration of a Helmholtz resonator attached to a tube

Gerges (2000) considers the HR as an acoustic system of one degree of freedom that is separated into three distinct elements: (1) Mass element: the fluid in the neck is incompressible working as the system mass; (2) Stiffness element: the air in the cavity is compressible storing potential energy; (3) Resistance element: the system resistance is responsible for the acoustic energy dissipation, which is represented by two factors: the neck opening radiates sound that introduces the radiation resistance and the flow inside the neck produces a viscous resistance.

It needs to consider an effective neck length (l_{eff}), which includes a correction factor (δ) for mass-loading due to air entrainment, near the neck extremities (de Bedout *et al*, 1997):

$$l_{eff} = l + \delta \ . \tag{1}$$

For this study, the area ratio ($RA = A_p/A_c$) is less than 0.16, where A_c is the cavity area of HR and A_p is the neck area. Hence the correction is (Santana Jr., 2008):

$$\delta = 0.85d(1 - 0{,}7\sqrt{RA}\ ) \ , \tag{2}$$

where d is the neck diameter.

2.2 Acoustic Impedance

The conception of acoustic impedance (Z) (Fahy, 2001) is related to how coupled regions of fluid or the contact of fluid and solid system interacts between them, indicating their acoustic properties. From this point of view, it is determined quantitatively the reflection and transmission of the sound wave between two fluids and also the sound absorption, reflection and transmission occurred in the fluid and solid system.

The acoustic impedance of a tube, with a source at one end inducing a velocity inside it, can be obtained at a central point on the surface (the same one it is applied the excitation). Fahy (2001) expressed it as Eq. (3):

$$Z_{x=0} = \left(\frac{\rho c}{S}\right) j \cdot \cot K l_T \ , \tag{3}$$

K is wave number represented for $K = \omega/c$, S is the area of the reference tube in m^2, c is the sound velocity and l_T is the length of tube of interest.

The formulation for acoustic impedance of HR depends on each application, but it is basically expressed by the ratio of complex amplitude of harmonic pressure (p) to the associated volume velocity (Q). Thus the acoustic impedance can be expressed in $Pa/m^3 \cdot s$ by the Eq. (4) (Fahy, 2001):

$$Z = \frac{p}{Q} = \frac{p}{v.S} \ , \tag{4}$$

where v is the particle velocity and S the surface area associated with the velocity. In the same manner the acoustic impedance is conventionally denoted by a complex quantity (Fahy, 2001):

$$Z = R + jX \ , \tag{5}$$

where R (real part) is the acoustic resistance and X (imaginary part) is the acoustic reactance. R is related to the dissipation of energy, as above mentioned, it is represented by the sum of two resistances, the viscous resistance (R_v) and the radiation resistance (R_r) (Santana Jr., 2008):

$$R = R_v + R_r \ , \tag{6}$$

where the viscous resistance (R_v) and the radiation resistance (R_r) are:

$$R_v = \frac{8\pi\mu l}{A_p} \text{ and } R_r = \frac{\rho\omega^2}{A_p} \ , \tag{7}$$

wherein μ is the kinetic viscosity, l is the neck length, ρ is the air density, ω is the frequency in rad/s. The acoustic reactance X is resulted of the association of the effective mass and stiffness of the acoustic system (Kinsler, *et al.*, 1962):

$$X = \omega M - \frac{1}{\omega C} \ , \tag{8}$$

where M is the acoustic inertance, which Kinsler, *el al.*, 1962 associated with the kinetic energy stocked in the resonator's neck, and C is the acoustic compliance, which can be defined as the volume displacement that is produced by the application of unit pressure. Accordingly, they can be expressed as:

$$M = \frac{\rho l_{eff}}{A_p} \text{ and } C = \frac{V_c}{\rho c^2} \ , \tag{9}$$

V_c is the cavity of the HR. Substituting Eq. (7) into Eq. (6), Eq. (9) into Eq. (8), and finally the Eq. (8) and Eq. (6) into the Eq. (5), it obtains:

$$Z = \frac{8\pi\mu l}{A_p} + \frac{\rho\omega^2}{A_p} + j\left(\omega\frac{\rho l_{eff}}{A_p} - \frac{1}{\omega\frac{V_c}{\rho c^2}}\right) . \tag{10}$$

A Helmholtz resonator impedance is purely resistive (i.e. the total reactance is zero) at the resonance frequency (Fahy, 2001):

$$0 = \omega\left(\frac{\rho l_{eff}}{A_p}\right) - \frac{1}{\omega}\left(\frac{V_c}{\rho c^2}\right)^{-1} \rightarrow \omega_{RH} = c\sqrt{\frac{A_p}{l_{eff}V_c}} \ . \tag{11}$$

From the Eq. (11) it is clear to see that the natural frequency depends directly on HR dimensions.

2.3 Considerations about ASTM E1050-12

The standard test method ASTM E1050-12 is studied to understand exactly the way that the acoustic impedance is obtained, and to take the standard considerations made in order to dimension the reference cavity and the fundamental parameters that are used during the simulation.

This standard method uses a cylindrical tube. At one end is mounted the specimen and a sound source is placed at another end, the sound source is broad band signal and inside the tube there is plane waves.

The tube diameter (d_T) is limited by the upper working frequency (f_u) in order to maintain plane wave propagation inside it, so d_T can be specified by:

$$d_T < \frac{Kc}{f_u} \ , \tag{12}$$

where K is a constant equal to 0.586.

The plane wave should reach the pressure point, i.e. the specimen, fully developed, so the length from the source until the specimen must have a minimum of three tube diameters.

The environment parameters interfere directly in the results, thus it is required to assume a value for the temperature (T) and pressure (P). The speed of sound (c) and air density (ρ) are dependent of those parameters as show in Eq. (13) and Eq. (14):

$$c = 20.047\sqrt{273.15 + T} \ , \tag{13}$$

$$\rho = 1.290\left(\frac{P}{101.325}\right)\left(\frac{273.15}{273.15+T}\right) . \tag{14}$$

2.4 Impedance function in FEM

The Finite element method is the mainly used numerical prediction technique for solving engineering problems. FEM splits complex geometries into smaller portions (finite elements); these small parts are governed by boundary conditions and a set of partial differential equations (Desmet and Vandepitte, 2012).

As it was previously illustrated the acoustic system can be described like a mechanical system. In the same way, the finite element model will contain an acoustic stiffness matrix $[K_a]$, an acoustic mass matrix $[M_a]$, an acoustic damping matrix $[C_a]$ and an acoustic excitation vectors $\{F_{ai}\}$. Desmet and Vandepitte (2012) showed a solution for the unknown nodal pressure values $\{p_i\}$, which consists of combination of these components

$$([K_a] + j\omega[C_a] - \omega^2[M_a]) \cdot \{p_i\} = \{F_{ai}\} . \quad (15)$$

The acoustic force vector $\{F_{ai}\}$ can be represented for the sum of the input pressure vector $\{P_i\}$, input velocity vector $\{V_{ni}\}$ and the acoustic source vector at node i $\{Q_i\}$. Desmet and Vandepitte (2012) illustrated:

$$([K_a] + j\omega[C_a] - \omega^2[M_a]) \cdot \{p_i\} = \{Q_i\} + \{V_{ni}\} + \{P_i\} . \quad (16)$$

But for the boundary condition of the normal impedance and a sound source applied on the system (the input pressure vector and the input velocity vector are neglected), thus the frequency response to harmonic excitation at steady-state is (Guimarães, 2013):

$$([K_a] + j\omega[C_a] - \omega^2[M_a]) \cdot \{p(\omega)\} = j\rho\omega\{Q_i(\omega)\} . \quad (17)$$

In this manner, from an excitation, with an volume velocity $Q(\omega)$, at a (or some) node(s), the Eq. (17) allows to find the frequency response of the pressure for each frequency at all nodes of the FEM model. Considering $Q = 1m^3/s$, the impedance function (Z) can be achieved by the pressure frequency response itself:

$$\{Z(\omega)\} = \{p(\omega)\}_{Q=1} = j\rho\omega\big(([K_a] + j\omega[C_a] - \omega^2[M_a])\big)^{-1} . \quad (18)$$

3. METHODOLOGY

The research considered some applications in the engineering area at which the HR has been used lately, in favor of determining its working range of frequency and the volume of the reference cavity as well.

The frequency range of attenuation encompasses a large range, for example, in cases of rocket combustion chamber the frequency range is 200 Hz to 8000 Hz (Guimarães *et al*, 2012). Or considering an automotive cavity the frequency range analyzed is between 20 Hz to 1000 Hz (Wyckaert *et al*, 1996). The working frequency range considered in this research is from 0 Hz to 600Hz, due to the repetition of the amplitudes of the natural frequencies after 600Hz, in such tube of simplified form.

The volume of interest is based on the ASTM E1050-12 standards. Hence, adopting the Eq. (12) and the upper frequency of 600 Hz, the tube's diameter must be less than 0.33 m, the study opted for a security diameter equal to 0.15 m, as well as the tube length that must be a minimum of three times tube's diameter longer. Both dimensions are specified on Tab. 1.

Table 1. Dimensions of the tube of interest

Parameter	
d_T (m)	0.15
l_T (m)	1.0

In order to determine the HRs dimensions, the first step that needs to be done is to find the HRs fundamental frequencies.

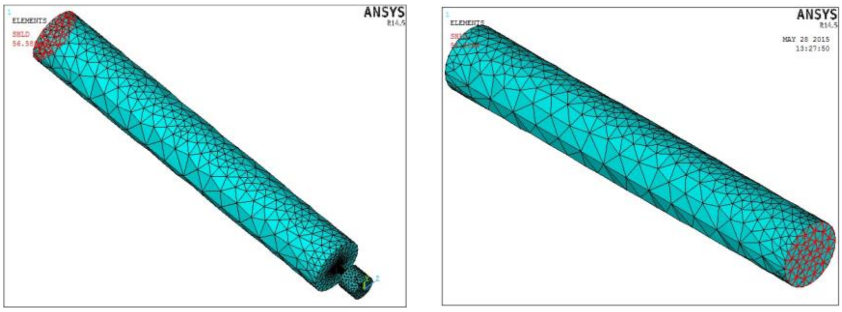
To determine the HRs dimensions it was used a routine which was considered RA<0.16 and the HR natural frequency was obtained from Eq. (11). Hence, it was conducted several attempts until the proper configuration to be reached. Table 2 describes the essential attributes achieved.

Table 2. Helmholtz Resonator attributes

	170-A	170-B	250-A	250-B
d(m)	0.005	0.016	0.005	0.016
D(m)	0.02	0.040	0.02	0.040
l (m)	0.045	0.065	0.045	0.065
L(m)	0.05315	0.12174	0.0232	0.055
Natural Frequency (Hz)	170.01	170.00	257.33	252.93
RA	0.01235	0.06059	0.01235	0.06059

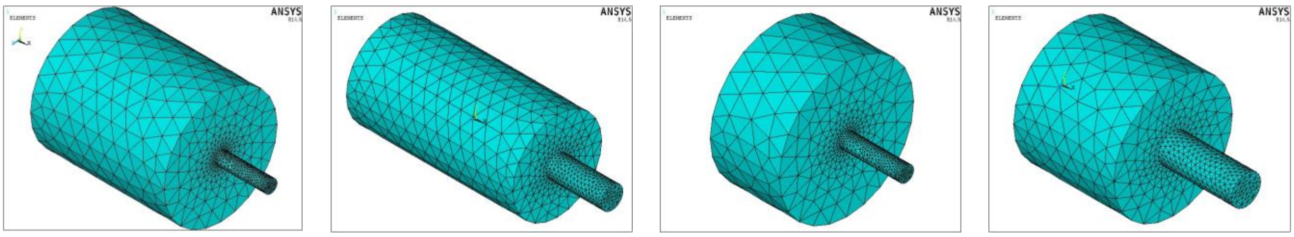
The numerical model was developed in ANSYS. The Helmholtz resonator is attached to the tube of interest and the sound source is designed as a velocity applied on a tube surface. The system configuration with the velocity highlighted with red arrows is shown in Fig. 2. The finite element models consider the cavities' surfaces as rigid walls but the surface that represents an acoustic source. The acoustic fluid type used in ANSYS is the FLUID30, with tetrahedral-shaped elements.

The size of the biggest element dimension in the mesh is 0.06667 m. Assuming 7 elements per wavelength, it is correspondent to a wavelength of 0.46667 m, considering it the higher frequency analyzed accurately is 736 Hz. The FEM models of the reference cavity alone and with a HR attached are illustrated in Fig. 2. The models of four resonators are illustrated in Fig. 3.



(a) (b)

Figure 2. a) FEM model of a typical HR attached to the tube; b) FEM model of the reference cavity without HR



(a) (b) (c) (d)

Figure 3. a) HR 170-A; b) HR 170-B; c) HR 250-A; d) HR 250-B

The meshes are generated as free mesh, thus their size and number of nodes and elements are within a range of values. In all simulations the reference tube is designed with the same dimensions, hence its mesh parameters are equal. Summarizing, the average number of nodes of the tube (without HR) is 2300 nodes, the number of elements is 10000 elements and, as aforesaid, the size of an element is 0.06667 m. The Table 3 clarifies the meshes data of the four Helmholtz Resonators.

The fluid inside the cavities is assumed as air at 20°C and at a barometric pressure of 101.3KPa, thus the sound velocity, using the Eq. (13), is equal to 343.28 m/s and the air density is 1.2017kg/m^3, adopting Eq. (14). Besides at the neck's cavity was considered a kinetic viscosity (μ) equal to 1.983e^{-5} $kg/m.s$ in order to represent the viscous resistance in the neck walls.

From Eq. (4) the volume velocity is obtained as $Q = v.S$. Assuming the sound source with unitary volume velocity, i.e. $Q = 1\,m^3/s$, and the surface area equal $0.01767\,m$, the complex velocity considered on the surface (v) is $40.014 + i40.014\,m/s$.

Table 3. Meshes data

	Parameters	Mesh of Helmholtz Resonator	
		Neck	**Cavity**
HR 170-A	Number of elements	6883	6432
	Number of nodes	1480	6884
	Element medium size (m)	0.00133	0.0059
HR 170-B	Number of elements	4962	7583
	Number of nodes	1091	1570
	Element medium size (m)	0.00267	0.00869
HR 250-A	Number of elements	6944	2851
	Number of nodes	1493	682
	Element medium size (m)	0.00133	0.0058
HR 250-B	Number of elements	4784	3944
	Number of nodes	1062	874
	Element medium size (m)	0.00267	0.00786

Before considering all numerical results it was performed a validation of the FEM model. An excitation surface velocity was applied in one end of the tube. The impedance function was taken at a central point placed on the same surface of the excitation. The validation consists of the comparison between the analytical impedance function, Eq. (3), and the response obtained in ANSYS, after the modal and harmonic analysis.

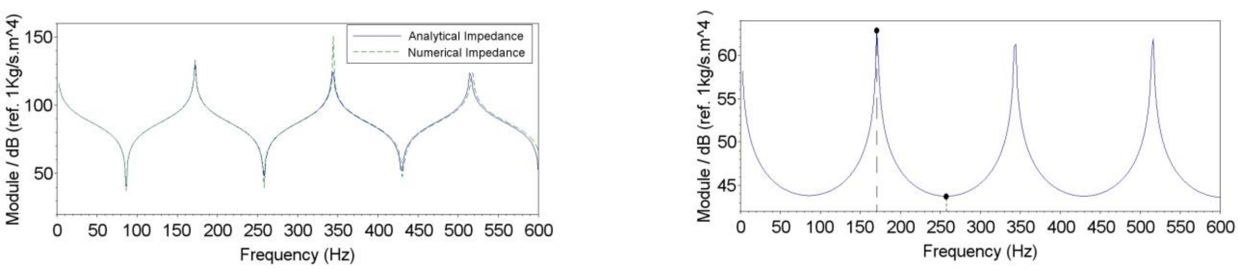
To obtain the natural frequencies and the modes of the models was performed a modal analysis. Desmet and Vandepitte (2012) explained the rule of thumb, which says that an accurate prediction of the steady-state dynamic behavior in a certain frequency range is obtained by using all modes with natural frequencies, smaller than twice the upper frequency limit of the considered frequency range. The frequency range of consideration is limited to 600Hz, so considering a security range, the modal analysis range is from 0 to 2000 Hz.

The harmonic analysis has been run to get the pressure frequency response of a central point at the opposite end where the harmonic excitation was applied. It was set substeps of 2Hz from 0 to 600 Hz, with stepped analysis, which makes sure the same velocity value for all substeps.

4. RESULTS AND DISCUSSION

The validation of the FEM model is illustrated in the Fig. 4 (a), through a point impedance function (excitation and response at the same point. It is clear to see that the numerical model is working as the analytical model expects, perceiving high agreement level.

The natural frequency of the Helmholtz resonators is established under considerations of the harmonic response of the reference cavity being excited by unitary volume velocity source. It was simulated a harmonic analysis on the tube alone (without HR attached to it), and the transfer impedance function (excitation at one end and response at the other end) is being represented on Fig. 4 (b). In order to achieve the objectives of the study, the fundamental frequency of the HRs must match two values: the first natural frequency of the tube (170 Hz) and a value placed between two resonances peaks of the tube's frequency response (250 Hz).



(a) (b)

Figure 4. a) Validation of the numerical model of the tube without HR; b) Harmonic response of the tube without HR

It is possible to visualize the HR influence on the impedance functions of the reference cavity, in Fig. 5 and Fig. 6. Four simulations were performed with the HRs (one at a time) attached to the tube of interest. The transfer impedance functions were obtained, considering the response point at the connection of the HR with the tube (the opposite end to the excitation source).

The first comparison, illustrated in the Fig. 5, shows the impedance functions of the tube without HR and the effect of the two resonators, both tuned to 170 Hz (the volume of each other is different). It could be noted the resonators performance in the impedance function of the tube. Considering the smaller HR (170-A), two resonance peaks became present around the original tube resonant frequency for the first mode. This behavior was expected, considering the HR as an additional one-degree-of-freedom system attached to the original tube. The larger HR (170-B) changes the impedance function behavior of the tube enhancing the effect of the last HR, due to larger volume, i.e. larger mass. In this case the new resonance peaks present values more distant from the first natural frequency of the original tube.

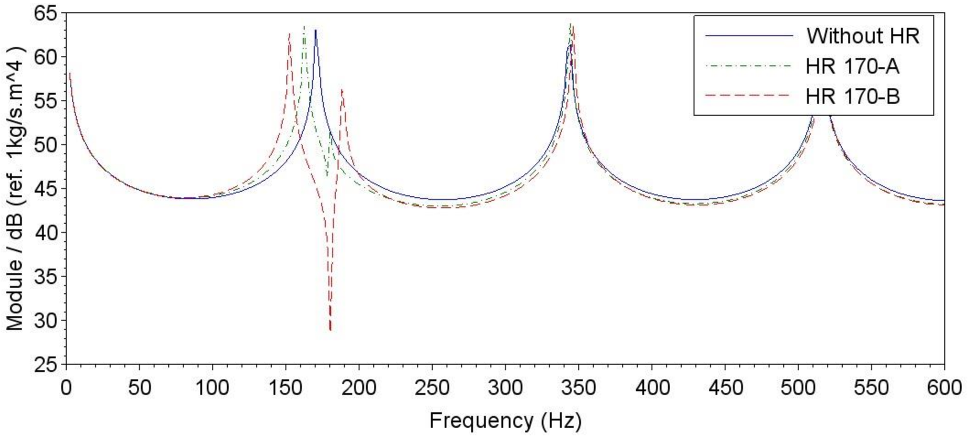


Figure 5. Impedance Function of the HR 170-A, 170-B and the tube without HR

The comparison of the Helmholtz resonators tuned to 250 Hz attached to the tube and the response of the reference tube being excited without HR is represented on Fig. 6. It can be noted that one of the main proposal is verified and ensures its issue. The influences of both 250 Hz resonators are confirmed varying the impedance function of the original tube function, even in the lowest levels where none of the resonance peaks are present. The difference between both HR could also be verified and can be associated to the HR volume. Some peaks in the 250-A HR were overestimated. It could be related to the very low levels of dissipative elements present in the model.

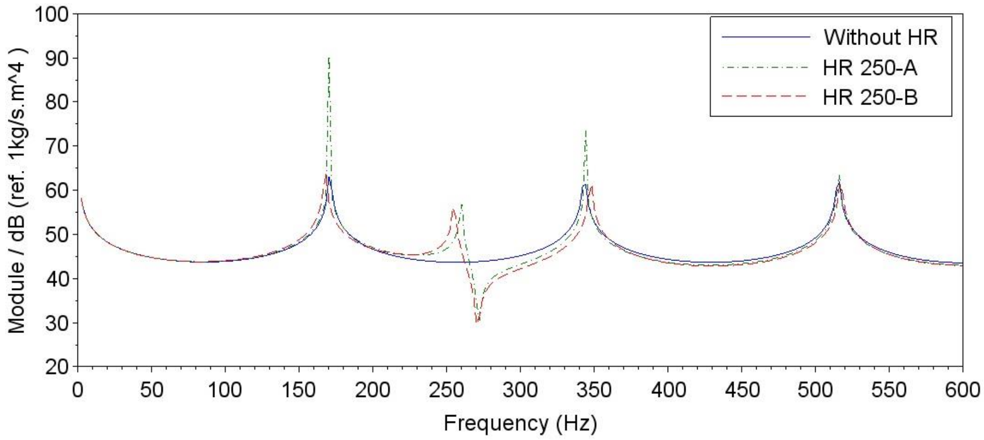


Figure 6. Impedance Function of the HR 250-A, 250-B and the tube without HR

5. CONCLUSIONS

The model which is built in the Finite Element Method proved, via validation with analytical method, to be appropriated to characterize the influence on the impedance function of the reference cavity with a Helmholtz resonator attached to it.

The complex velocity with unitary module used as excitation on the tube's surface made the challenge resolution easier, once the frequency response of the pressure is already the impedance function itself. It was possible to identify the acoustic behavior of the tube using the acoustic impedance function.

The dimensions of the HRs looked to be adequate with the references. Considering a HR tuned to the same frequency of the tube resonance, the attenuation of the impedance function is significant.

For this paper conclusion it is verified a representative and expected attenuation on the impedance function of the tube in the regions where the Helmholtz resonator is tuned, and ensuring the main attempt of this study: a Helmholtz resonator tuned to the lowest levels of the impedance function (not coinciding with a resonance) of the reference tube has a direct effect on the tube response.

Such results encourage the next step of this research, which is to construct a HR parametric model from the acoustic impedance functions of the cavity where the HR is connected to.

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