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MODEL-FREE CONTROL OF MAGNETIC LEVITATION SYSTEMS THROUGH ALGEBRAIC DERIVATIVE ESTIMATION

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Abstract. *The recent model-free control approach via algebraic derivative estimation is applied to a magnetic levitation system. Instead of relying on adaptation laws or neural networks, this novel technique takes advantage of the fast algebraic parameter estimation combined with a ultra local generic model, avoiding using complex non-linear models in tracking control. This approach allows to reduce non-linearities, uncertainties and perturbations to a single parameter, which is constantly identified on real time through algebraic techniques. Since the model-free approach doesn't specify the estimation method for velocity and acceleration, the algebraic derivative method is used to provide a fast, non-asymptotic state estimation in closed loop control. Experimental results were obtained in a laboratory prototype showing excellent tracking performance of model-free control combined with the algebraic derivative method. Comparisons are made between the proposed approaches and a model-based feedback linearization control law.*

Keywords: *model-free control, nonlinear control, derivative estimation, magnetic levitation...*

1. INTRODUCTION

The control of magnetically suspended objects has received a great attention in the recent years, mainly due to advances on the magnetic bearing technology and the development of Maglev trains. Magnetic Levitation can provide a frictionless operation in a large range of systems, avoiding mechanical wear and regular maintenance.

In the classical approach, a PID control is usually projected for a fixed levitation gap, taking account the linearized model of the system dynamics. The model free approach, first presented in Fliess and Join (2008) and Fliess and Join (2009), rely on a ultra local generic model, which is updated in real-time using the algebraic parameter estimation techniques of Fliess and Sira-Ramirez (2004). These techniques can be also used for derivative estimation as in Mboup *et al.* (2007).

In this paper, the basic concepts of model-free control will be presented in a way that encourage the implementation by the practitioner, showing the practical details of the digital implementation. A more complete theory including several applications can be found in Fliess and Join (2013).

2. ULTRA-LOCAL MODEL

The main idea of model-free control is the substitution of the SISO dynamic system model by a generic ultra local model:

$$y^{(\nu)} = \phi + \alpha u \tag{1}$$

- $y^{(\nu)}$ is the ν-order derivative of y. ν is a positive integer selected by the practitioner. In the known examples, ν may be chosen to be 1, or seldom 2, to provide a good tracking performance.
- ϕ represents the unknown parts of the plant, including possible disturbances. These effects are condensed in this single parameter, which is identified in real time through algebraic techniques due to Fliess and Sira-Ramirez (2004). Another possible approach is the determination of ϕ through algebraic derivative estimation (Mboup *et al.* (2007)).
- $\alpha \in \mathbb{R}$ is a constant parameter chosen by the practitioner such that αu and $y^{(\nu)}$ are of the same magnitude. This choice is obtained by trials and errors until a good closed loop performance is achieved.
- The ultra-local model may be seen as an approximate dynamics valid in a short period of time. The online identification of ϕ allows a real time update of this model, which is used for control synthesis. This approach aims the simplification of nonlinear control by abandoning the need of a global and complex nonlinear model.

3. CONTROL LAW

Assume that $\nu = 2$ in Eq. 1:

$$\ddot{y} = \phi + \alpha u \tag{2}$$

Closing the loop with a linearizing control including a Proportional-Integral-Derivative action:

$$u = \frac{-\phi + \ddot{y}_d - K_I \int e - K_P e - K_D \dot{e}}{\alpha} \tag{3}$$

y_d is the desired trajectory, $e = y - y_d$ is the tracking error and K_I,K_P,K_D are the usual PID tuning gains. The resulting closed-loop dynamics will be:

$$\ddot{e} + K_I \int e + K_P e + K_D \dot{e} = 0 \tag{4}$$

3.1 PD Tuning

Choosing $K_I = 0$, $K_P = \lambda^2$ and $K_D = 2\lambda$, $\lambda \in \mathbb{R}^+$ results in a stable closed loop dynamics with two real negative poles equal to $-\lambda$:

$$\ddot{e} + \lambda^2 e + 2\lambda \dot{e} = 0 \tag{5}$$

3.2 PID Tuning

Choosing $K_I = \lambda^3$, $K_P = 3\lambda^2$ and $K_D = 3\lambda$, $\lambda \in \mathbb{R}^+$ results in a stable closed loop dynamics with three real negative poles equal to $-\lambda$:

$$\ddot{e} + \lambda^3 \int e + 3\lambda^2 e + 3\lambda \dot{e} = 0 \tag{6}$$

4. ONLINE IDENTIFICATION OF THE ULTRA-LOCAL MODEL PARAMETER

4.1 Identification through Algebraic Estimation (ALG)

Applying the Laplace transform on the ultra-local model of Eq. 2, assuming that ϕ is constant in a short period of time:

$$s^2 Y(s) - s y(0) - \dot{y}(0) = \frac{\phi}{s} + \alpha U(s) \tag{7}$$

Differentiating with respect to s in order to eliminate $\dot{y}(0)$:

$$2Y(s) + s^2 \frac{d}{ds} Y(s) - y(0) = -1 s^{-2} \phi + \alpha \frac{d}{ds} U(s) \tag{8}$$

Differentiating one more time with respect to s in order to eliminate $y(0)$:

$$2Y(s) + 4s \frac{d}{ds} Y(s) + s^2 \frac{d^2}{ds^2} Y(s) = 2s^{-3} \phi + \alpha \frac{d^2}{ds^2} U(s) \tag{9}$$

Multiplying by $\frac{1}{s^3}$ for the elimination of time derivatives and filtering, letting every factor be integrated at least one time[1]:

$$2\frac{1}{s^3} Y(s) + 4\frac{1}{s^2} \frac{d}{ds} Y(s) + \frac{1}{s} \frac{d^2}{ds^2} Y(s) = 2\phi \frac{1}{s^6} + \alpha \frac{1}{s^3} \frac{d^2}{ds^2} U(s) \tag{10}$$

[1] In this context, the iterated integrals are low-pass filters which attenuate noises.

The terms containing the factor $\frac{1}{s^\alpha}$ are transformed to time domain using the relation:

$$\frac{c}{s^\alpha}\ ,\ \alpha \geq 1\ ,\ c \in \mathbb{C}\ \longleftrightarrow\ c\frac{t^{\alpha-1}}{(\alpha-1)!} \tag{11}$$

Remembering that the differentiation of $Y(s)$ with respect to s is related to a multiplication by $-t$ in time domain and left multiplying $Y(s)$ by $\frac{1}{s^\alpha}$, $\alpha \geq 1$ corresponds to iterated integrals, the Cauchy formula for repeated integration is used to provide a closed-form expression for the operator $\frac{1}{s^\alpha}\frac{d^n}{ds^n}$ in a single integral:

$$\frac{1}{s^\alpha}\frac{d^n}{ds^n}Y(s)\ \longleftrightarrow\ \frac{(-1)^n}{(\alpha-1)!}\int_0^t (t-\tau)^{\alpha-1}\tau^n y(\tau)\,\mathrm{d}\tau \tag{12}$$

The resulting expression of Eq. 10 in time domain will be:

$$\phi = \frac{60}{t^5}\int_0^t (t^2 - 6t\tau + 6\tau^2)y(\tau)\,\mathrm{d}\tau - \frac{30\alpha}{t^5}\int_0^t (t-\tau)^2\tau^2 u(\tau)\,\mathrm{d}\tau \tag{13}$$

The above expression can be digitally implemented resulting in a FIR filter. For this purpose, the method is modified and we integrate backwards in a small fixed length window T to provide a feasible implementation in real-time. After some change of variables, Eq. 14 is the final expression for the algebraic estimator of ϕ (ALG) :

$$\phi = \frac{60}{T^5}\int_0^T (T^2 - 6T\tau + 6\tau^2)y(t-\tau)\,\mathrm{d}\tau - \frac{30\alpha}{T^5}\int_0^T (T-\tau)^2\tau^2 u(t-\tau)\,\mathrm{d}\tau \tag{14}$$

Each integral in this expression can be digitally implemented using the trapezoidal integration. Notice that the filter coefficients are independent of the current time t and can be computed a priori to reduce the computational effort. An efficient implementation of each integral is to store the pre-calculated discrete filter values in a vector (taking account the coefficients of trapezoidal integration). At each sample time, the buffer vector containing the samples of the last T seconds is multiplied by the filter coefficients vector resulting in the integral in the window T. To avoid an algebraic loop, the previous value of ϕ is used for the calculation of the actual control input in Eq. 3. In this implementation, the assumption that ϕ is constant is no longer a problem, since the estimation is made in a short sliding window T. If this were not the case, the estimator would have to be periodically reseted due to estimation drift.

4.2 Identification through Derivative Estimation (ACC)

Consider the third order approximation of the noisy signal $\tilde{y}(t)$ around $t = 0$:

$$y(t) = a_0 + a_1 t + a_2\frac{t^2}{2!} + a_3\frac{t^3}{3!} \tag{15}$$

Applying the Laplace Transform:

$$Y(s) = \frac{a_0}{s} + \frac{a_1}{s^2} + \frac{a_2}{s^3} + \frac{a_3}{s^4} \tag{16}$$

The identification of a_2 follows the same method of the previous subsection. Applying the differential operator $\frac{1}{s^4}\frac{d^2}{ds^2}\frac{1}{s}\frac{d}{ds}s^4$ and transforming back to time domain using the well known rules of the Laplace transform and the Cauchy formula for repeated integration::

$$a_2 = \ddot{y}(t) = \frac{120}{t^6}\int_0^t (4t^3 - 45t^2\tau + 108t\tau^2 - 70\tau^3)y(\tau)\,\mathrm{d}\tau \tag{17}$$

The second order derivative estimation is obtained substituting $y(\tau)$ by the noisy observation $\tilde{y}(\tau)$ in the above expression. The estimator is implemented as a FIR filter as showed in the previous section. This way, the unknown parameter ϕ can be estimated in real time using the acceleration estimation (ACC) through Eq. 2:

$$\phi = \ddot{y} - \alpha u = \frac{120}{t^6}\int_0^t (4t^3 - 45t^2\tau + 108t\tau^2 - 70\tau^3)y(\tau)\,\mathrm{d}\tau - \alpha u \tag{18}$$

To avoid an algebraic loop in the digital implementation, the last control input is used to calculate the actual parameter:

$$\phi[k] = \ddot{y}[k] - \alpha u[k-1] \tag{19}$$

More details regarding to algebraic derivative estimation including digital implementation can be found in Zehetner *et al.* (2007) and Mboup *et al.* (2009).

5. FEEDBACK LINEARIZATION CONTROL LAW

For comparison purposes, experiments with the feedback linearization control will also be made. Consider the following dynamic model of the magnetic levitation system shown in Fig. 1, where F_{mag} is de magnetic force:

$$m\ddot{x} = mg - F_{mag} \tag{20}$$

$$F_{mag} = \frac{K_{mag} i^2}{x^2} \tag{21}$$

m is the levitated mass, g denotes the gravity acceleration, x is the levitation gap, i is the eletric current through the coil and K_{mag} is the magnetic parameter of the system.

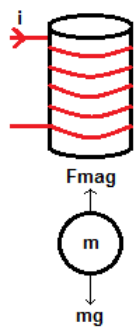

Figure 1. Magnetic Levitation System

Consider the following change of variables:

$$\theta = -\frac{m}{K_{mag}} \qquad u = \frac{i^2}{x^2} \tag{22}$$

The system dynamics can be written:

$$\theta(\ddot{x} - g) = u \tag{23}$$

If the parameter θ is exactly known, one can consider the following tracking control law:

$$\begin{aligned} u &= \theta(v - g) \\ v &= \ddot{x}_d - \lambda^3 \int \tilde{x} - 3\lambda^2 \tilde{x} - 3\lambda \dot{\tilde{x}} \end{aligned} \tag{24}$$

x_d is the desired trajectory, $\tilde{x} = (x - x_d)$ is the tracking error and $\lambda \in \mathbb{R}^+$. While u can be considered the linearization control, v is the stabilization part related to pole placement. The resulting asymptotically stable linear error dynamics is shown below, leading $\tilde{x} \to 0$. This formulation results in three negative closed-loop poles equal to $-\lambda$.

$$\dddot{\tilde{x}} + 3\lambda \ddot{\tilde{x}} + 3\lambda^2 \dot{\tilde{x}} + \lambda^3 \tilde{x} = 0 \tag{25}$$

A stability proof when an estimation $\hat{\theta}$ is used instead the real parameter θ is given in Moraes (2015). The result is that for $\frac{\hat{\theta}}{\theta} > \frac{1}{9}$ the closed loop system is still stable, considering trajectories such that $\dddot{x}_d = 0$. Trajectories that don't match this requisite can still be imposed to the system since the higher order term $\dddot{x}_d$ is usually transitory.

6. ALGEBRAIC DERIVATIVE ESTIMATORS AS NON-ASYMPTOTIC STATE OBSERVERS

In this paper, $\dot{x}$ (velocity of the levitated mass) will be estimated through algebraic derivative techniques, discarding the need of a model-based observer. For this purpose, consider the second order approximation of the noisy signal $\tilde{y}(t)$ around $t = 0$:

$$y(t) = a_0 + a_1 t + a_2 \frac{t^2}{2!} \tag{26}$$

Applying the Laplace Transform:

$$Y(s) = \frac{a_0}{s} + \frac{a_1}{s^2} + \frac{a_2}{s^3} \tag{27}$$

Multiplying by s^3 to isolate a_2:

$$s^3 Y(s) = s^2 a_0 + s a_1 + a_2 \tag{28}$$

Differentiating with respect to s, in order to eliminate a_2:

$$3s^2 Y(s) + s^3 \frac{d}{ds} Y(s) = 2s a_0 + a_1 \tag{29}$$

Multiplying by $\frac{1}{s}$, isolating a_0:

$$3sY(s) + s^2 \frac{d}{ds} Y(s) = 2a_0 + \frac{a_1}{s} \tag{30}$$

Differentiating with respect to s, in order to eliminate a_0:

$$3Y(s) + 5s \frac{d}{ds} Y(s) + s^2 \frac{d^2}{ds^2} Y(s) = -a_1 \frac{1}{s^2} \tag{31}$$

Multiplying by $\frac{1}{s^3}$:

$$3 \frac{1}{s^3} Y(s) + 5 \frac{1}{s^2} \frac{d}{ds} Y(s) + \frac{1}{s} \frac{d^2}{ds^2} Y(s) = -a_1 \frac{1}{s^5} \tag{32}$$

Transforming back to time domain:

$$\frac{3}{2} \int_0^t (t-\tau)^2 y(\tau)\,\mathrm{d}\tau - 5 \int_0^t (t-\tau)\tau y(\tau)\,\mathrm{d}\tau + \int_0^t \tau^2 y(\tau)\,\mathrm{d}\tau = -a_1 \frac{t^4}{4!} \tag{33}$$

Rearranging the expression, the estimation of a_1 is obtained:

$$a_1 = -\frac{12}{t^4} \int_0^t (3t^2 - 16t\tau + 15\tau^2) y(\tau)\,\mathrm{d}\tau \tag{34}$$

The first order derivative estimation is obtained substituting $y(\tau)$ by the noisy observation $\tilde{y}(\tau)$ in the above expression. The estimator is implemented as a FIR filter in a window T as showed in section 4.

7. EXPERIMENTAL SETUP AND RESULTS

The experimental setup used for control tests include a National Instruments PCI-6259 acquisition board working together with Matlab Realtime + Simulink Model. The power electronics amplify the control signal and generate the electrical current in the coil, which is rolled on a ferromagnetic core. The levitation gap is measured through an optic sensor and the measure is sent back to the acquisition board.

Figure 2. Coil + Levitated Mass

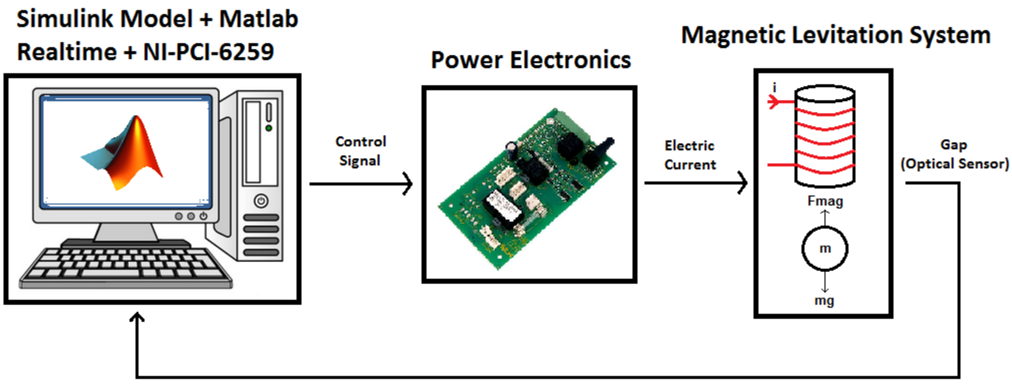

Figure 3. Experimental Setup Diagram

All the experiments were realized with a 10kHz sample frequency, control poles -40 defined by $\lambda = 40$ (both in model-free PD and PID control) and levitated mass $m = 0.240Kg$. In the algebraic identification of ϕ through (Eq. 14) a window $T = 0.012s$ was used. $y = x$, where x is the levitation gap. The model-free control effort $u = i$, where i is the electric current in the coil. The desired trajectory $x_d = y_d$ was generated by a step on $t = 1s$ filtered by the third order filter $\frac{8}{(s+2)^3}$ to ensure that the desired trajectory is of class $\mathbf{C}^2$. The first order derivative used in Eq. 3 is also estimated through an algebraic derivative estimator. On the following experiments, ACC denotes the estimation of ϕ through acceleration (Eq. 18) and ALG denotes the algebraic identification of ϕ (Eq. 14). For each estimation method, a different value of α was tuned to provide a good response: $\alpha_{ACC} = -4400$ and $\alpha_{ALG} = -50$.

In the feedback linearization control, the estimated magnetic parameter is $K_{mag} = 1.6 \times 10^{-4}$. The closed loop poles -40 are defined by $\lambda = 40$. The first order derivative used in the control law was estimated through an algebraic estimator.

The experimental results show an excellent tracking performance of model-free control, with a sightly better tracking when a integral action is included. The control signal was more noisy in the ALG scheme. Increasing the estimation window may increase the filtering properties, but the resulting estimation delay may lead to instability of the closed loop. The ACC scheme also have the estimation window of the second order derivative estimator, leading to a different tuning of α for each scheme.

It's interesting to notice that model-free control presented a better transitory tracking than feedback linearization, which is a model-based control law. Since the magnetic parameter varies with the gap, the feedback linearization relies only on the integral action to compensate the tracking error. In model-free control, the local model is re-identified at each sample time, leading to a better performance.

Although model-free control presents a better performance, several tries were made to reach a good tuning in ALG and ACC schemes. The fact that the magnetic force produced by the same current strongly varies with the levitation gap since $F_{mag} = K_{mag}\frac{i^2}{x^2}$ may difficult the tuning of α, both in ALG and ACC schemes. Despite this fact, there is a considerably large range of values of α which result in good performance.

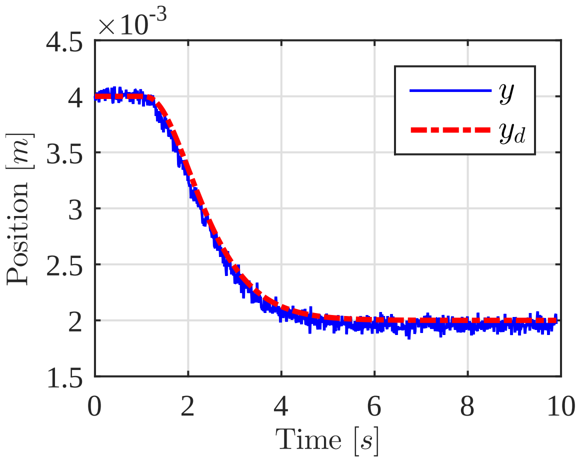

Figure 4. Model-free (ALG+PD): Levitation Gap

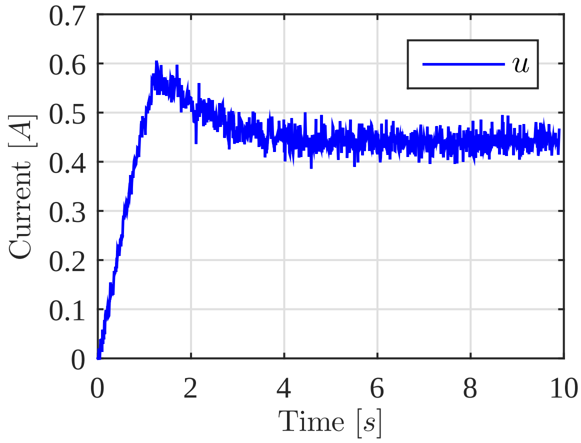

Figure 5. Model-free (ALG+PD): Control Effort

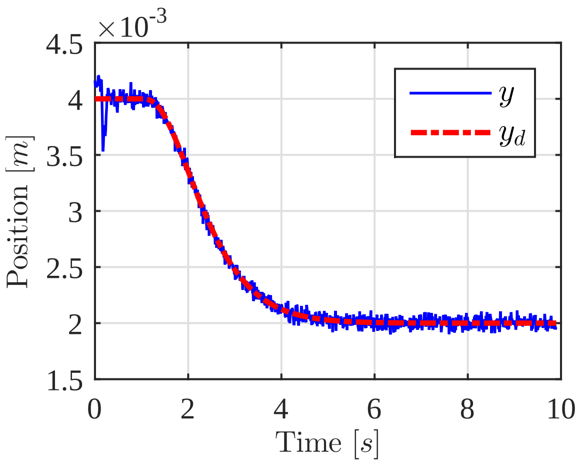

Figure 6. Model-free (ALG+PID): Levitation Gap

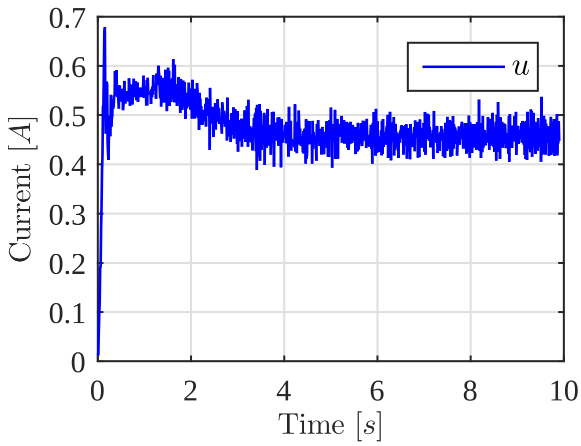

Figure 7. Model-free (ALG+PID): Control Effort

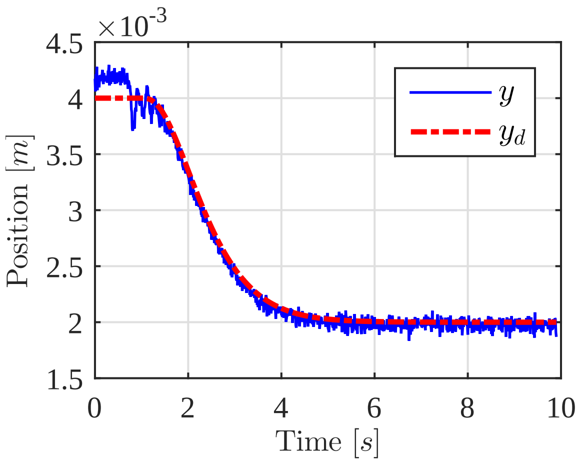

Figure 8. Model-free (ACC+PD): Levitation Gap

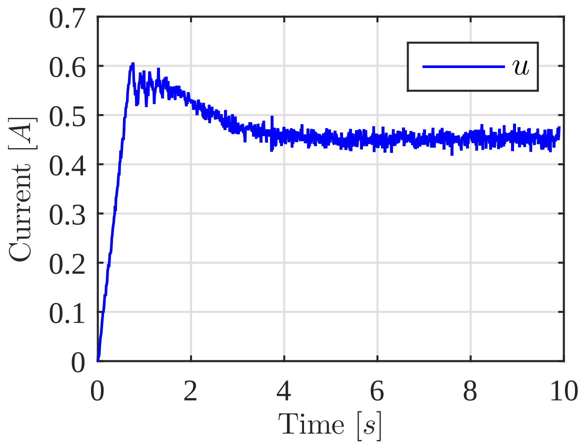

Figure 9. Model-free (ACC+PD): Control Effort

8. CONCLUSION

The experimental results show an excellent tracking performance of model-free control. At the same time, the algebraic derivative estimator proved to be a feasible alternative to the classic state observers, providing a fast and reliable estimation with a good closed loop performance without the need of a model-based observer. The model-free control presented a tracking performance even better than the feedback linearization, which is a model-based control law. This way, model-free control can be a feasible alternative for controlling systems where the obtention of an accurate dynamical model is a hard task.

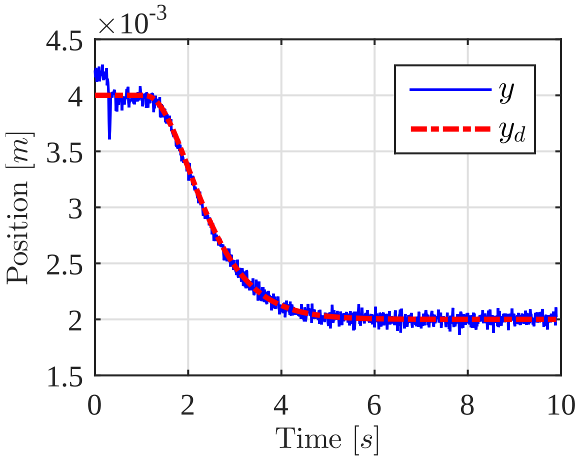

Figure 10. Model-free (ACC+PID): Levitation Gap

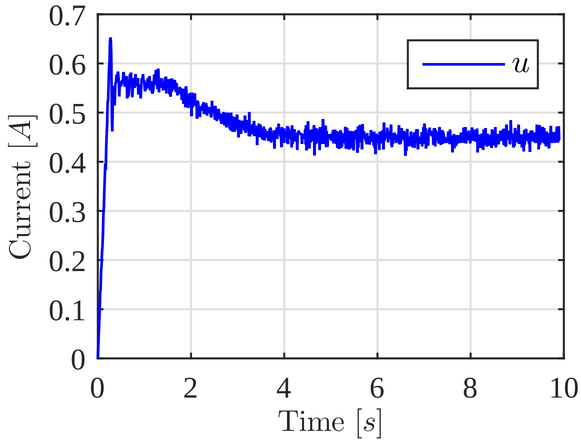

Figure 11. Model-free (ACC+PID): Control Effort

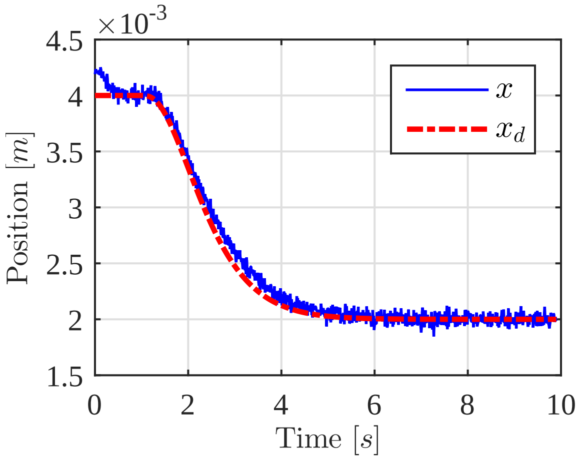

Figure 12. Feedback Linearization: Levitation Gap

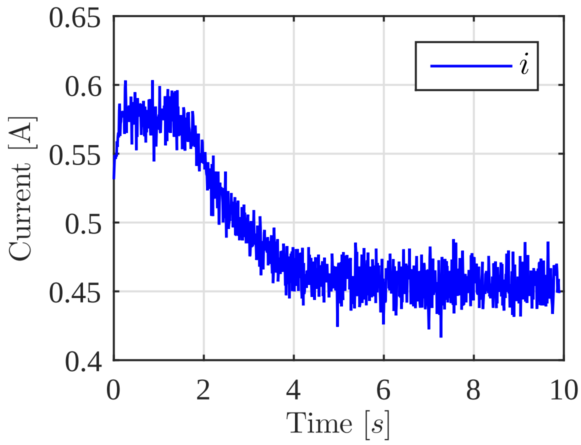

Figure 13. Feedback Linearization: Control Effort

9. ACKNOWLEDGEMENTS

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11. RESPONSIBILITY NOTICE