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TANGENT MODULUS ESTIMATION OF ACHILLES TENDON FROM ULTRASOUND IMAGES

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Abstract. *The nonlinear nature of tendon elastic properties suggest that tangent modulus is a property that better describe its behavior than a fixed value of the modulus of elasticity. Here, Achilles tendon tangent modulus is estimated in vivo, calculating the integral of incremental strains along the tendon length change and the true area, estimated from tendon diameter measurements and Poisson's ratio. Passive mobilization was applied at the ankle of the volunteers, with a dynamometer at a slow velocity (5°/s). Tendon elongation was tracked with ultrasound images, by monitoring the displacement of the myotendinous junction. Maximum values for tangent modulus were greater than the modulus of elasticity calculated from engineering stress and strain. Such differences increase for larger deformations.*

Keywords: *Biomechanics, tendon, mechanical deformation, tangent modulus*

1. INTRODUCTION

The passive mechanical properties of human muscle-tendon unit (MTU) are usually estimated in terms of angular stiffness (Nm/rad), from the passive torque-angle relationship, measured by dynamometry (Gajdosik, 2002). Being a structural property, an intensive material property, such as the Modulus of elasticity (*E*), provides more accurate information about the biological material behavior and its actual mechanical state than joint rotational stiffness. However, determining *E* of biological materials is highly problematical and the results varies widely depending on the experimental approach. For the Achilles tendon (AT), for example, modulus of elasticity spans from 220 to 1671 MPa (Zhao et al., 2009; Arya and Kulig, 2010).

AT modulus of elasticity can be determined *in* vivo by imposing a controlled angular displacement to the ankle keeping muscles relaxed. Ankle passive reaction torque can be measured by a dynamometer. Under small angular velocities, viscoelastic effects can be regarded as small. At the maximum and minimum limits of ROM, passive stiffness of other structures than the tendon can become relevant.

Tendon force-length curve follows a typical pattern such that at small forces large tendon deformation is observed, following a non-linear but still small stiffness behavior (Maganaris and Paul, 1999). At intermediate forces, tendon stiffness is approximately linear. In large tendon deformations, which is achieved normally only *in vitro* with axial testing machines (Vergari et al., 2011) stiffness is highly nonlinear, showing a sort of yielding. Thus, a single value of *E* is not appropriate to represent the overall tendon stress-strain curve. In such cases the tangent modulus, corresponding to the local derivative of the stress with respect to the strain is a more appropriate parameter to describe the material mechanical behavior (Popov, 1990). Additionally, if tendon cross-sectional area shrinks due to positive Poisson's ratio, the *true* stress is calculated by dividing the axial force by the *true* area, that can measured or estimated from Poisson's coefficient definition. In this case, the *true* strain should be calculated as the integral of incremental strains along the specimen length change.

Here, we estimate AT tangent modulus *in vivo* using a dynamometer to simultaneously measuring passive torque and imposing a controlled angular velocity at the ankle joint. Ultrasound image is used to track miotendineous junction (MTJ) displacement, AT insertion at the calcaneus and tendon cross-sectional area (CSA). Thus, using these data, true stress, true strain and tangent modulus are estimated for a number of subjects.

2. METHODOLOGY

Tangent modulus (E_{true}) is defined as the local derivative of the axial true stress (σ_{true}) relatively to the axial true strain (ε_{true}):

$$E_{true} = \frac{d\sigma_{true}}{d\varepsilon_{true}} \quad (1)$$

Considering a small tendon axial incremental length variation $L^T = (L^T - L_0^T)$, where L^T is the actual and L_0^T the initial (reference) tendon length. True strain comes from integrating the differential $d\varepsilon_{true} = dL^T/L^T$:

$$\varepsilon_{true} = \int_{L_0^T}^{L^T} d\varepsilon_{true} = \ln(1 + \varepsilon_{eng}) \quad \textbf{(2)}$$

where the *engineering* strain is given by $\varepsilon_{eng} = (L^T - L_0^T)/L_0^T$. Tendon cross-sectional true stress is given by:

$$\sigma_{true} = \frac{F}{CSA_{true}} \quad \textbf{(3)}$$

where F is the tendon force and CSA_{true} the true cross-sectional area.

Tendon force (F) is given by

$$F = M/r(AA) \quad \textbf{(4)}$$

where M is the ankle torque, r the AT moment arm with respect to ankle joint center and AA the ankle angle. Initial (reference) tendon cross sectional area CSA_0 is calculated directly from ultrasound measurements, being approximated by an ellipse with semi-axis a_0 and b_0. Actual CSA is determined from the following expression ($\nu = 0.55$) (Vergari et al., 2011).

$$CSA_{true} = CSA_0(1 + \varepsilon_{eng})^{-2\nu} \quad \textbf{(5)}$$

The relationship $\sigma_{true}\ x\ \varepsilon_{true}$ can be modelled though a nth order polynomial,

$$\sigma_{true} = a_1\varepsilon_{true}^{n} + a_2\varepsilon_{true}^{n-1} + \cdots + a_n\varepsilon_{true} + a_{n+1} \quad \textbf{(6)}$$

and tangent modulus follows from eq.**(1)**,

$$E_{true} = na_1\varepsilon_{true}^{n-1} + (n-1)a_2\varepsilon_{true}^{n-2} + \cdots + 2a_{n-1}\varepsilon_{true} + a_n \quad \textbf{(7)}$$

Musculotendon length is the sum tendon and muscle lengths ($L^{MT} = L^T + L^M$). L^{MT} can be found from regression curves (Grieve et al., 1978). To find muscle initial and actual lengths, a series of experiments with volunteers were performed. The foot was fastened to the dynamometer at 30° of plantar flexion position ($PF_{initial}$). The ultrasound probe was held by the researcher such that the MTJ was visualized approximately in the center of the image frame. The horizontal distance from the MTJ to the right border of the image frame was measured . The ankle was moved by the dynamometer from $PF_{initial}$ to maximal dorsiflexion at constant angular velocity of 5^o/s, keeping US probe fixed. The distance from MTJ to the right border of the image decreased, and was measured at intermediate joint angles with 5^o steps. Muscle lengthening was calculated for each US frame analyzed, from $PF_{initial}$ to maximum dorsiflexion. A fifth-order polynomial was adjusted for the muscle length variation. After a series of corrections for the initial tendon length, actual tendon length L^T was then estimated as $L^T(AA) = L^{MT}(AA) - L^M(AA)$. Finally, $\varepsilon_{true}(AA)$ was calculated from eq. **(2)**.

Nineteen healthy males volunteered for the study (18.94 ± 0.45 years, 73.14 ± 12.42 kg and 174.78 ± 7.39 cm) with no history of skeletal muscle injury. The study was approved by the UFRJ University Hospital ethics committee. An ultrasound (T2000 – *Terason Ultrasound*, 6-11 MHz, 40 mm) was used for MTU image acquisition. Lateral gastrocnemius (LG) surface EMG was recorded (EMG System, analog band-pass filtered 10 to 500 Hz 1kHz sampling frequency), to verify myoelectric silence during passive ankle movement. The passive ankle mobilization was done by the isokinetic dynamometer (Biodex System4Pro).

The subjects sat at the dynamometer chair with the right knee in full extension, hip flexed, and trunk flexed at 85°. The right foot was attached to the dynamometer footplate, aligning the lateral malleolus dynamometer axis. The individual range of motion (ROM) was determined by passive ankle dorsiflexion from 30° of plantar flexion up to the limit of dorsiflexion. The test consisted of three consecutive passive dorsiflexions at 5°/s keeping the ultrasound probe in contact to the skin over the MTJ, assuring it remained fixed during the whole ankle movement. Ultrasound videos of the MG MTJ displacement were synchronized with EMG and torque signals.

MTJ displacement was measured with *ImageJ* software (NIH, Maryland, USA). Total MTU length was considered equal to leg length (Nakamura et al., 2011). It was determined as the distance between the lateral malleolus and the popliteal line. MTU length as function of ankle angle were estimated using the regression equation proposed by Grieve et al. (1978). AT moment arm was calculated using regression equations (Menegaldo et al., 2004). Tendon CSA was measured in the US images obtained with the ankle at the rest position. True CSA was calculated by eq.(8).

3. RESULTS AND DISCUSSION

Considering strain values of 2%, which is compatible with literature reported modulus of elasticity *in vivo*, the average modulus of elasticity and stress are shown in Table 1, for engineering and true stress, strain and modulus of elasticity. Statistical tests demonstrated significant differences between the maximum true and engineering values all variables.

Table 2: Average and standard deviation of modulus of elasticity, strain and stress.

Variable	Mean	Standard deviation
E_{eng} (MPa)	188.56	99.19
E_{true} (MPa)	312.38	171.95
σ_{eng}(MPa)	17.53	9.88
σ_{true}(MPa)	22.01	12.96

True stress x true strain relationships for all subjects were adjusted by 5th order polynomials and differentiated to calculate tangent modulus expressions. Figure 1 presents an example, for one subject, of the difference among stress x strain curve and tangent modulus, calculated considering true and engineering stress and strain.

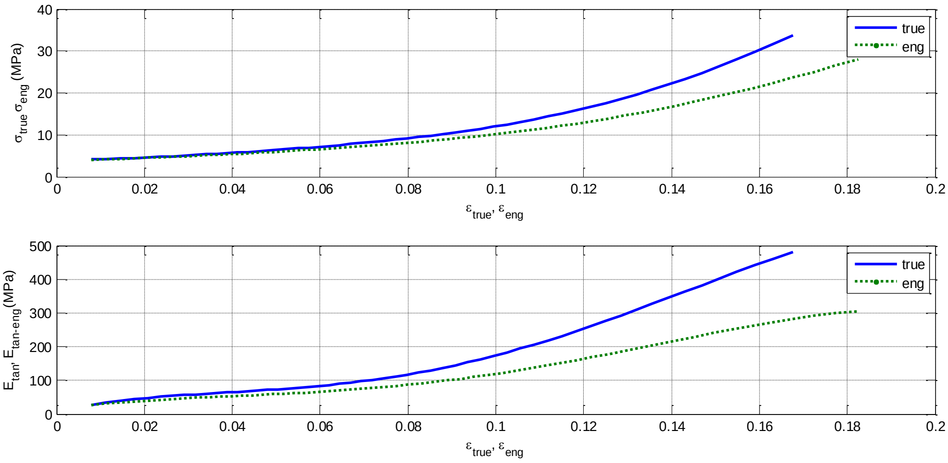

Figure 1 – Example for one subject of the difference between tension x deformation and tangent modulus calculated with engineering tension and deformation and their 'true' counterparts.

It can be observed the presence of large differences between the tangent modulus calculated from true and engineering stress and strain. These differences grow at larger deformations. Figure 1 (and similarly for the other 18 subjects) suggests that the modulus of elasticity of AT is approximately constant below $\varepsilon_{true} = 0.07$. Above this part of the curve, tendon tissue stiffness increases sharply for most of the subjects.

To conclude, tangent modulus of Achilles tendon is highly variable with respect to tendon strain, especially at strain values above approximately 10%. Large of tangent modulus dispersion among subjects was also observed, suggesting that the Achilles tendon tissue properties present high variability, especially at larger deformations.

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