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ASSESSING THE IMPACT OF PHENOMENOLOGICAL VISCOSITY MODEL UNCERTAINTIES ON THE NUMERICAL SIMULATION OF NON-DILUTE TURBIDITY CURRENTS

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Abstract. *Turbidity currents are particular particle-laden flows driven by the presence of sediments, which generate local density gradients responsible for the motion. Besides being transported by the fluid, the sediments, due to gravity, are able to move downwards and settle. The study of turbidity currents aims to improve the understanding about the formation of sedimentary basins that might host oil reservoirs. The non-standard rheological behavior, appearing with the presence of sediments, especially for non-dilute flows, might be taken into account in the numerical model, which, often, is carried out through phenomenological constitutive-like equations for the effective viscosity of the mixture (fluid + sediments). Therefore, the reliability of such models are evaluated into the balance equations, with special concern about uncertainties in those models. In this sense, Uncertainty Quantification (UQ) can provide a rational framework to assist in this task. Here, an initial study about the impact of the so-called error model, in which uncertainties are supposed to be generated by the form of the phenomenological equations, is presented. Different viscosity laws taking into consideration high sediments concentration head a probabilistic perspective in the context of the UQ analysis. More specifically, the viscosity law is built upon a nominal mathematical form with an embedded stochastic parameter that is responsible for addressing the uncertainty in the model. The UQ analysis uses a deterministic high-optimized finite element parallel solver based on the Residual Based Variational Multiscale Method. Then, a non-intrusive stochastic collocation method is employed to handle the stochastic dimension of the problem in order to lead in a number of simulations emphasizing the impact of the model uncertainties on quantities of interest as the sediment deposition patterns.*

Keywords: *non-dilute turbidity currents, model uncertainties, uncertainty quantification*

1. INTRODUCTION

Turbidity currents are mainly responsible for sedimentary basins formation that might host oil reservoirs. The understanding of this particular particle-laden flow driven by the presence of sediments is difficult. This is due to the high complexity of these flows, in which sediments generate local density gradients responsible for the motion (often turbulent) and, by the effect of gravity, move downwards and settle. Besides, non-dilute flows has a non-standard behavior showing out the necessity of coupling the flow with viscosity, increasing complexity, and thus, multiscale variables are concomitantly being affected one by the others.

In this work, to simulate deposition patterns with reliability, we put together three different disciplines: computational fluid dynamics (CFD) (Guerra *et al.*, 2013), uncertainty propagation (UQ) and computer science, particularly scientific workflow managing systems (Guerra *et al.*, 2012). The remainder of this paper is organized as follows. In the next section we briefly introduce the model governing equations for non-dilute turbidity currents, the CFD approach used in the simulations and the computational set-up for the numerical experiments. The next section discusses the viscosity-sediment coupling and defines the UQ scenario for studying model discrepancies between phenomenological laws relating concentration-dependent viscosities. Section 4 reports the main UQ results for this scenario and the paper ends with a summary of our main conclusions.

2. GOVERNING EQUATIONS

In Guerra *et al.* (2013), Navier Stokes (NS), continuity and sediment transport equations (Equations 1 , 2 and 3 respectively) are solved together for dilute suspension of sediment. Indeed, in this approach, the velocity-pressure non-conservative form of the NS equations are considered in order to describe the incompressible turbulent flow carrying dilute suspensions of sediment.

In addition of being driven by fluid motion, particles are endowed with extra mobility modeled by the settling velocity w_{st} in gravity direction $\mathbf{e^g}$. The settling velocity is assumed as constant as in Guerra *et al.* (2013), even if heuristic models that take into account others effects such as hindering settling and flocculation are found in literature (Winterwerp, 2002), more accurate modeling for poli-disperse or non-dilute solutions (Huppert, 1998; Winterwerp, 2002). Considering that the NS equations are based on Boussinesq approximation, valid for low concentrations, but sufficiently high to be considered non-dilute solutions, and that the purpose of this work is to assess the impact of phenomenological viscosity model uncertainties, treating settling velocity as constant is expected to be a good approach, even if it should be adapted in future works.

Thus, following Guerra *et al.* (2013), the particle transport is governed by an advection-diffusion equation involving $c = \frac{C}{C_0}$, the scaled concentration, expressing the fraction of volume occupied by the particles. C and C_0 are, respectively, the actual concentration and a normalization value, the latter typically taken as the initial concentration. Diffusion is supposed to be quite small, and its inclusion in the modeling is often motivated by numerical reasons. Thus, the dimensionless equations that govern the particle-laden flow are

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\sqrt{Gr}} \Delta \mathbf{u} + c\mathbf{e^g} \tag{1}$$

$$\nabla \cdot \mathbf{u} = 0 \tag{2}$$

$$\frac{\partial c}{\partial t} + (\mathbf{u} + w_{st}\mathbf{e^g}) \cdot \nabla c = \nabla . \left(\frac{1}{Sc\sqrt{Gr}} \nabla c \right) \tag{3}$$

where $\mathbf{u}$, p, and t are, respectively, non-dimensional velocity, pressure, and time. Previously, p, many times referred in the literature as the dynamic pressure, results after removing the hydrodynamic component of the pressure. Moreover, the Grashof number Gr expresses the ratio between buoyancy and viscous effects given by (Guerra *et al.*, 2013)

$$Gr = \left(\frac{u_b}{\upsilon H} \right)^2 \tag{4}$$

with υ as the fluid kinematic viscosity, H as a characteristic length of the flow and $u_b = \sqrt{gHC_0(s-1)}$ as the buoyancy velocity; g stands for the gravity acceleration and s for the quotient of particle and fluid densities. A second dimensionless number resulting from turning the governing equations into a non-dimensional form is the Schmidt number, Sc, that expresses the ratio between diffusion and viscous effects given by

$$Sc = \frac{\upsilon}{\kappa} \tag{5}$$

where κ is the diffusion coefficient that is quite small.(Guerra *et al.*, 2013)

The deposition pattern, which is a quantity of interest in UQ analysis, expressed in function of the longitudinal dimension x and time t, is modeled by

$$D_{i(x,t)} = \int_0^t w_{st} c_{i(x,t)} d\tau \qquad i = 1, ..., N \tag{6}$$

applied in the bottom surface where N is the number of bottom elements. Following this proceeding for deposition, topology is not modified among time and erosion, a possible source of sediments particle, is then not considered (Nasr-Azadani *et al.*, 2013). Moreover, modification of topology imposes an adaptive grid, pointing out more others computational problems to perform the uncertainty quantification. Going further, coupling between viscosity and sediment concentration is needed in non-dilute flows because the influence of suspended particles on flow is not any more negligible (Widera, 2011).

The computational fluid dynamics simulations are performed with a high performance finite element code based on the Residual Based Variational Multiscale Method (RBVMS). For more information about RBVMS formulation and solver procedures, see Guerra *et al.* (2013). Here, will be pointed out only the essential of the fluid dynamics and the flow-sediment coupling with viscosity laws, which is not considered in Guerra *et al.* (2013). Looking for simplicity and to avoid unnecessary computational work, simulations are carried out on a simple parallelepiped channel with $6 \times 0.4 \times 0.5$ non-dimensional length units. The sediments are injected with a constant rate of a non-dilute solution through a rectangular window as can be seen in Figure 1. Initial mixture concentration is 0.1.

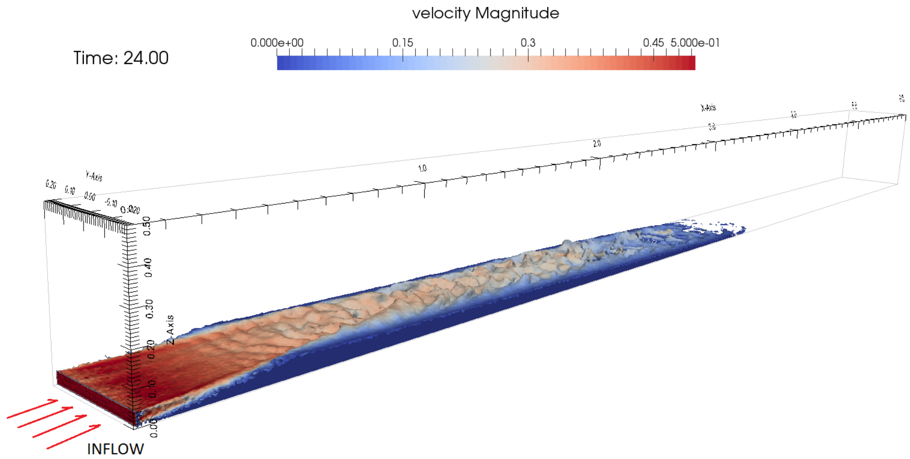

Figure 1. Channel setup for computational simulations

Boundary conditions at the bottom are $\frac{\partial c}{\partial t} = w_{st}\frac{\partial c}{\partial t}$ with initial conditions $c(.,0) = 0$. The top has free boundary conditions and non-slip conditions are applied to the other walls. Due to the viscosity-sediment coupling, the input flow is considered as a mono-disperse mixture of water and sediment, the latter being inert rigid spheres.

3. VISCOSITY-SEDIMENT COUPLING

3.1 Viscosity Models

Several laws relating viscosity as a function of sediment concentration have been developed and they generally are similar in nature to Eistein's equation (Einstein, 1906). These laws intend to decrase errors observed in labortory and in the field, extending their applicability range for higher concentrations. Indeed, Einstein's equation is valid only for rigid spheres of sediments and very low concentrations, up to 5 %. Simulating the Brownian movement with body forces equilibrium instead of the equations of motion as Einstein, Batchelor and Green (1972) came up with a more accurate polynomial equation, valid for volumetric concentrations up to 0.3. As inferred in Arhenius (1917), we may find in Krieger and Dougherty (1959), Mooney (1951) and Brady (1993) polynomial equations representing the viscosity in function of the volumetric concentration. In the present work we restrict ourselves to the relations listed in Table (1).

Table 1. Different viscosity equations in function of concentration taken from Widera (2011) where μ_m stands for the viscosity of mixture, μ for the viscosity of water and c_m for the maximum packing fraction.

Author	Equation
Einstein (1906)	$\mu_m = \mu\,(1 + 2.5c)$
Mooney (1951)	$\mu_m = \mu\left[\exp\left(\frac{2.5c}{1-\frac{c}{c_m}}\right)\right]$
Krieger and Dougherty (1959)	$\mu_m = \mu\left[1 - \frac{c}{c_m}\right]^{-2.5c_m}; c_m = 0.74$
Thomas (1965)	$\mu_m = \mu\left[1 + 2.5c + 10.05c^2 + 0.00273\exp(16.6c)\right]$
Batchelor (1977)	$\mu_m = \mu\left[1 + 2.5c + 6.2c^2\right]$
Brady (1993)	$\mu_m = 1.3\mu\left[1 - \frac{c}{c_m}\right]^{-2.0}$
Toda and Hisamoto (2006)	$\mu_m = \mu\left[\frac{1-0.5c}{(1-c)^3}\right]$
Toda and Hisamoto with k (2006)	$\mu_m = \mu\left[\frac{1+0.5kc-c^2}{(1-kc)^2(1-c)}\right]; k = 1 + 0.6c$

Krieger and Dougherty's equation (equation (7)) is particularly interesting because, intuitively, the relative viscosity of a suspension will approach infinity as the volume fraction (c) approaches some maximum value (c_m) (Stickel and Powell, 2005)

$$\mu_m = \mu\left[1 - \frac{c}{c_m}\right]^{-\lambda c_m} \tag{7}$$

$$lim\frac{\mu_m}{\mu} = \infty \tag{8}$$

Therefore, more accurate results are expected with an adequate value of the maximum packing fraction, that is, 0.74 for mono-dispersed spherical rigid particles, embraced in this work. As can be seen in Figure (2), the use of Krieger and Dougherty's equation presents changes in the variables distributions. Indeed, the new viscosity distribution affects the global rheology, showing differences between patterns.

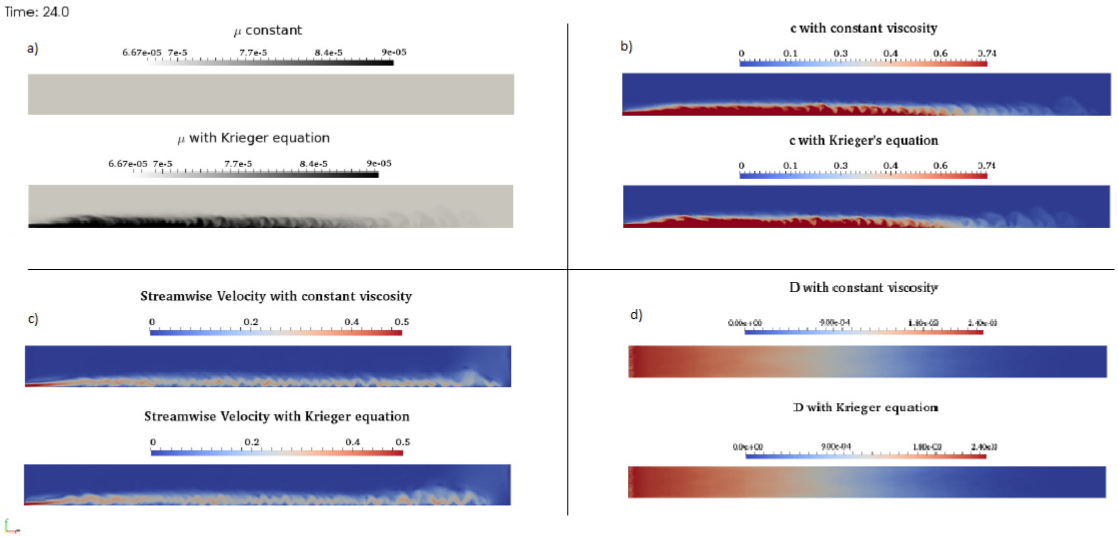

Figure 2. Comparing quantities of interest reached by simulations with constant viscosity and with Krieger and Dougherty's equation in a center cut at time t = 24

We may observe clearly in Figure (2) differences in viscosity (a), concentration (b) and velocity (c). However, differences are not perceptible in the deposition plots (d). Several reasons as constant settling velocity or a small time of simulation might explain the small differences among both approaches. But considering that the deposition mass is only around 10 % of the injected mass of sediment, the particles remains mainly in the suspention and differences are not expected to be high. More insight on these changes might be provided using UQ techniques, building a rational framework to assist in the task of giving reliability and quantifying those differences.

3.2 Defining the stochastic approach

On the other hand, the parameter λ is used in Krieger and Dougherty (1959) as 2.5 with the purpose of fitting Einstein's equation for low concentration. However, there is a discussion on the literature upon this point. Some authors follow Einstein, saying that it could not be another way. Others as in Brady (1993) indicate that this parameter can be different from Einstein's value. However, it seems that this value has an influence on the form of sediment (shape, size, homogeneity, rigidity) and their properties, as interactions among particles from distinct natures, for instance, cohesive forces. Citing Widera (2011), "all above mentioned problems still have to be investigated more deeply and unfortunately for the time being here there is no solution for most of them." Thus, as the nature and reasons of these interactions are not in the scope of this work, only the variability of this parameter takes an important place here since the purpose is to quantify uncertainties generated by the insertion of concentration-dependent viscosity laws to understand better the flow when considering this complex rheology.

As lack of knowledge upon what is the most suitable law, we may use a stochastic approach to somehow address this issue. Figure (3) points out that a variation of λ parameter of Krieger and Dougherty's equation represents quite well all viscosity laws evaluated. Thus, a one–order non–intrusive stochastic approach, applied on λ with uniform distribution between $[1.4; 3.8]$, can support the model uncertainty analysis of the viscosity coupling model. In fact, the interest is to perform Krieger and Dougherty's functional form with an embedded stochastic parameter that is responsible for addressing the uncertainty in the model, that is, model discrepancy.

The stochastic model follows Nobile *et al.* (2008), a classical Smolyak approximation using nested Clenshaw-Curtis abscissas, where convergence was handled by the deposition values at the simulation final time. This convergence was reached in level 8. To perform a CFD simulation for each abscissa, achieving convergence in a reasonable computer time, the computational code deal with two level of parallelism. The first is the internal CFD solver parallelism, optimizing the flow solution and the second, an external parameter sweep that enables the simulation of several instances of the solver at the same time, optimizing the evolution in the stochastic dimension.

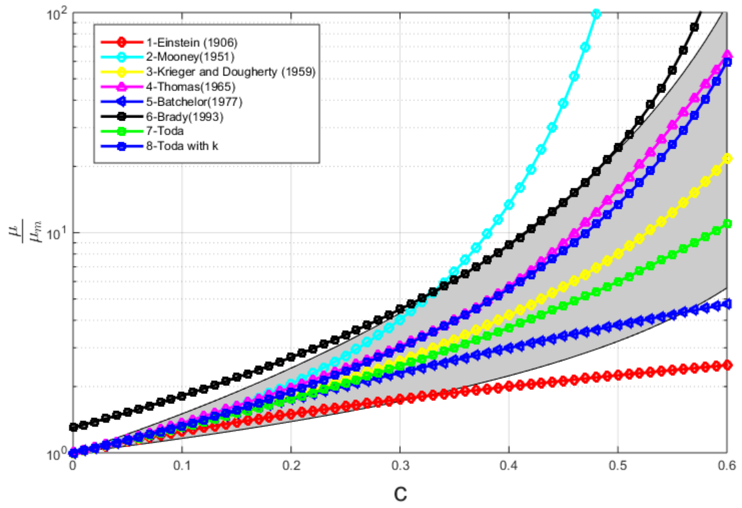

Figure 3. Range of Krieger and Dougherty's equation with variation of λ parameter in $[1.4; 3.8]$ and others phenomenological formulations

4. UQ RESULTS

The quantities of interest obtained by the UQ approach help to understand the global behavior of turbidity currents. In the observed viscosity, concentration and streamwise velocity given in figures (4), (5) and (6) respectively), are notably smoother when compared to individual simulations concerning an unique value of λ parameter or without viscosity-sediment coupling. Both mean and standard deviation does not show a strong turbulent behavior and uncertainties are concentrated in the channel bottom or carried by the current, being more pronounced in the current front.

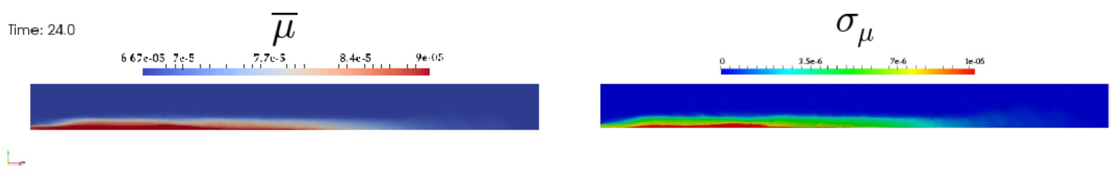

Figure 4. Viscosity at time 24 in a longitudinal center channel cut

Figure 5. Concentration at time 24 in a longitudinal center channel cut

Figure 6. Velocity at time 24 in a longitudinal center channel cut

In the case of the streamwise velocity, there is a severe annihilation of turbulence when we look at the stochastic expected values, as given by the Q-criterion plot in figure (7). Although each simulation presents turbulence, the mean is smooth, since vortices may cancel themselves when we compute the mean. From this fact, we can also infer the importance of viscosity in turbidity current. By modifying significantly the flow rheology, the global velocity behavior can be found with reliability, but locally, there are missing informations as a direct consequence of the stochastic approach.

The deposition pattern requires also attention. Its standard deviation increases in a central zone. From left to right, in the flow direction, deposition decreases, but different rheological behaviors for each stochastic case make uncertainty

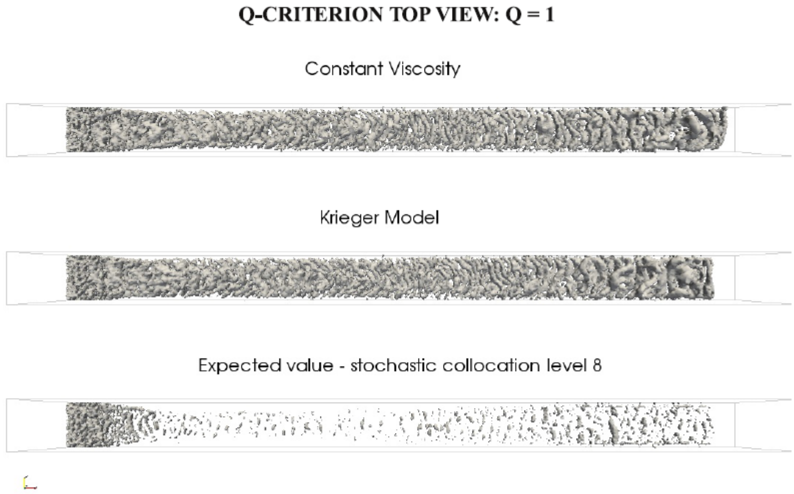

Figure 7. Topview Q-criterion at time 24 for constant viscosity case, Krieger'equation and stochastic collocation at level 8

Figure 8. Deposition at time 24 in the bottom

grow until there is no more sediments to be deposited. Since the settling velocity is constant, this phenomenon affecting fluid flow may point to different ways to drag sediment particles generating a greater deviation in intermediate zones, not so far as the current front, where dragging is predominant, and not so close, where deposition is similar for all cases.

Even if there are some phenomena that need deeper investigation for non-dilute turbidity currents, this initial study about the impact of the so called error model, in which uncertainties are supposed to be generated by the functional form of the phenomenological equations, seems to be very helpful in the understanding of the rheology of gravity flow. However, those simulations stress the impact of viscosity model uncertainties providing good statistical results for each quantity, over spatial and temporal dimensions. It is then possible to suggest that this framework may contribute to increase the reliability of CFD simulations where concentration-dependent phenomenological viscosity laws are fundamental to the understanding of complex cases of non-dilute turbidity currents.

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