

WRENCH CAPABILITY POLYTOPES IN REDUNDANT PARALLEL MANIPULATORS

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Abstract. In this paper the characteristic wrench capability polytope of three different planar parallel manipulators is obtained from the optimization of its static equations. In order to solve the problem regarding the global optimization, an evolutionary algorithm known as differential evolution (DE) is used. The objective function of the optimization problem of force capability is defined by employing the screw theory and Davies method as a primary mathematical tool. Finally, some force capability polytopes are obtained for each studied manipulator.

Keywords: Wrench capability, redundant parallel manipulators, optimization.

1. INTRODUCTION

Robots are now widely used in factories, and applications of robots in space, the oceans, nuclear industries, and other fields are being actively developed. Also, nowadays, the use of robots in every facet of society, including the home, is being seriously considered. In this context, creating autonomous robots that can learn to act in unpredictable environments has been a long-standing goal of robotics, artificial intelligence, and cognitive sciences (Romanelli, 2011).

An important step towards the autonomy of robots is the need to provide them with a certain level of independence in order to face quick changes in the environment surrounding them; to get robots operating outside rigidly structured environments, such as research centres or universities facilities and beyond the supervision of engineers or experts, it is necessary to face different technological challenges, amongst them, the development of strategies that allow robots to interact with the environment (Romanelli, 2011).

In regard to the nature of the interaction between a robot and its environment, robotic applications can be categorized in two classes. The first class is referred to non-contact tasks (unconstrained motion in a free space, without any environmental influence on the robot). In these tasks, the robot dynamics is the most important aspect as regards its performance, several industrial applications such as pick-and-place, spray painting, gluing, and arc or spot welding belong to this category. In contrast to the non-contact tasks, many complex advanced robotic applications as packaging, assembling, or machining require the manipulator to be coupled with other objects which can move, this kind of applications can be categorized as contact tasks. In the future of robotics, the interaction with the environment is fundamental and more and more tasks will include and require interaction (Romanelli, 2011).

In this paper, we will focus on the contact task class, in this context, the *wrench capability* in redundant planar manipulators is studied herein. In robotics, although the terms *wrench capability* or *force capability* of a robotic mechanism can be used as synonymous, in the current document is preferably used the term *wrench capability* because is broader.

The *wrench capability* is defined as the maximum wrench that can be applied (or sustained) for a given pose, based on the limits of its actuators (Nokleby *et al.*, 2005). The *wrench capability* of a robotic mechanism is dependent on its design, posture, actuation limits and redundancies (Weihmann *et al.*, 2011). By considering all possible directions of the applied wrench, a wrench capability plot can be generated for the given pose (Nokleby *et al.*, 2005, 2007).

The *wrench capability* analysis is essential for the design and performance evaluation of manipulators. For a given pose, the end effector is required to move with a desired force/moment and to sustain a specified wrench. Thus, the information of the joint torques that will produce such conditions could be investigated. This study is referred as the inverse static force problem. An extended problem can be formulated as the analysis of the maximum force or wrench that the end effector can apply in the force or wrench spaces.

Parallel manipulators usually consist in a moving platform connected to a fixed platform by several legs in order to transmit the movement (Tsai, 1999). when the number of actuations in a parallel manipulator is higher than the dimension

of its taskspace, it is said a redundant parallel manipulator (RPPM). **Redundant actuation** in parallel manipulators can be divided into three categories. The first category features actuating some of the passive joints within the branches of a parallel manipulator. *The second category of redundant manipulators are those that feature additional branches beyond the minimum necessary to actuate the device.* Finally, the third category of redundantly-actuated parallel manipulators are devices that are a hybrid of the first two categories (Nokleby *et al.*, 2005). Herein only manipulators within the second category of redundancies are treated, and in order to create a comparative basis three different manipulators were studied, a 3RRR non-redundant **planar parallel manipulator (PPM)** and two **redundant planar parallel manipulators (RPPM)**, a 4RRR, and a 5RRR.

In parallel manipulators with redundant actuation, the solution to the inverse force problem (given the desired wrench to be applied by the platform, what are the required joint torques/forces) no longer has a unique solution. An infinity of possible solutions exists to the inverse force problem. This infinity of possible solutions allows the joint torques/forces to be optimized (Nokleby *et al.*, 2005).

The main objective of this study is to solve the inverse force problem, and obtain the force capability polytopes of different parallel manipulators in static or quasi-static conditions. The studied manipulators in the current paper are a 3RRR non-redundant planar parallel manipulator (PPM) and two redundant planar parallel manipulators (RPPM), a 4RRR, and a 5RRR.

2. GEOMETRIC REPRESENTATION OF THE STUDIED MANIPULATORS

In this paper three parallel manipulators are studied, a 3RRR non-redundant planar parallel manipulator and two redundant parallel manipulators, a 4RRR and a 5RRR. The planar parallel manipulators studied herein are shown in Figs. 1(a), 1(b) and 1(c) respectively. In these parallel manipulators, the fixed and moving platforms are formed by regular polygons with 3, 4 and 5 sides joined by using the same number of legs. Each leg has three rotational joints whose axes are perpendicular to the $(x - y)$ plane, and the first joint in each leg is actuated.

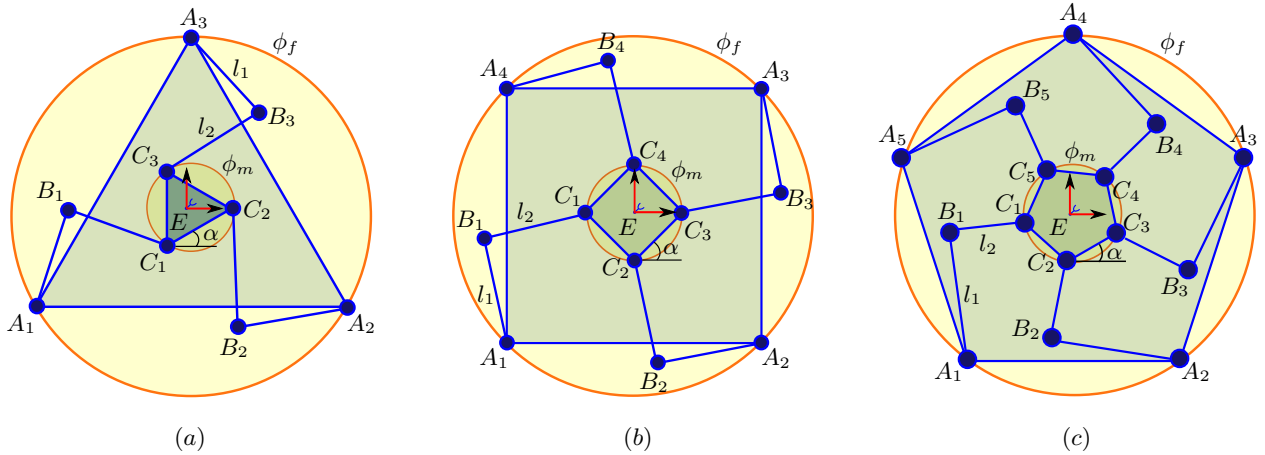


Figure 1. Geometric representation of the studied PPM's: (a) 3RRR, (b) 4RRR, (c) 5RRR

Notice that in the planar parallel manipulators shown in Figs. 1(a), 1(b) and 1(c), all them have a circular envelopment around their moving and fixed platform. This circular envelopment was constructed in order to normalize the geometry of the studied manipulators and the influence that the number of legs can have over its force capability. In this study, the moving and fixed platform of the parallel manipulators are circumscribed about a circle with the same diameter. In all the studied manipulators were used the diameters $\phi_f = 1$ [m] and $\phi_m = 0.3$ [m] for the circles surrounding the fixed and moving platform respectively.

In all the studied manipulators herein, the legs are formed by two links with lengths l_1 and l_2 , the end effector of the manipulator is located in the geometrical center of the moving platform (E) and the angle of orientation (α) represents the orientation of the moving platform (Mejia *et al.*, 2014a,b). Here, the link lengths in each leg were specified as $l_1 = l_2 = 0.6$ [m], the end effector of the manipulator is located in $E = (0, 0)$ [m], the moving platform is oriented in $\alpha = 0^\circ$ and the maximum torque for each actuated joint of the manipulator is imposed as $\tau_{max} = \pm 100$ [Nm].

3. STATICS OF ROBOTIC MECHANISMS

In the static analysis of robotic mechanisms, the goal is to determine the force and moment requirements for the joints in relation to the wrenches applied at the end effector. It is possible to apply forces and moments at the joints of the mechanism to analyze the wrenches obtained at the end effector, or to apply external wrenches at the end effector to calculate the forces and moments required at the joints to balance these external forces.

There are several methodologies which allow us to obtain a complete static analysis of robotic mechanisms; however, in this paper the formalism presented by Davies (Davies, 1983c) is used as the primary mathematical tool to analyze the mechanisms statically. The Davies method appears in many publications in the literature and further explanations regarding its use can be found in (Davies, 1983a,b,c; Cazangi, 2008; Weihmann *et al.*, 2011, 2012; Mejia *et al.*, 2015).

The Davies method provides a systematic way to relate the joint forces and moments in closed kinematic chains (Cazangi, 2008). This method is based on **graph theory**, **screw theory** and the **Kirchhoff cut-set law** and it can be used to obtain the statics of a robotic mechanism as a matricial expression (Mejia *et al.*, 2015). The Davies method for static analysis can be described in a simplified way through the following steps:

1. Given a mechanism, draw its kinematic chain identifying all of its “ n ” links and “ e ” direct couplings.
2. Draw the coupling graph “ G_C ” using the links of the mechanism as the vertices of the graph, and the joints of the mechanism as the edges of the graph.
3. Generate the action graph “ G_A ” from “ G_C ” through unfolding single actions from direct couplings. In this step, each edge of “ G_C ” representing a coupling is replaced in “ G_A ” by c constraint edges in parallel.
 - Assign positive directions to each edge with an arrow pointing from the minor to major vertex.
 - Locate the number of cuts ($k = n - 1$) and chords ($l = e - n + 1$) in the action graph and depict them.
4. Write the cut-set matrix $[Q_N]_{k,e}$ with suitable signs.
5. Write a wrench $\$J$ for each edge from G_A as follows:

$$\$J = \begin{bmatrix} -y \\ 1 \\ 0 \end{bmatrix} J_{F_x} + \begin{bmatrix} x \\ 0 \\ 1 \end{bmatrix} J_{F_y} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} J_{M_z} \quad (1)$$

6. Replace each wrench $\$J$ in the cut-set matrix $[Q_N]_{k,e}$ in order to obtain the generalized action matrix $[A_N]_{3k,e}$.
7. Operate algebraically the generalized action matrix $[A_N]_{3k,e}$ in order to statically solve the system.

Once the Davies method has been applied in order to obtain the backward statics in planar parallel manipulators, it is possible to represent the wrenches in the end effector $[F_x, F_y, M_z]^T$ as a generalized function of a coefficient matrix $[A]$ and a vector containing the N primary actions $[\tau_{A_1}, \tau_{A_2}, \dots, \tau_{A_N}]^T$ as shown in Eq. (2). In this equation the $a_1, \dots, a_{3N}$ elements represent kinematic expressions as a function of the manipulator joint positions, and the $\tau_{A_1}, \tau_{A_2}, \dots, \tau_{A_N}$ elements represent the primary actions of the actuated joints.

$$\begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & \dots & a_N \\ a_{N+1} & a_{N+2} & \dots & a_{2N} \\ a_{2N+1} & a_{2N+2} & \dots & a_{3N} \end{bmatrix} \cdot \begin{bmatrix} \tau_{A_1} \\ \tau_{A_2} \\ \vdots \\ \tau_{A_N} \end{bmatrix} \quad (2)$$

In this way, the static model of the the 3RRR, 4RRR and 5RRR PPM's studied herein, can be expressed in a generalized way as shown in Eqs. (3), (4) and (5) respectively.

$$\begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \cdot \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} \cdot \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \end{bmatrix} \cdot \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \end{bmatrix} \quad (5)$$

4. CONSTRAINED OPTIMIZATION PROBLEM

The mathematical optimization (**MO**) is formally defined as a process which involves: (i) the formulation and (ii) the solution of a constrained optimization problem of the general mathematical form (Storn *et al.*, 2005):

$$\text{minimize } f(x), \quad x = [x_1, x_2, x_3, \dots, x_n]^T \in \mathbb{R}^n \quad (6)$$

$$\text{subject to } h_i(x) = 0, \quad i = 1, \dots, n_e \quad (7)$$

$$g_j(x) \leq 0, \quad j = 1, \dots, n_g \quad (8)$$

where $f(x)$, $h_i(x)$ and $g_j(x)$ are scalar functions of the real column vector x .

In general, constrained optimization problems can have several local minima and the existence of a single global minimum is guaranteed only under certain circumstances. For nonlinear, multimodal, multivariate functions, this is not an easy task. In addition, some functions may have discontinuities, and thus derivative information is not easy to obtain. This may pose various challenges to many traditional methods and the use of a heuristic or meta-heuristic method is necessary in order to solve this kind of problems.

The aim of the optimization problem studied in this paper, is to maximize the force F_{app} applied or sustained for the manipulator in a given direction (α_d), while the imposed moment at the end effector of the manipulator (M_k) is changed within a prescribed interval $[M_{k_{min}} \leq M_k \leq M_{k_{max}}]$. The optimization problem can be described as the process in which the torque in the actuators $\tau_{A1}, \tau_{A2}, \dots, \tau_{AN}$ must be optimized in order to maximize the pure force and minimize the error in the imposed value for the moment at the end effector of the manipulator. This optimization must be done in all possible directions given for the angle α_d . In our simulations, were used 360 repetitions (one per angle $[0^\circ; 359^\circ]$), and the imposed moment is changed in each iteration of α_d within the imposed interval $[M_{k_{min}} \leq M_k \leq M_{k_{max}}]$.

4.1 Objective function

The objective function used in the optimization process is shown in Eq. (9). In this equation the terms F_x and F_y are the components of the force at the end effector of the manipulator, α_d is the desired angle of the application of the force, α_o is the obtained angle of the application of the force F_{app} as a function of the F_x and F_y components, M_z is the moment obtained at the end effector of the manipulator, M_k is the constant moment imposed at the end effector of the manipulator, N is the number of legs of the manipulator ($N = 3$ for the 3RRR PPM, $N = 4$ for the 4RRR RPPM and $N = 5$ for the 5RRR RPPM). Finally the “P” term is the penalization of the objective function.

$$F_{obj} = \left| \frac{\alpha_d - \alpha_o}{\alpha_d} \right| + \left| \frac{\tau_{max}}{N \cdot l_1 \cdot \sqrt{F_x^2 + F_y^2}} \right| + \left| \frac{M_z - M_k}{M_k} \right| + P \quad (9)$$

In Eq. (9), the $|(\alpha_d - \alpha_o)/\alpha_d|$ term minimizes the normalized error between the obtained and desired force direction, the $|\tau_{max}/(N \cdot l_1 \cdot \sqrt{F_x^2 + F_y^2})|$ term maximizes the normalized force obtained, and the $|(M_z - M_k)/M_k|$ term minimizes the normalized error between the obtained moment and the desired moment at the end effector of the manipulator.

The penalization term “P” included in Eq. (9) is activated when the condition $[\tau_{min} \leq \tau_{An} \leq \tau_{max}]$ is not satisfied, this condition is imposed as the maximum admissible torque in the actuators τ_{An} . In the present paper were used $\tau_{min} = -100$ [Nm] and $\tau_{max} = 100$ [Nm].

4.2 Differential Evolution (DE) algorithm

In order to solve the problem regarding the global optimization, an evolutionary algorithm known as Differential Evolution (DE) was used. DE is a very simple population based, stochastic function minimizer and very powerful at

the same time. This algorithm is commonly accepted as one of the most successful algorithms for the global continuous optimization problem (Weihmann *et al.*, 2012).

DE optimizes a problem by maintaining a population of candidate solutions, and creating new candidate solutions by combining existing ones, according to its simple formula, and then keeping whichever candidate solution has the best score or fitness on the optimization problem at hand. In this way, the optimization problem is treated as a black box that merely provides a measure of quality given a candidate solution, and therefore, the gradient is not needed (Weihmann *et al.*, 2012).

The performance of the DE algorithm is sensitive to the mutation strategy and respective control parameters, such as the population size (NP), crossover rate (CR), and the mutation factor (MF). The best settings for control parameters can change for different optimization problems. (Weihmann *et al.*, 2011). In this study the parameters $NP = 30$, $CR = 0.8$, and $MF = 0.5$ were used, as suggested in (Storn and Price, 2005), and the maximum iteration number was established in 4000.

5. WRENCH CAPABILITY OPTIMIZATION

The optimization of the objective function shown in Eq. (9) by using the Differential Evolution (DE) algorithm allow us to know the maximum force (F_{app}) with a prescribed moment (M_k) that can be applied or sustained in a given direction (α_d). If all the possible directions of the desired angle (α_d) are considered as $0^\circ \leq \alpha_d \leq 360^\circ$, a force capability polygon can be constructed as a polar representation of the maximum force with a prescribed moment at the end effector of the manipulator. Considering the imposed moment $M_k = 0$, the force capability polygon for the 3RRR PPM is obtained as shown in Fig. 2(a), the force capability polygon for the 4RRR RPPM is obtained as shown in Fig. 2(b) and the force capability polygon for the 5RRR RPPM is obtained as shown in Fig. 2(c).

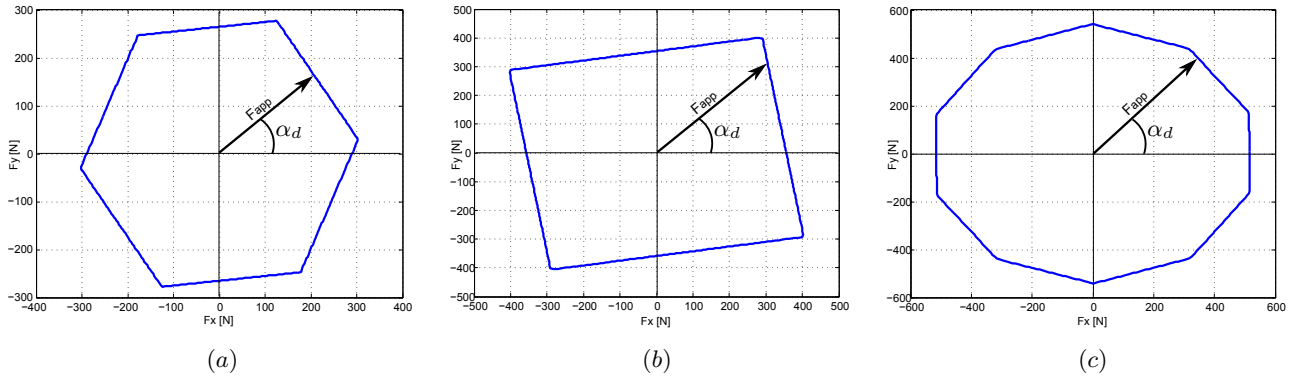


Figure 2. Force capability polygons for: (a) 3RRR PPM, (b) 4RRR RPPM and (c) 5RRR RPPM

5.1 Wrench capability polytopes in parallel manipulators

Following the same strategy that we used previously, it is possible to obtain different wrench capability polygons for different values of the imposed moment at the end effector of the manipulator. If a three-dimensional representation of several wrench capability polygons is plotted, a complete mapping of the wrench capability at the end effector of the manipulator can be obtained. This kind of graphic representation is called the **wrench capability polytope**. The wrench capability polytope of the studied manipulators are shown in Figs. 3, 4 and 5. In Fig. 3 the wrench capability polytope of the 3RRR PPM is shown, In Fig. 4 the wrench capability polytope of the 4RRR RPPM is shown, and finally, in Fig. 5, the wrench capability polytope of the 5RRR RPPM is shown.

From Figs. 3, 4 and 5 it is possible to observe that the wrench capability polytopes are really polyhedrons. These polyhedrons have the property that the number of edges constituting them, can be calculated as a function of the number of legs (N) connecting the moving and the fixed platform in the manipulator as shown in Eq. (10). In Eq. (10) $E_p(N)$ is the number of edges of a polytope as a function of the number of legs (N) in the studied manipulators. In this way the 3RRR PPM containing $N = 3$ legs, give as a resultant polytope, a polyhedron with $E_p = N * (N - 1) = 3 * (3 - 1) = 6$ edges. In a similar way the 4RRR and the 5RRR RPPM's give as result a polyhedron with $E_p = 12$ and $E_p = 20$ respectively.

$$E_p(N) = N * (N - 1) \quad (10)$$

At this point it must be emphasized that Eq. (10) cannot be generalized, due that the wrench capability polytopes of a robotic mechanism are dependent on its design, posture, actuation limits and redundancies. These dependencies could

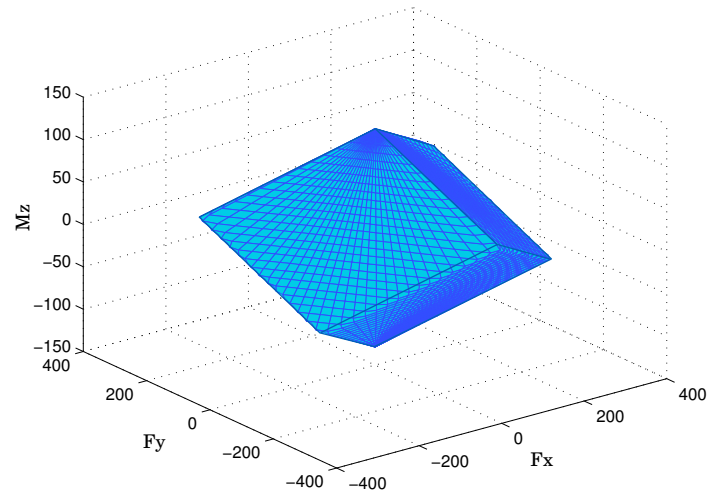


Figure 3. Force capability polytope for the studied 3RRR PPM

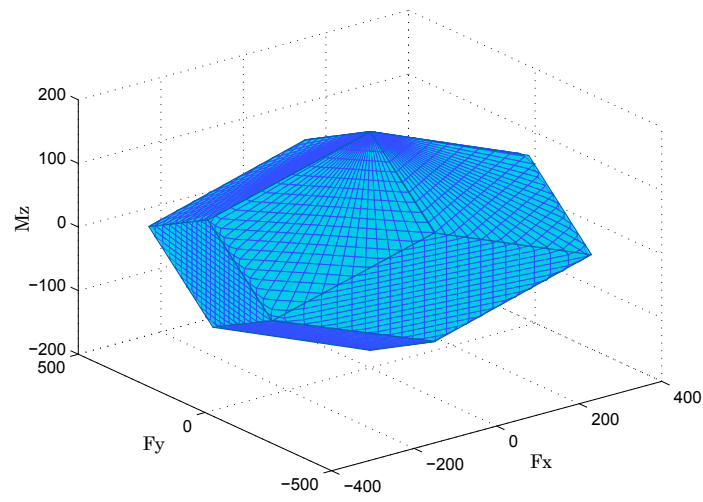


Figure 4. Force capability polytope for the studied 4RRR RPPM

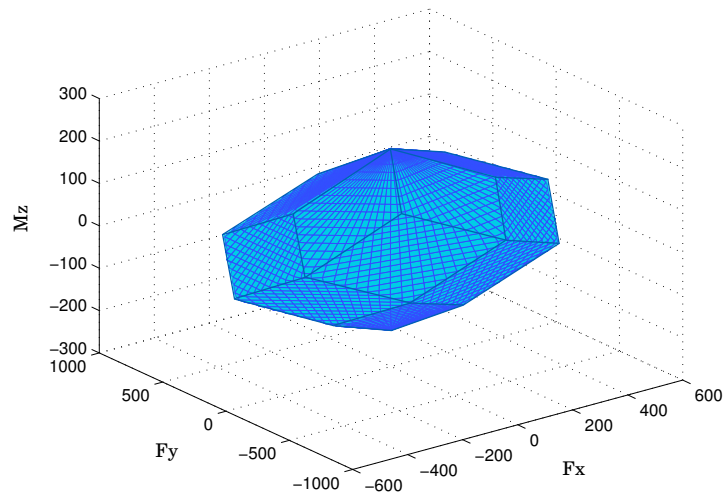


Figure 5. Force capability polytope for the studied 5RRR RPPM

make that in presence of a singularity, for instance, the wrench capability polytopes change. The Eq. (10) presented in this paper must be used only for the described examples.

6. CONCLUSIONS

This paper presented a method to obtain the force capability polytope in three different planar parallel manipulators, a 3RRR non-redundant planar parallel manipulator and two redundant parallel manipulators, a 4RRR, and a 5RRR. The method requires the optimization of the torque in the actuated joints of the manipulator by imposition of a moment at the end effector of the manipulator. The optimization problems were solved using *DE* algorithms.

The force capability polytopes obtained herein are composed by the superposition of several force capability polygons and depend on several parameters as the manipulator's kinematic position, orientation, working mode and redundancies.

The present study may be extended in various ways. Manipulators with different DOFs, kinematic chains and including dynamic behavior may be studied, and the minimization of the force and the maximization of the moment may be considered in future researches.

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