COBEM2015

ABCM Associação Brasileira de Engenharia e Ciências Mecânicas

23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

INFLUENCE OF THE ASSEMBLY MODE ON THE FORCE CAPABILITY IN PARALLEL MANIPULATORS

Leonardo Mejia
Julio Frantz
Henrique Simas
Daniel Martins
Federal University of Santa Catarina (UFSC), Florianópolis, Brazil, CEP: 88040-900.
leonardo.mejia.rincon@posgrad.ufsc.br
julio.frantz@posgrad.ufsc.br
henrique.simas@posgrad.ufsc.br
daniel.martins@posgrad.ufsc.br

Abstract. *This paper shows how different assembly modes in parallel manipulators can influence their force capability. Herein, the force capability problem in parallel manipulators is solved by the optimization of its static equations and the objective function used to optimize the force capability is defined by employing the screw theory and the Davies method as primary mathematical tools. In order to solve the problem regarding the global optimization, an evolutionary algorithm known as Differential Evolution (DE) is used. Finally, some force capability polygons are obtained for different assembly modes in two different planar parallel manipulators in order to analyse their influence.*

Keywords: *Force capability, assembly mode, parallel manipulators, optimization*

1. INTRODUCTION

The task space capabilities of a manipulator to perform motion and/or to exert forces are of fundamental importance in robotics (Chiacchio *et al.*, 1996). In the design phase, their evaluation can be useful to determine the structure and the size of a manipulator that best fit the designer's requirements. In the operational phase, they can be used to find a better configuration or a better operation point for a manipulator to perform a given task. (Chiacchio *et al.*, 1996).

*The **force capability** of a robotic mechanism is defined as the maximum wrench that can be applied (or sustained) for a given pose, based on the limits of its actuators* (Nokleby *et al.*, 2005). The force capability of a robotic mechanism is dependent on its design, posture, actuation limits and redundancies (Weihmann *et al.*, 2011) and by considering all possible directions of the applied wrench, a force capability plot can be generated for the given pose (Nokleby *et al.*, 2005, 2007). This force capability plot is called *the **force capability polygon***, and is a polar representation of the force capability in all directions.

The main objective of this paper is to study the influence of different assembly modes in parallel manipulators on their force capability. In order to carry out this study two cases were considered, the first is a *3$\underline{R}$RR* planar parallel manipulator and the second is a *4$\underline{R}$RR* redundant planar parallel manipulator.

2. GEOMETRIC REPRESENTATION OF THE STUDIED MANIPULATORS

Parallel manipulators usually consist in a moving platform connected to a fixed platform by several legs in order to transmit the movement (Tsai, 1999). In this paper two parallel manipulators are studied, a *3$\underline{R}$RR Planar Parallel Manipulators (PPM)* and a *4$\underline{R}$RR Redundant Planar Parallel Manipulators (RPPM)*. In these parallel manipulators, the fixed and moving platforms are connected by using three and four legs respectively. Each leg has three rotational joints whose axes are perpendicular to the $(x - y)$ plane, and the first joint in each leg is actuated. The *3$\underline{R}$RR* PPM and the *4$\underline{R}$RR* RPPM studied herein are shown in Figs. 1(a) and 1(b) respectively.

In the first studied manipulator (the 3$\underline{R}$RR planar parallel manipulator), the moving and fixed platforms are formed by equilateral triangles with sides l_m and l_f respectively. The legs are formed by two links with lengths l_1 and l_2, the end effector of the manipulator is located in the geometrical center of the moving platform (E) and the angle of orientation (ϕ) represents the orientation of the moving platform (Mejia *et al.*, 2014a). In the studied manipulator the link lengths in each leg are specified as $l_1 = l_2 = 0.6$ [m], the moving platform edge and the fixed platform edge are defined as $l_m = 0.5196$ [m] and $l_f = 1.7321$ [m], the end effector of the manipulator is located in $E = (1\ [\text{m}],\ 1\ [\text{m}])$, the moving platform is oriented in $\phi = 0°$ and the maximum torque for each actuated joint of the manipulator is imposed as $\tau_{max} = \pm\ 1$ [Nm].

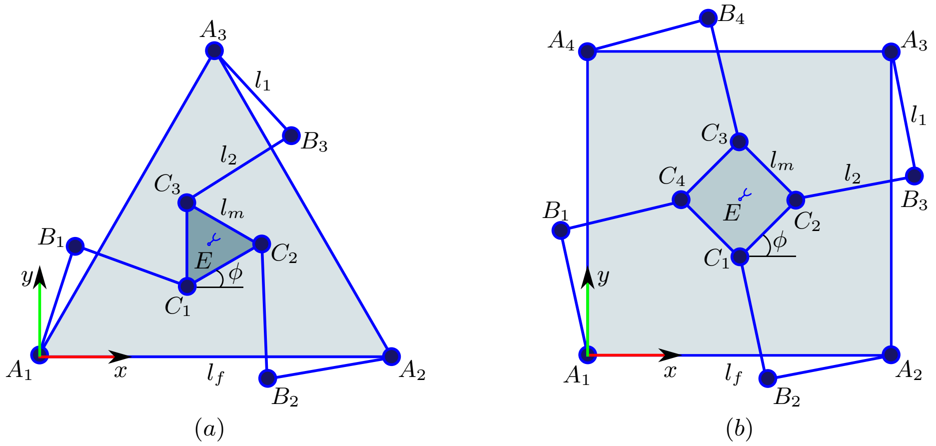


Figure 1. Geometric representation of the studied PPM's: (a) 3RRR, (b) 4RRR

In the other hand, the second studied manipulator (the 4RRR redundant planar parallel manipulator), is a manipulator with four legs and the moving and fixed platforms are formed by squares with sides l_m and l_f respectively. Their legs are formed by two links with lengths l_1 and l_2, the end effector of the manipulator is located in the geometrical center of the moving platform (E) and the angle of orientation (ϕ) represents the orientation of the moving platform (Mejia *et al.*, 2014b). In this studied manipulator, the link lengths in each leg are specified as $l_1 = l_2 = 0.6$ $[\mathrm{m}]$, the moving platform edge and the fixed platform edge are defined as $l_m = 0.4243$ $[\mathrm{m}]$ and $l_f = 1.4142$ $[\mathrm{m}]$, the end effector of the manipulator is located in $E = (1\ [\mathrm{m}],\ 1\ [\mathrm{m}])$, the moving platform is oriented in $\phi = 0°$ and the maximum torque capability for each actuated joint of the manipulator is imposed as $\tau_{max} = \pm\ 1$ $[\mathrm{Nm}]$.

3. STATICS OF ROBOTIC MECHANISMS

In the static analysis of robotic mechanisms, the goal is to determine the force and moment requirements for the joints in relation to the wrenches applied at the end effector. It is possible to apply forces and moments at the joints of the mechanism to analyze the wrenches obtained at the end effector, or to apply external wrenches at the end effector to calculate the forces and moments required at the joints to balance these external forces.

There are several methodologies which allow us to obtain a complete static analysis of robotic mechanisms; however, in this paper the formalism presented by Davies (Davies, 1983c) is used as the primary mathematical tool to analyze the mechanisms statically. The Davies method appears in many publications in the literature and further explanations regarding its use can be found in (Davies, 1983a,b,c; Cazangi, 2008; Weihmann *et al.*, 2011, 2012; Mejia *et al.*, 2015).

The Davies method provides a systematic way to relate the joint forces and moments in closed kinematic chains (Cazangi, 2008). This method is based on ***graph theory***, ***screw theory*** and the ***Kirchhoff cut-set law*** and it can be used to obtain the statics of a robotic mechanism as a matricial expression (Mejia *et al.*, 2015). The Davies method for static analysis can be described in a simplified way through the following steps:

1. Given a mechanism, draw its kinematic chain identifying all of its *"n"* links and *"e"* direct couplings.
2. Draw the coupling graph **"G_C"** using the links of the mechanism as the vertices of the graph, and the joints of the mechanism as the edges of the graph.
3. Generate the action graph **"G_A"** from **"G_C"** through unfolding single actions from direct couplings. In this step, each edge of **"G_C"** representing a coupling is replaced in **"G_A"** by **"c"** constraint edges in parallel.
    - Assign positive directions to each edge with an arrow pointing from the minor to major vertex.
    - Locate the number of cuts ($k = n - 1$) and chords ($l = e - n + 1$) in the action graph and depict them.
4. Write the cut-set matrix $[Q_N]_{k,e}$ with suitable signs.
5. Write a wrench $\$_J$ for each edge from G_A.
6. Replace each wrench $\$_J$ in the cut-set matrix $[Q_N]_{k,e}$ in order to obtain the generalized action matrix $[A_N]_{3k,e}$.
7. Operate algebraically the generalized action matrix $[A_N]_{3k,e}$ in order to statically solve the system.

Once the Davies method has been applied in order to obtain the backward statics in PPM's, it is possible to represent the wrenches in the end effector $[F_x, F_y, M_z]^T$ as a generalized function of a coefficient matrix [A] and a vector containing the N primary actions $[\tau_{A_1}, \tau_{A_2}, ..., \tau_{A_N}]^T$ as shown in Eq. (1). In this equation the $a_1, \ldots, a_{3N}$ elements represent kinematic expressions as a function of the manipulator joint positions, and the $\tau_{A_1}, \tau_{A_2}, ..., \tau_{A_N}$ elements represent the primary actions of the actuated joints.

$$\begin{bmatrix} F_x \\ F_y \\ M_z \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & \cdots & a_N \\ a_{N+1} & a_{N+2} & \cdots & a_{2N} \\ a_{2N+1} & a_{2N+2} & \cdots & a_{3N} \end{bmatrix} \cdot \begin{bmatrix} \tau_{A_1} \\ \tau_{A_2} \\ \vdots \\ \tau_{A_N} \end{bmatrix} \tag{1}$$

4. CONSTRAINED OPTIMIZATION PROBLEM

In simple terms, optimization is an attempt to maximize the desirable properties of a system while simultaneously minimizing the undesirable characteristics (Storn *et al.*, 2005). Formally, mathematical optimization ***(MO)*** is defined as a process which involves: *(i)* the formulation and *(ii)* the solution of a constrained optimization problem of the general mathematical form (Storn *et al.*, 2005):

$$minimize \;\; f(x), \;\; x = [x_1, x_2, x_3, \ldots, x_n]^T \in \mathbb{R}^n \tag{2}$$

$$subject \; to \quad h_i(x) = 0, \quad i = 1, \ldots, n_e \tag{3}$$

$$g_j(x) \leqslant 0, \quad j = 1, \ldots, n_g \tag{4}$$

where $f(x)$, $h_i(x)$ and $g_j(x)$ are scalar functions of the real column vector x.

In general, constrained optimization problems can have several local minima and the existence of a single global minimum is guaranteed only under certain circumstances. For nonlinear, multimodal, multivariate functions, this is not an easy task. In addition, some functions may have discontinuities, and thus derivative information is not easy to obtain. This may pose various challenges to many traditional methods and the use of a heuristic or meta-heuristic method is necessary in order to solve this kind of problems.

The aim of the optimization problem studied in this paper, is to maximize the force F_m applied or sustained for the manipulator in a given direction (θ), while the moment is imposed as a constant (M_k). The optimization problem can be described as the process in which the torque in the actuators $\tau_{A_1}, \; \tau_{A_2}, ..., \tau_{A_N}$ must be optimized in order to maximize the pure force and minimize the error in the imposed value for the moment at the end effector of the manipulator. This optimization must be done in all possible directions given for the angle θ. In our simulations, were used 360 repetitions (one per angle $[0°; 359°]$), and the imposed moment is considered as a constant along all the values of the angle θ.

4.1 Objective function

The objective function used in the optimization process is shown in Eq. (5). In this equation the terms F_x and F_y are the components of the force at the end effector of the manipulator, α_d is the desired angle of the application of the force, α_o is the obtained angle of the application of the force as a function of the F_x and F_y components, M_z is the moment obtained at the end effector of the manipulator, M_k is the constant moment imposed at the end effector of the manipulator, N is the number of legs of the manipulator ($N = 3$ for the 3$\underline{\text{R}}$RR PPM and $N = 4$ for the 4$\underline{\text{R}}$RR PPM) and finally the "P" term is the penalization of the objective function.

$$F_{obj} = \left| \frac{\alpha_d - \alpha_o}{\alpha_d} \right| + \left| \frac{\tau_{max}}{N \cdot l_1 \cdot \sqrt{F_x^2 + F_y^2}} \right| + \left| \frac{M_z - M_k}{M_k} \right| + P \tag{5}$$

In Eq. (5), the $|(\alpha_d - \alpha_o)/\alpha_d|$ term minimizes the normalized error between the obtained and desired force direction, the $\left|\tau_{max}/(N \cdot l_1 \cdot \sqrt{F_x^2 + F_y^2})\right|$ term maximizes the normalized force obtained, and the $|(M_z - M_k)/M_k|$ term minimizes the normalized error between the obtained moment and the desired moment at the end effector of the manipulator.

The penalization term "P" included in Eq. (5) is activated when the condition $[\tau_{min} \leq \tau_{An} \leq \tau_{max}]$ is not satisfied, this condition is imposed as the maximum admissible torque in the actuators τ_{An}. In the present paper were used $\tau_{min} = -1$ [Nm] and $\tau_{max} = 1$ [Nm].

4.2 Diferential Evolution (DE) algorithm

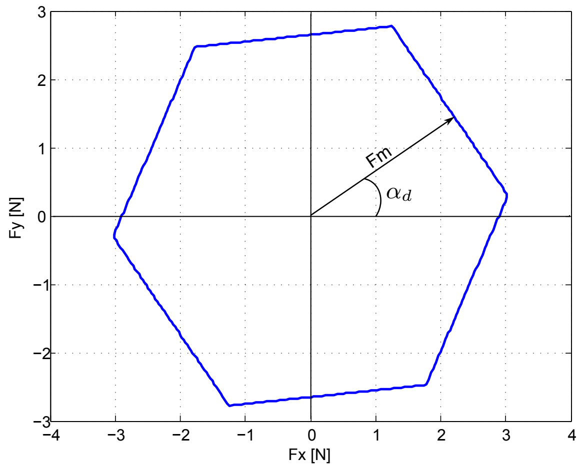
In order to solve the problem regarding the global optimization, an evolutionary algorithm known as Differential Evolution (*DE*) was used. *DE* is a very simple population based, stochastic function minimizer and very powerful at the same time. This algorithm is commonly accepted as one of the most successful algorithms for the global continuous optimization problem (Weihmann *et al.*, 2012).

DE optimizes a problem by maintaining a population of candidate solutions, and creating new candidate solutions by combining existing ones, according to its simple formula, and then keeping whichever candidate solution has the best score or fitness on the optimization problem at hand. In this way, the optimization problem is treated as a black box that merely provides a measure of quality given a candidate solution, and therefore, the gradient is not needed (Weihmann *et al.*, 2012).

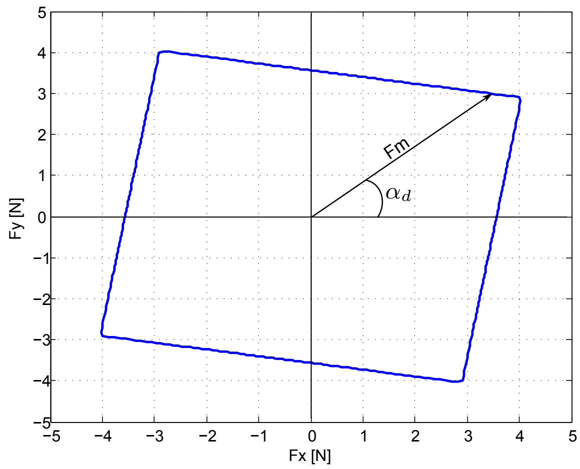
The performance of the DE algorithm is sensitive to the mutation strategy and respective control parameters, such as the population size *(NP)*, crossover rate *(CR)*, and the mutation factor *(MF)*. The best settings for control parameters can change in different optimization problems, and the same functions with different requirements, for consumption time and accuracy (Weihmann *et al.*, 2011). In this study the parameters $NP = 30$, $CR = 0.8$, and $MF = 0.5$ were used, as suggested in (Storn and Price, 2005), and the maximum iteration number was established in 4000.

5. FORCE CAPABILITY OPTIMIZATION

The optimization of the objective function shown in Eq. (5) by using the Differential Evolution *(DE)* algorithm allow us to know the maximum force (F_m) with a prescribed moment $(M_k = 0)$ that can be applied or sustained in a given direction (α_d). If all the possible directions of the desired angle (α_d) are considered as $0° \leq \alpha_d \leq 360°$, a force capability polygon can be constructed as a polar representation of the maximum force with a prescribed moment at the end effector of the manipulator. The force capability polygon for the 3RRR PPM (see Fig. 1(a)) is shown in Fig. 2(a), and the force capability polygon for the 4RRR RPPM (see Fig. 1(b)) is shown in Fig. 2(b).



(a)



(b)

Figure 2. Force capability polygons for: (a) 3RRR PPM, (b) 4RRR RPPM

5.1 Influence of the assembly mode on the force capability in the studied PPM's

The direct and inverse kinematics problem of parallel robots have been study in many papers to define first the maximal numbers of solution for each problem and secondly to characterize the joint space and workspace. The moving platform can admit several positions and orientations in the workspace for one given set of input joint values. Conversely, the robot can admit several input joint values for a given moving platform configurations (Chablat *et al.*, 2011).

The notion of ***assembly modes*** has been defined to represent the different solutions to the direct kinematic problem while the notion of ***working mode*** has been introduced to separate the solutions to the inverse kinematic problem (Chablat and Wenger, 1998). In this paper we only studied the influence of the *assembly mode* on the force capability in PPM's. The eight possible assembly modes (AM_1 to AM_8) for the 3RRR PPM are shown in Fig. 3, while the sixteen possible assembly modes (AM_1 to AM_{16}) for the 4RRR RPPM are shown in Fig. 4.

Following the same methodology previously used in order to obtain the force capability polygons shown in Fig. 2(a) and 2(b), it is possible to obtain a force capability polygon for each new assembly mode previously shown in Figs. 3 and 4 respectively. These new force capability polygons are now shown in Figs. (5) and (6) for the 3RRR and 4RRR PPM's.

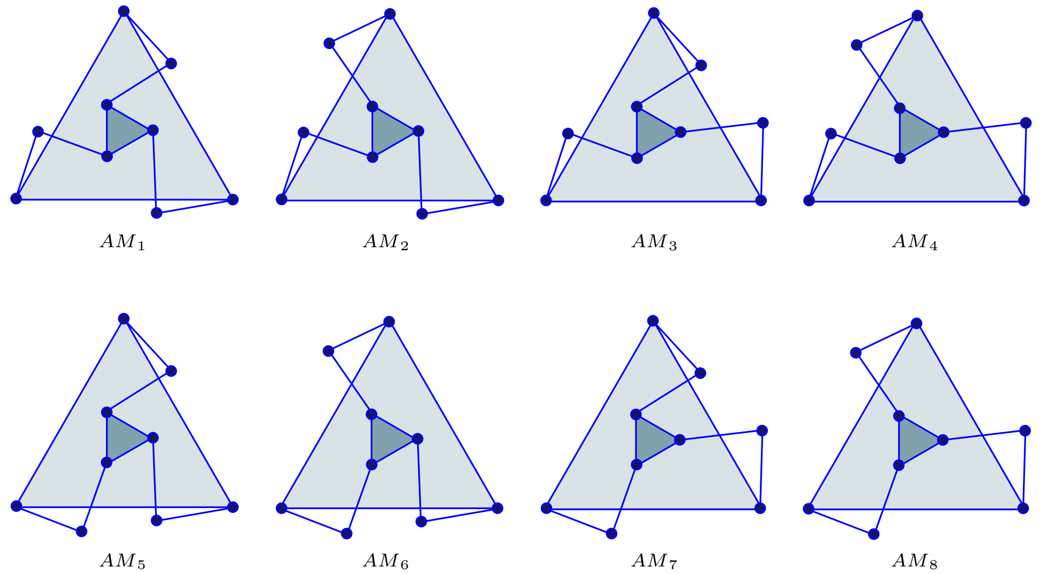


Figure 3. The eight possible assembly modes for the 3$\underline{\text{R}}$RR PPM

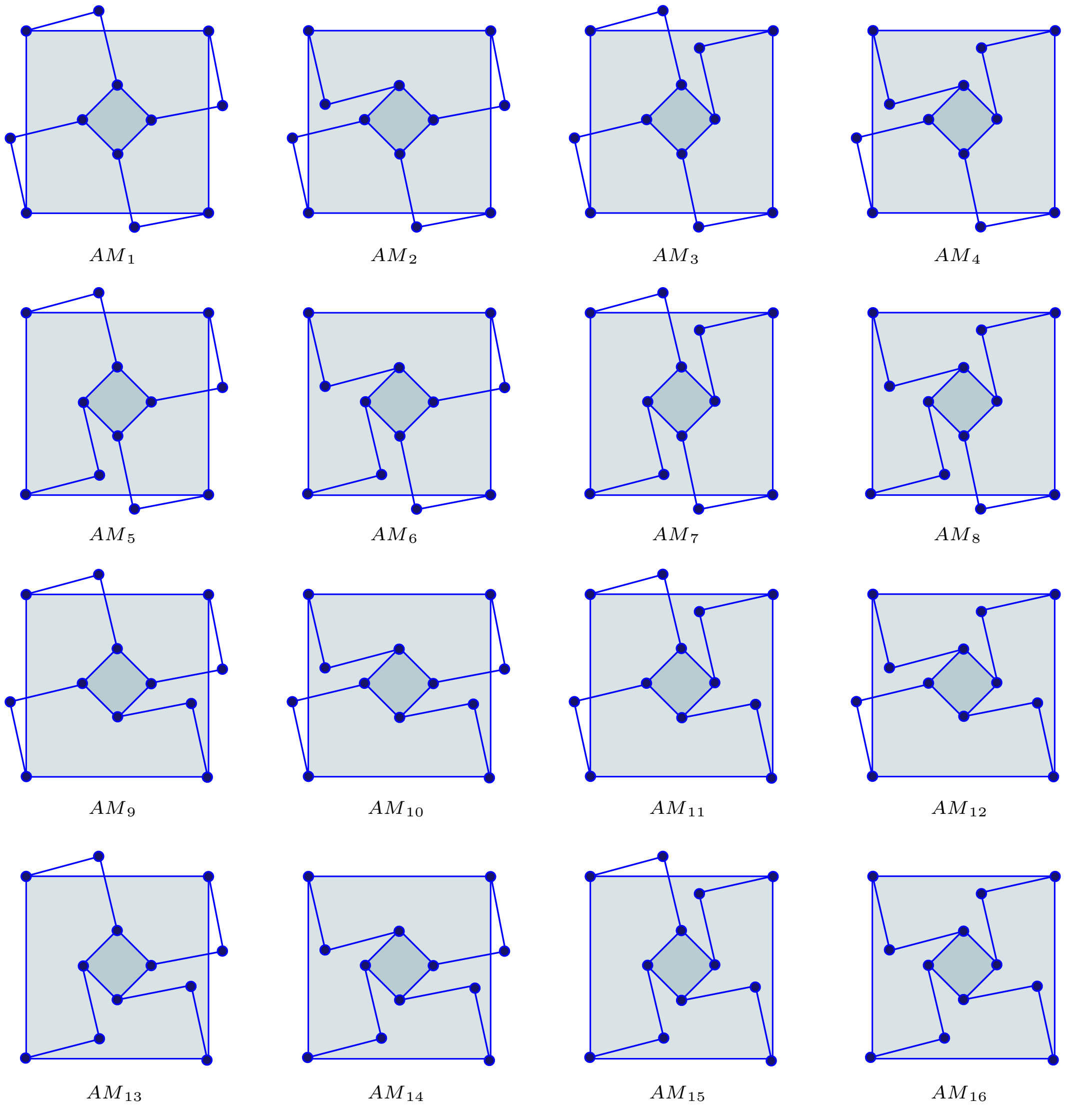


Figure 4. The sixteen possible assembly modes for the 4$\underline{\text{R}}$RR PPM

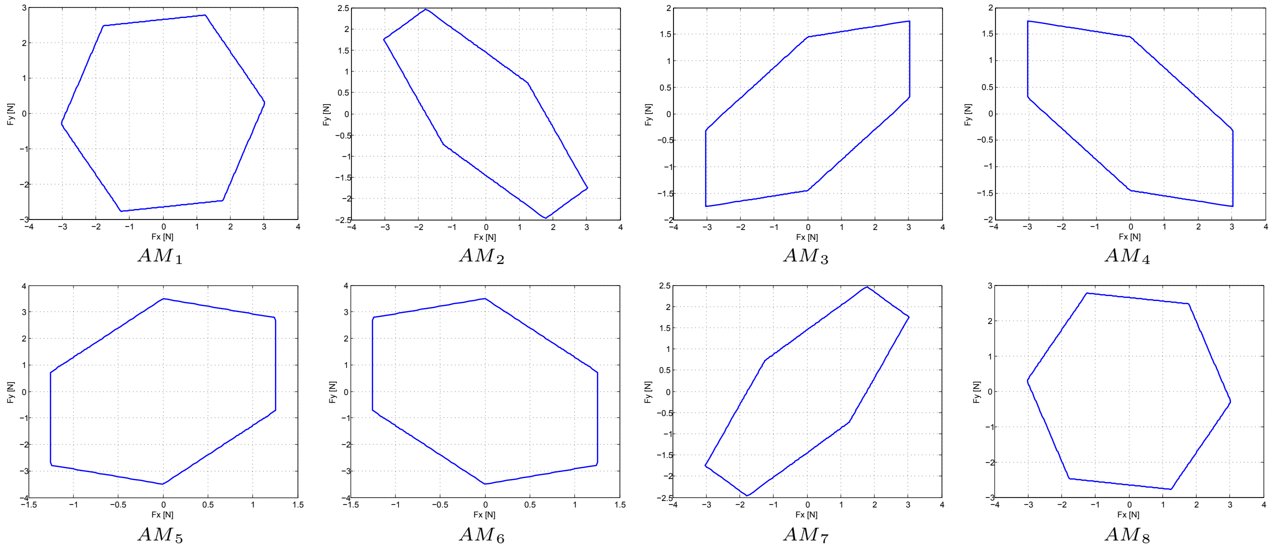


Figure 5. Force capability polygons for the eight assembly modes in the 3RRR PPM

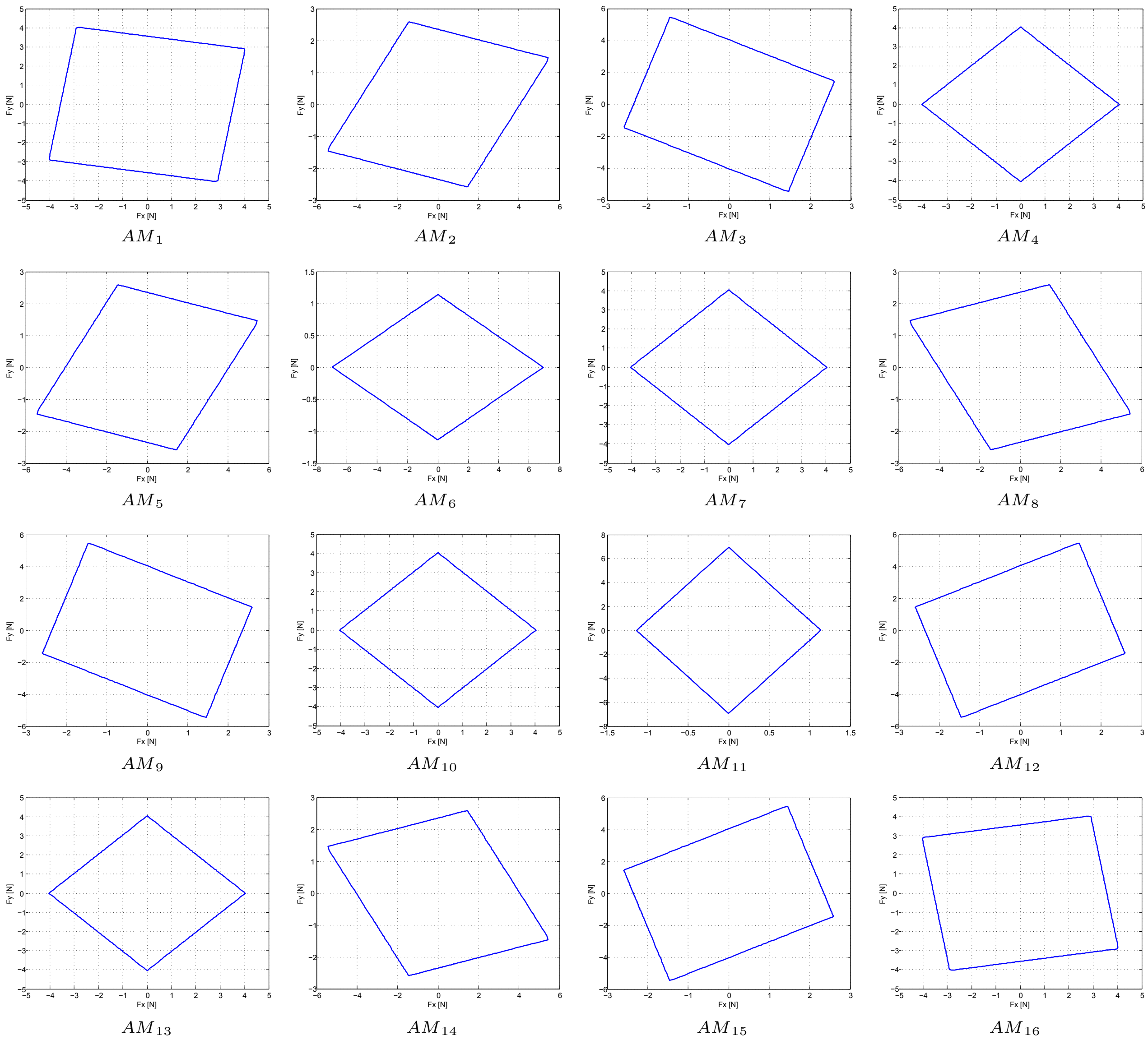
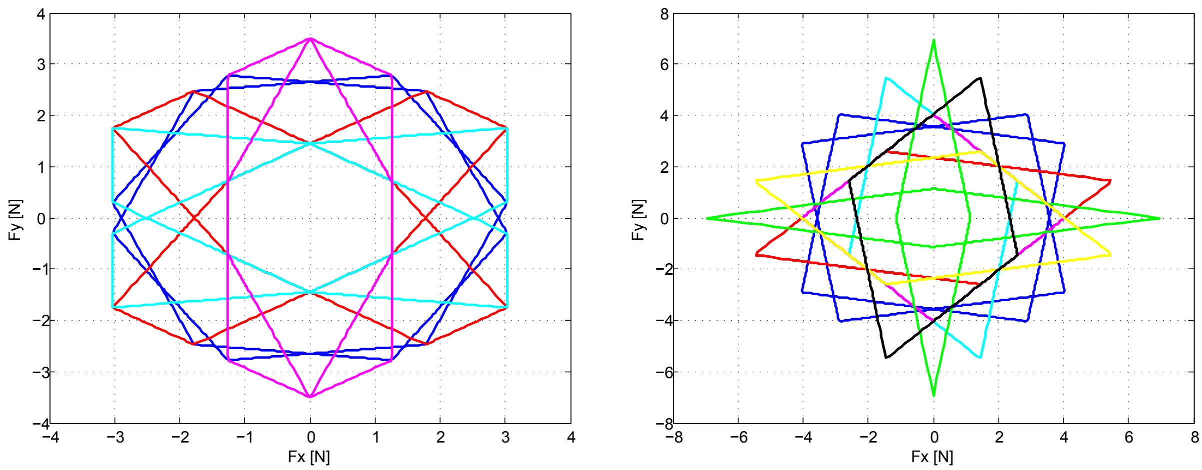


Figure 6. Force capability polygons for the sixteen assembly modes in the 4RRR PPM

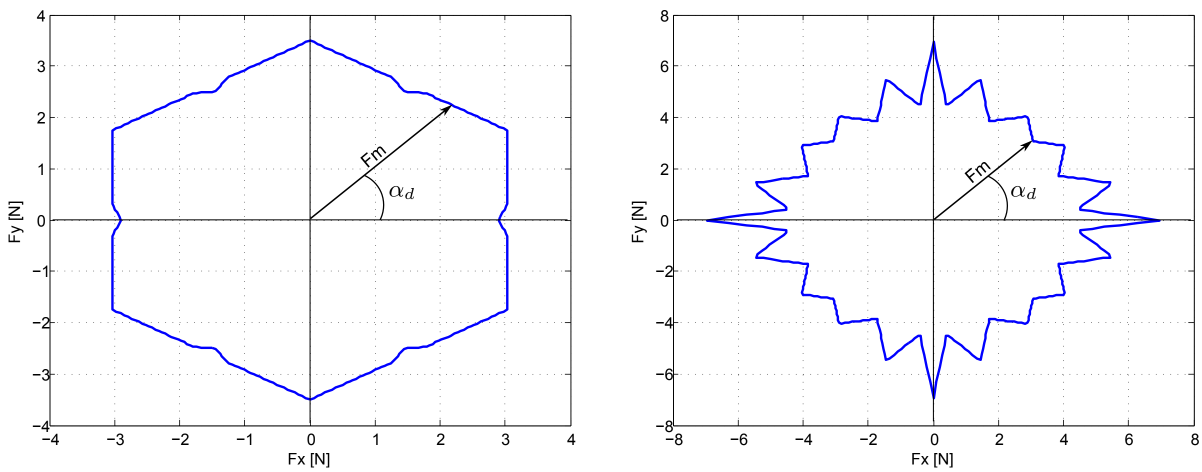
From Figs. 3 and 4, it is possible to observe that parallel manipulators have several symmetries that influence in a direct way the geometrical representation of the force capability polygons. In order to explain this phenomenon, suppose the assembly modes AM_1 and AM_8 shown in Fig. 3. Observe that these manipulators have all their legs located in an opposite direction, and as result of this kind of variation, their corresponding force capability polygons have a symmetry of reflection (as shown in AM_1 and AM_8 in Fig. 5). Other kind of phenomenon found when the assembly mode in parallel manipulators is changed is the superposition, In order to explain this phenomenon, suppose the force capability polygons AM_{12} and AM_{15} shown in Fig. 6. Observe that these results are exactly the same (they can be superposed) and they are corresponding to the assembly modes of the manipulators shown in AM_{12} and AM_{15} shown in Fig. 4 where their legs are rotated around the z axis by $-90°$.

In order to improve the graphical presentation of the force capability polygons shown in Figs. (5) and (6), they are now plotted again, but this time using overlapping of the force capability polygons on the same reference frame. The result of the overlapping is shown in Figs. 7(a) and 7(b) for the force capability polygons previously obtained in Figs. 5 and 6. Finally, a union of the overlapped force capability plots shown in Fig. 7(a) and 7(b) is done in order to obtain the generalized force capability polygon for the 3RRR and 4RRR PPM's. This union is done considering the maximum force F_m that can be obtained for each assembly mode of the manipulators. The generalized force capability polygons obtained for the 3RRR and 4RRR PPM's are shown in Figs 8(a) and 8(b).



(a) (b)

Figure 7. Overlapping plots for the force capability polygons in: (a) the 3RRR PPM, (b) the 4RRR RPPM



(a) (b)

Figure 8. Generalized force capability polygons in: (a) the 3RRR PPM, (b) the 4RRR RPPM

6. CONCLUSIONS

This paper studied the influence that several variations in the assembly mode in parallel manipulators can have on their force capability, opening the possibility to amplify the maximum force at the end effector of the manipulator.

The force capability problem in parallel manipulators can be solved by the optimization of its static equations, and an objective function can be used to optimize the force capability. The screw theory and the Davies method were used in this paper as primary mathematical tools in order to define the backward statics of the studied manipulators.

The present study may be extended in various ways. Manipulators with different DOFs, kinematic chains and including dynamic behavior may be studied, and variations on the imposed moment may be considered in future researches.

7. ACKNOWLEDGEMENTS

Authors would like to thank to the Department of Mechanical Engineering of the Federal University of Santa Catarina and to the National Council for Scientific and Technological Development (CNPq) which has made the present work possible.

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