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ON THE INFLUENCE OF NONLINEAR MECHANICAL EFFECTS ON VIBRATION-BASED ENERGY HARVESTING

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Abstract*. Vibration-based energy harvesting has the main objective to convert available mechanical energy from the environment to electrical energy. Piezoelectric materials are usually employed to promote the mechanical-electrical conversion. This work investigates the influence of nonlinear effects in vibration-based energy harvesting. Mechanical nonlinearities are treated in monostable and bistable Duffing oscillator. System performance is evaluated by considering different system characteristics. Numerical simulations are carried out considering different kinds of responses, including periodic and chaotic regimes. The system performance is monitored considering the generated power showing the most efficient kinds of behaviors for energy harvesting purposes.*

Keywords: *Numerical simulation, energy harvesting, nonlinear dynamics, piezoelectric.*

1. INTRODUCTION

Vibration-based energy harvesting has the main objective to convert available mechanical energy into electrical energy. Piezoelectric materials are usually employed to promote mechanical-electrical conversion. The growing use of mobile devices is encouraging this procedure allowing the battery charges.

Several research efforts have been dealing the feasibility of energy harvesting and storage employing different mathematical models (Ajitsaria et al., 2007; Du Toit et al, 2005; Lumentut et al., 2009) and experimental tests (Erturk et al, 2009; Du Toit et al., 2006). The major challenge is to enhance the energy harvesting performance in order to be useful for more purposes.

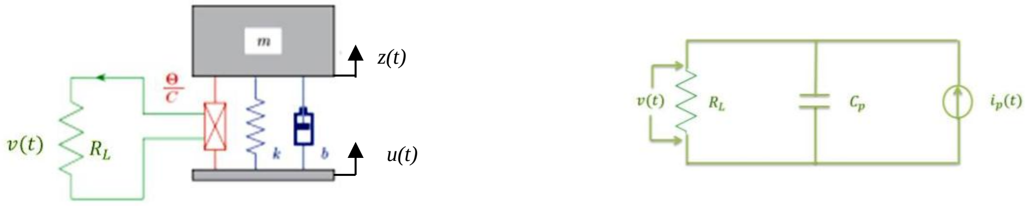
In this regard, nonlinear effects are of special interest. Different nonlinearities are can be imagined and it is important to highlight mechanical (Erturk et al., 2011; Leadenham and Erturk, 2015), electrical (Shun and Lien, 2006) and piezoelectric electro-mechanical coupling (Triplett and Quinn, 2009).

Stanton (2010) and Cottone (2009) investigated bistable configurations of energy harvesting system using an experimental approach. Nguyen (2013) investigated energy harvesting using MEMS. Ramlan (2010) treated monostable Duffing oscillator with hardening effect. Du Toit (2005) discussed nonlinear aspects related to the piezoelectric coupling constants showing that there is a significant dependence of strains. Triplett and Quinn (2009) investigated the nonlinear coupling behavior of piezoelectric materials and also some aspects related to the mechanical nonlinearities in a vibration-based energy harvesting. Silva et al. (2013) investigated the influence of hysterestic behavior of piezoelectric coupling. De Paula et al. (2015) investigated the random aspects on vibration-based energy harvesting.

This article analyzes nonlinear mechanical effects considering Duffing type oscillator with monostable and bistable systems. An archetypal model based vibration energy harvesting is considered, coupling to the mechanical system an electric circuit with linear piezoelectric element. Input and output powers are monitored evaluating the energy harvesting system. Periodic and chaotic responses are evaluated in order to verify system performance.

2. ENERGY HARVESTING SYSTEM - ELECTROMECHANICAL MODEL

A vibration-based energy harvesting system consists of a mechanical system connected to an electrical circuit by a piezoelectric element (Figure 1). Mechanical system is a Duffing type oscillator with displacement z and and base excitation is u. Besides, there is a mass m and a linear viscous damping with coefficient b. Electro-mechanical coupling is promoted by a piezoelectric element with coupling coefficient Θ, piezoelectric capacitance C_p and Q charge; V_p is the voltage, i is the current and R_L is the electrical resistance, with $V_p = iR_L$. By assuming a harmonic excitation of the base, $u(t) = F_0 \sin(\psi t)$.



(a) (b)

Figure 1. (a) Vibration-based energy harvesting system and (b) equivalent electrical circuit with a dependent current source.

By assuming that $x = z - u$ the equations of motion can be written as follows:

$$
\begin{aligned}
&x'' + 2\zeta x' + \beta x + \alpha x^3 - \Theta_m v = \gamma \mathrm{sen}(\omega\tau) \\
&v' + \lambda v + \Theta_e \dot{x} = 0
\end{aligned}
\tag{1}
$$

where the electric-mechanical coupling is provided by:

$$\Theta_m = \frac{\Theta}{m{\omega_n}^2}, \quad \Theta_e = \frac{\Theta}{C_p} \tag{2}$$

and the following parameters are defined:

$$\omega_n^2 = \frac{k}{m}, \quad (*)' = \frac{d(\cdot)}{d\tau}, \quad 2\zeta = \frac{b}{\sqrt{km}}, \quad \gamma = \frac{F_0\psi^2}{\omega_n^2}, \quad \tau = \omega_n t, \quad \omega = \frac{\psi}{\omega_n}, \quad \lambda = \frac{1}{R_L C_p} \tag{3}$$

Nonlinearity of the mechanical system is considered assuming a Duffing-type restitution force associated with the following potential energy:

$$U(x) = \frac{1}{2}\beta x^2 + \frac{1}{4}\alpha x^4 \tag{4}$$

where α and β characterize different behaviors. When β <0, the system has a double-well potential with bistable aspects. If β >0 and α >0, the system is nonlinear monostable. Linear system is characterized by α=0. Figure 2 shows the potential energy and the corresponding restitution force.

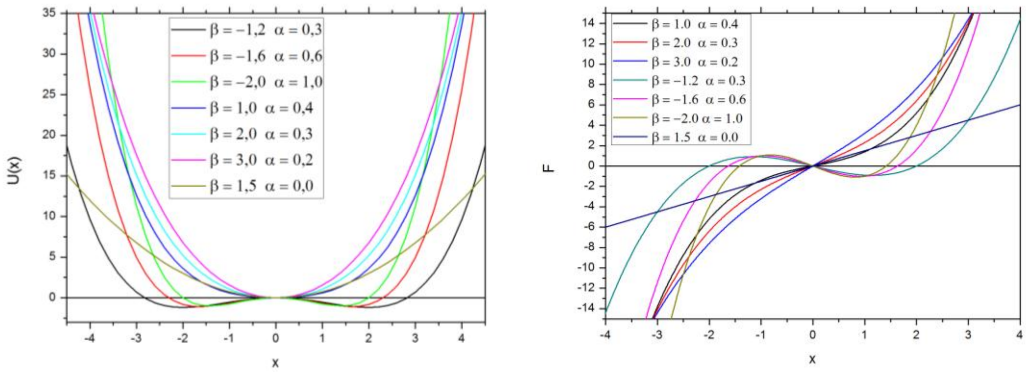


Figure 2. Duffing type system: potential energy and corresponding force for different values of α and β.

The generated and average powers of the energy harvesting system is defined as follows:

$$P = \lambda v^2 \tag{5}$$

$$P_{avg} = \frac{1}{T}\int_0^T P(t)d\tau \tag{6}$$

where $T = 2\pi$. The system performance can be evaluated by considering the conversion efficiency, η, that establishes a relation between electrical and mechanical powers, or in other words, output and input powers.

$$\eta = \frac{p_e}{p_m} \tag{7}$$

The definitions of electrical and mechanical power is as follows.

$$p_e = \sqrt{\frac{1}{\tau}\int_0^{\tau}(vi)^2 d\tau} \tag{8}$$

$$p_m = \sqrt{\frac{1}{\tau}\int_0^{\tau}(x'(\gamma \sin(\omega t)))^2 d\tau} \tag{9}$$

3. NUMERICAL SIMULATIONS

Numerical simulations are carried out exploring the dynamics of the energy harvesting system. Different nonlinearities and excitation aspects are investigated. Basically, different Duffing parameters, α and β, are evaluated considering linear, monostable and bistable characteristics. Moreover, forcing amplitudes and frequencies are varied evaluating the system response. Efficiency is analyzed establishing the main aspects of energy harvesting power. Fourth order Runge-Kutta method is employed and the following parameters are employed for all simulations: $\zeta = 0.01, \Theta_m = 0.05, \Theta_e = 0.5$ and $\lambda = 0.05$

3.1 INFLUENCE OF THE RESONANCE FREQUENCY

Resonant response is the most interesting situation for energy harvesting purposes since it is related to large amplitudes and therefore is associated with larger harvested power. Figure 5 shows the system efficiency considering different characteristics of the mechanical system. Figure 6-7 presents a comparison between input and output powers. Note that monostable system has better efficiency than the bistable for this set of parameters.

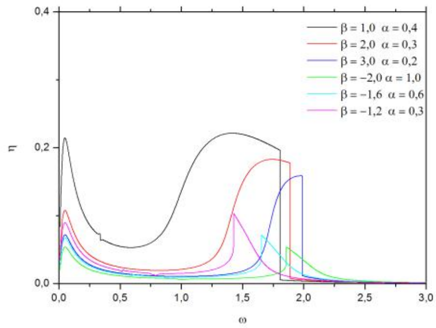
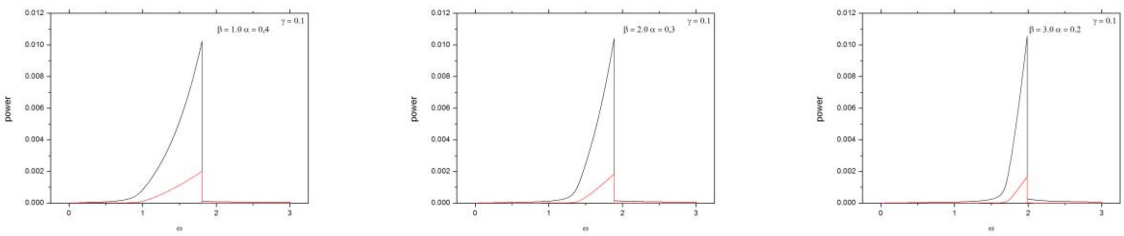


Figure 5. Efficiency versus forcing frequency.



(a) (b) (c)

Figure 6. Input power (black) and output power (red) for different parameters ($\beta > 0$).
(a) $\beta = 1.0, \alpha = 0.4$; (b) $\beta = 2.0, \alpha = 0.3$; (c) $\beta = 3.0, \alpha = 0.2$.

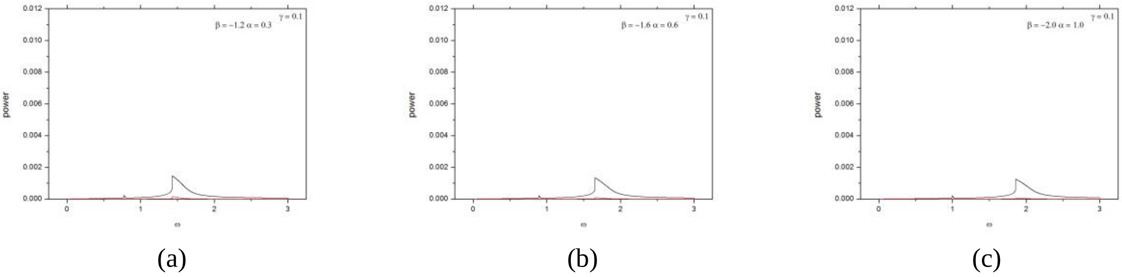


Figure 7. Input power (black) and output power (red) for different parameters ($\beta < 0$).
(a) $\beta = -1.2, \alpha = 0.3$; (b) $\beta = -1.6, \alpha = 0.6$; (c) $\beta = -2.0, \alpha = 1.0$.

3.2 INFLUENCE OF THE FORCING AMPLITUDE

Different conditions of excitation are now investigated. Basically, it is considered the system response under resonant conditions, identified from the previous simulations, and the influence of forcing amplitude is investigated. Initially, a monostable system is treated considering $\beta > 0$. Figure 8 presents bifurcation diagrams considering power energy and average power for different values of the excitation forcing amplitude. Different procedures are employed to build the bifurcation diagrams. The first procedure, Fig. 8(a) vanishes initial conditions for all parameters; on the other hand, the second procedure, Fig 8(b), adopts the previous parameter response as initial condition. The differences between both diagrams suggest multistability of solutions. Note that chaotic region presents a good performance in terms of generated power.

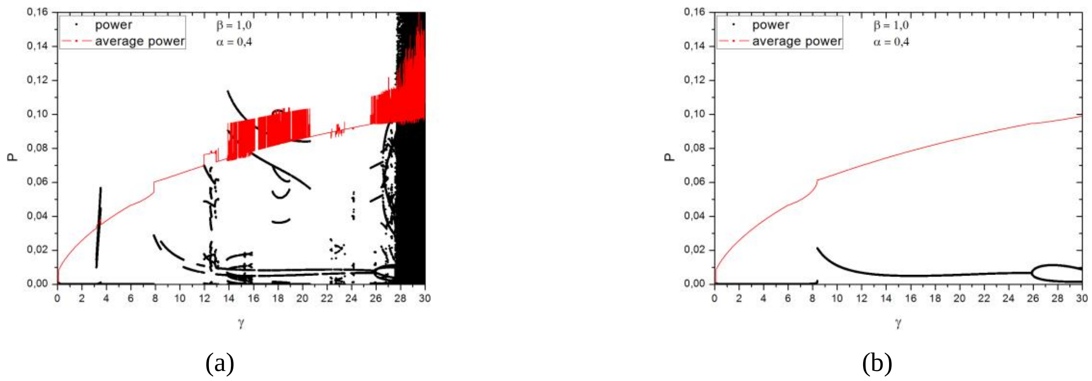


Figure 8. Power energy (P) and average power energy for different values of forcing amplitude.
(a) Initial conditions vanish for each parameter; (b) different initial conditions for each parameter.

Figure 9 represents efficiency of the monostable system. Resonant condition is the most efficient situation and for other cases, a very low efficiency is observed. Figure 10 presents some details of the system response. Based on these results it is possible to conclude that chaotic response generates a good amount of power, but with low efficiency for the considered set of parameters.

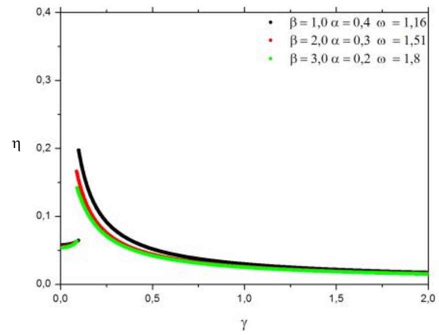


Figure 9. Monostable system performance ($\beta > 0$)

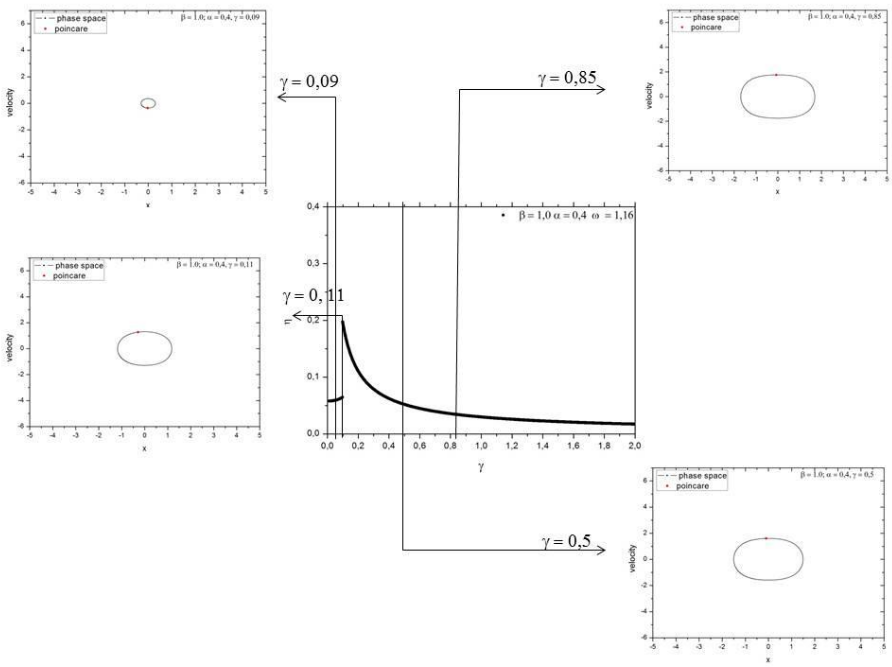
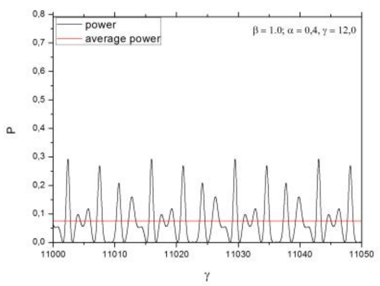
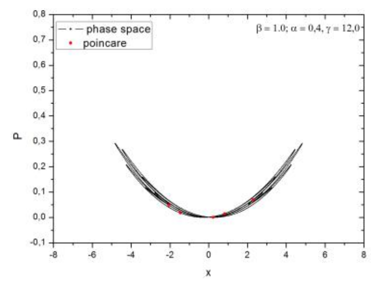


Figure 10. Details of monostable system dynamics (β = 1.0; α = 0.4; ω=1.16).

A forcing amplitude $\gamma = 12.0$ is now investigated. Figure 11-12 present results for different initial conditions. Note that Figure 11 presents a period-5 response while Figure 12 shows a period-3 response. Considering $\gamma = 29.0$, the same situation is observed as shown in Figure 13-14.

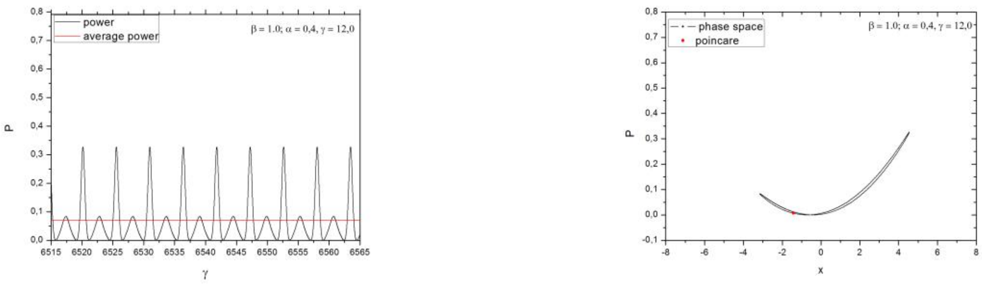


(a)



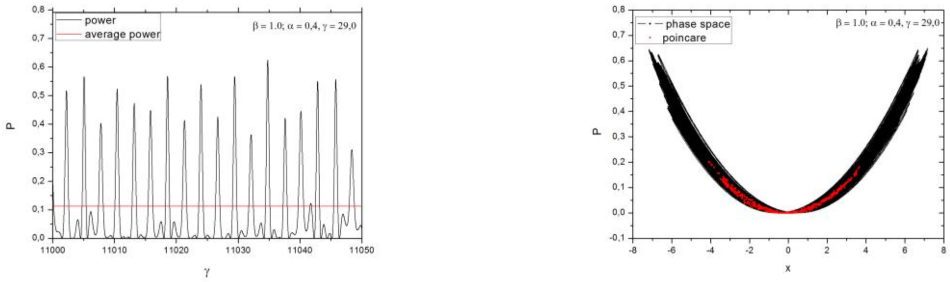
(b)

Figure 11. Energy harvesting for β = 1.0; α = 0.4; ω=1.16 and γ = 12.0, with initial conditions (x, x', v, P) = (0, 0, 0, 0).



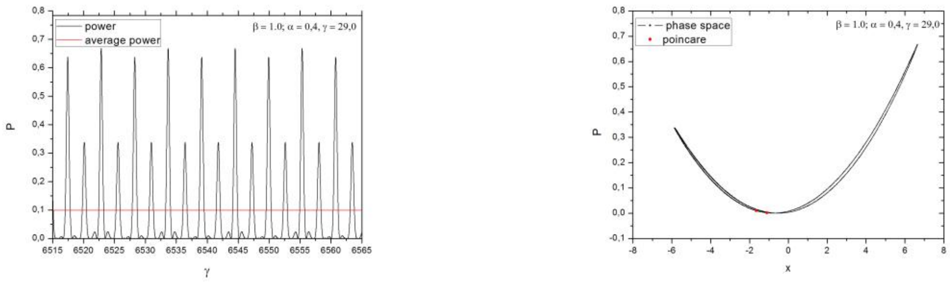
(a) (b)

Figure 12. Energy harvesting for β = 1.0; α = 0.4; ω=1.16 and γ = 12.0, with initial conditions (x, x', v, P) = (–1.43, 2.00, 0.37, 0.01).



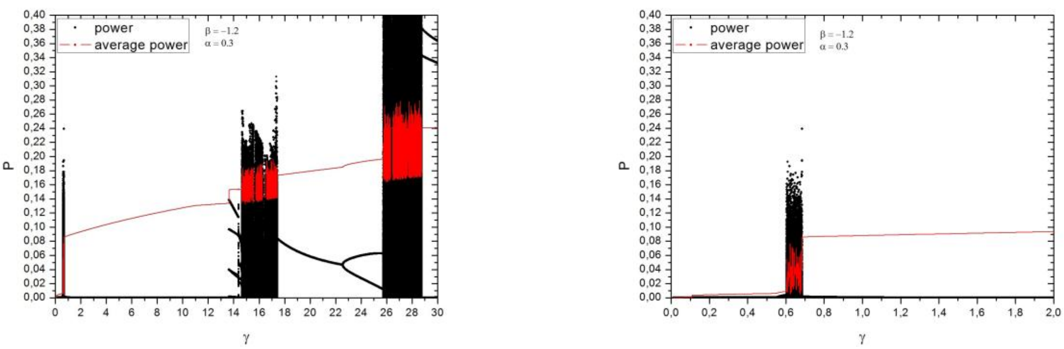
(a) (b)

Figure 13. Energy harvesting for β = 1.0; α = 0.4; ω =1.16 and γ = 29.0, with initial conditions (x, x', v, P) = (0, 0, 0, 0).



(a) (b)

Figure 14.Energy Harvesting for β = 1.0; α = 0.4;ω=1.16 and γ = 29.0, with initial conditions (x, x', v, P) = (–1.66, –2.77, 0.45, 0.01).



(a) (b)

Figure 15. Bifurcation diagrams showing amplitude power energy for different values of forcing amplitudes. (a) Different initial conditions for each parameter; (b) zoom for $0 \leq \gamma \leq 2.0$.

The bistable system is now investigated considering $\beta = -1.2$, $\alpha = 0.3$, $\omega = 1.42$. Figure 15 presents the bifurcation diagrams. Once again, chaotic regions are interesting in terms of harvested energy. Figure 16 shows bistable system performance, presenting two peaks. The first one is the resonant condition while the second is a chaotic response. Note that chaotic response has an interesting behavior for energy harvesting purposes, presenting a good harvested power and efficiency. Figure 17 presents details of some system responses. Note that bistable system has different response characteristics, presenting oscillations either around one equilibrium point or around three points.

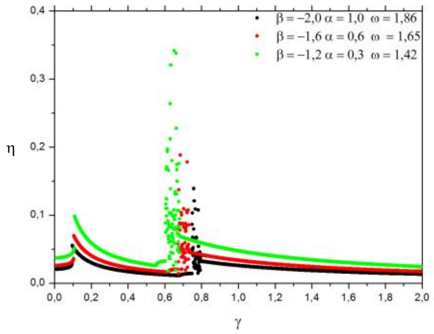


Figure 16. Bistable system performance ($\beta < 0$).

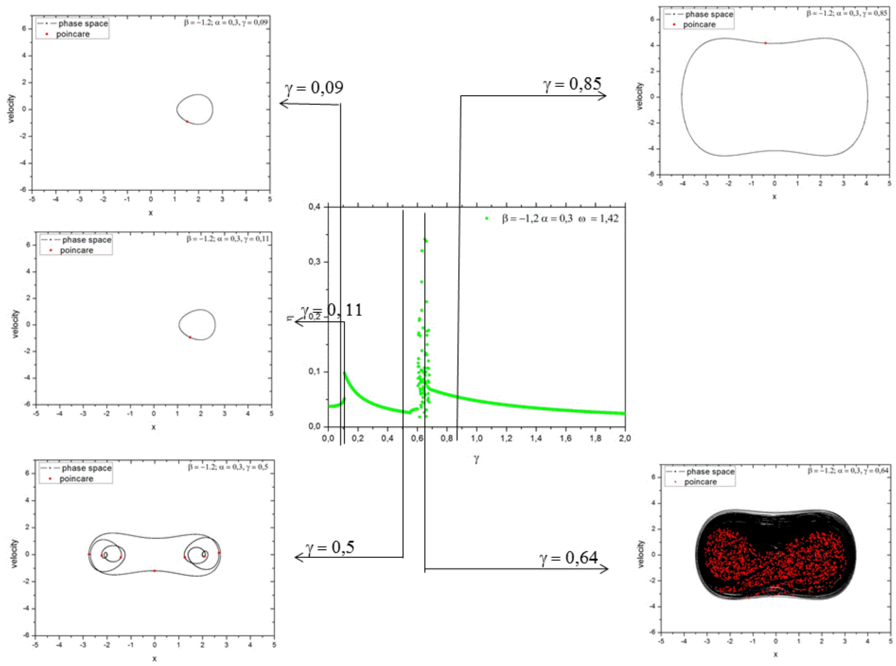


Figure 17. Details of bistable system dynamics system ($\beta = -1.2$; $\alpha = 0.3$; $\omega = 1.42$).

4. CONCLUSIONS

Mechanical nonlinear effects of the energy harvesting system is treated considering a Duffing-type oscillator connected to an electrical circuit by an piezoelectric element. Rich response is observed including chaotic and multistable solutions. Monstable system presents better performance under resonant conditions. On the other hand, bistable system presents better results on chaotic region. In general, chaotic response has an interesting amount of harvested power but bistable system presents an efficient generation.

5. ACKNOWLEDGEMENTS

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