23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

CONSTRUCTAL DESIGN OF I-SHAPED PATHWAY FOR COOLING A HEAT GENERATING BODY BY CONVECTION HEAT TRANSFER

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Abstract. *This work applies the "Constructal Design" method for the geometric optimization of a conductive "I" shaped pathway of high thermal conductivity material that is placed into a square-shaped heat-generating medium with low thermal conductivity. The main goal is to find out the optimized geometry, which provides the easier access to the heat currents, in order to minimize the maximal excess of temperature of the whole system, i.e. the hot spots. The conductive pathway is cooled in the rim by the convection heat transfer mechanism. The total area and the area of the high conductivity material are fixed, but their aspect ratios can vary. The results show great effect of area fraction,* ϕ*, i.e., the ratio between the area of the pathway and the total area, as well as the thermal characteristics of the system in the best calculated configurations. The results also show that dimensionless minimum maximal excess of temperature decreases as the value of the area fraction and the non-dimensional group increases.*

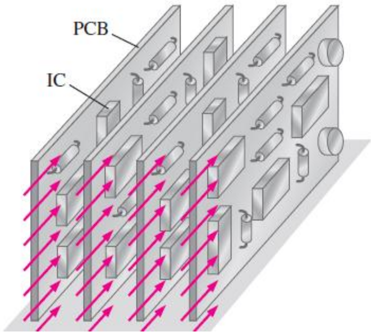
Keywords: *Constructal Design, geometric study, convection heat transfer.*

1. INTRODUCTION

The process of cooling integrated circuits (ICs, or chips), present in electronic devices, instruments, computers and numerous components (resistors, transistors, diodes and capacitors) (Çengel and Cimbala, 2006), in which the objective is to minimize the maximal temperature of a finite volume, considering the heat generation in each point has been widely investigated in the present literature (Bejan, 2000; Boichot *et al.*, 2009; Hajmohammadi *et al.*, 2013; Kakaç, 1994; Nag, 2007; Qi *et al.*, 2003 and Liu *et al.*, 2005). These chips and electronic components are welded in printed circuit boards (PCBs) which are usually displaced in parallel rows considering the heat dissipation. The cooling air is generally blown through the space between each PCB to keep temperature constant preventing the circuit super-heating. Based in statistical data concerning real devices, Qi *et al.*, 2003 and Liu *et al.*, 2005 say that 55% of fails are due to the exceeding temperature rise of the devices. The solution used for cooling the systems consists in building conductive structures, with high thermal conductive materials, connected to the external ambient, used as a heat sink (Bejan, 2000). The cooling of the system by the reduction of thermal resistance is achieved inserting small pieces of high conductivity material building channels in which the generated heat is conducted to the outside air by heat sinks with low temperatures located in the end of the body. This work presents the optimized results of a system which consists of a solid body with uniform heat generation where is placed an I-shaped high conductive pathway cooled by convection. The solid body considered in this problem is the chip used in GPU (Graphics Processing Unit) NVIDIA® GeForce® 9400 GT (Nvidia, 2009). The technical specification states 105°C as the maximal operating temperature to avoid failure. Here it was considered a single chip, from the IC, in which the conductive pathway is placed. Therefore, in the proposed problem was used only one chip inserted on the PCB, the width component was neglected. This is a problem involving heat transfer, assuming steady state condition and uniform heat generation and thermal conductivity. Thus, Constructal Design method was employed in the geometric optimization process.

2. MATHEMATICAL MODEL

The PCBs are generally displaced in parallel rows (Çengel, 2006) as shown in Fig. 1 (a), thus the air circulates thorough the region between the boards cooling the whole system. Figure 1 (b) shows the chip used in GPU NVIDIA® GeForce® 9400 GT (Nvidia, 2009). The geometric optimization considers the conductive pathway shown in Fig. 2, cooled by natural convection, with insulated external surfaces. The configuration used was a two-dimensional model once the material width was not considered for the sake of simplicity. In the solid body with low thermal conductivity, $\tilde{k}$, it is placed I-shaped conductive pathways with a high conductivity material $\tilde{k}_p$. The solid body generates heat uniformly in a volumetric rate, $q'''(W/m^3)$.

(a) 

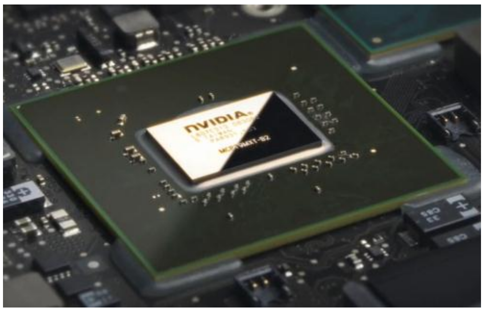
(b) 

Figure 1. System with the integrated chip used in this work.

The external surfaces of the body are insulated and the heat current, $q'''(AW)$, is removed thorough convection heat transfer in the region with temperature, T_∞.The heat transfer coefficient, h, is uniform over the whole contact area with the solid body, as well as the environment temperature, T_∞.

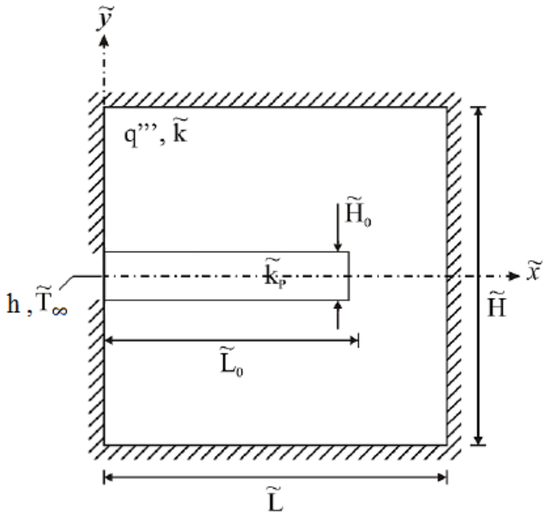


Figure 2. Domain with different thermal conductivity values and the dimensions for the high conductive pathway.

The analysis which states the global thermal resistance as a function of geometry's shape consists by solving numerically the steady heat conduction equation through the domain of the problem. The maximal dimensionless temperature, θ_{max}, is defined by:

$$\theta_{max} = \frac{T_{max} - T_\infty}{q'''A/k_p} \tag{1}$$

where T_{max} is maximal temperature (K), the environment temperature is T_∞, q''' is volumetric heat generation rate (Wm^{-3}), A is area (m^2) and k_p is high thermal conductivity $(Wm^{-1}K^{-1})$.

The present study aims to calculate the maximal excess of dimensionless temperature, θ_{max}, and to observe what the geometry, $(H/L; H_0/L_0)$, facilitates the heat removal. Another point of analysis was the influence of the dimensionless group, $\lambda = hA^{1/2}/k_p$, on changes in system temperature. According to Constructal Design view, the search may be subject to two area constraints. The constraint of the total area is:

$$A = HL \tag{2}$$

and the constraint of the area occupied by the high thermal conductivity material:

$$A_p = H_0 L_0 \tag{3}$$

where H is height (m), L is length (m), A_p area covered by k_p material (m^2), H_0 is height (m) and L_0 is length (m) of I-shaped conductive pathway. Thus, the area fraction, ϕ, is given by:

$$\phi = \frac{A_p}{A} \tag{4}$$

The analysis that determines the overall thermal resistance as a function of the geometry consists by solving numerically the heat conduction equation over the body region with uniform heat generation:

$$\frac{\partial^2 \theta}{\partial \tilde{x}^2} + \frac{\partial^2 \theta}{\partial \tilde{y}^2} + 1 = 0 \tag{5}$$

as well as for the region without heat generation:

$$\frac{\partial^2 \theta}{\partial \tilde{x}^2} + \frac{\partial^2 \theta}{\partial \tilde{y}^2} = 0 \tag{6}$$

The dimensionless variables shown in Eq. (7) are used to describe in dimensionless terms the studied configuration and the heat conduction equation for each region:

$$\tilde{x}, \tilde{y}, \tilde{H}_0, \tilde{L}_0, \tilde{H}, \tilde{L} = \frac{x, y, H_0, L_0, H, L}{A^{1/2}} \qquad \tilde{k}_p = \frac{k_p}{k} \qquad \theta = \frac{T - T_\infty}{q''' A / k_p} \tag{7}$$

Eq. (2) is described in dimensionless form by:

$$\tilde{H}\,\tilde{L} = 1 \tag{8}$$

and the area fraction shown in Eq. (4) has the following dimensionless form:

$$\phi = \tilde{H}_0\,\tilde{L}_0 \tag{9}$$

Equations (4) and (5) are solved numerically using the boundary conditions for the domain regions, considering the whole body adiabatic except by the rim, i.e., the region, $\left(\tilde{x} = 0\,; -\tilde{H}_0/2 \le \tilde{y} \le \tilde{H}_0/2\right)$, which is cooled by convection heat transfer. The balance between convection and conduction in this region gives:

$$\frac{\partial \theta}{\partial \tilde{x}} = -\lambda \theta \tag{10}$$

where λ is defined as:

$$\lambda = \frac{h A^{1/2}}{k_p} \tag{11}$$

In the adiabatic regions representing the outer edges on the chip, the boundary condition is given by,

$$\frac{\partial \theta}{\partial \tilde{n}} = 0 \tag{12}$$

where n indicates the direction perpendicular the insulated surface. Once the equations were defined, the next step is to perform the optimization, i.e., minimizing the dimensionless excess of temperature given by Eq. (2) subjected to constraints (Eq. (8) and (9)).

3. NUMERICAL MODEL

The function defined by Eq. (1) can be solved using a numerical form of the steady heat conduction equation. Thus, it is possible to obtain the temperature field for each configuration depending on the chosen degrees of freedom. The mesh used was determined by means of successive refinements, increasing the number of elements, until the criterion $\left|\left(\theta_{\max}^{j} - \theta_{\max}^{j+1}\right)/\theta_{\max}^{j}\right| < 1.0\times10^{-4}$ is achieved. In this context, $\theta_{\max}^{j}$ represents the excess of maximal dimensionless temperature evaluated with the actual mesh and $\theta_{\max}^{j+1}$ is the excess of maximal dimensionless temperature obtained for the next refinement. In order to obtain the numerical solution for the steady heat conduction equation for each region of the domain it was used a finite element code, based on triangular elements, developed in MatLab® (MatLab, 2012) environment, using the toolbox PDE (partial-differential-equations). Table 1 shows the mesh independency test performed for a configuration using an I-shaped conductive pathway. The tests were performed using meshes with a range of 330 to 86000 triangular elements for $\phi = 0.1$, $\lambda = 1$, $H/L = 1$, $H_0/L_0 = 1$, $\tilde{k} = 1$ and $\tilde{k}_p = 400$.

Table 1. Mesh refinements to obtain results independent from the number of elements.

Number of elements	$\theta_{\max}^{j}$	$\left|\left(\theta_{\max}^{j} - \theta_{\max}^{j+1}\right)/\theta_{\max}^{j}\right| \le 1.0\times10^{-4}$
336	3.1536	1.1111×10^{-3}
1334	3.1571	4.3949×10^{-4}
5376	3.1585	1.7353×10^{-4}
21504 (1)	3.1591	6.8635×10^{-5}
86016	3.1593	-

(1) Independent mesh

The mesh considered ideal for the purposed problem is composed of 21504 triangular elements, representing four successive refinements in the mesh.

4. RESULTS

In this section it is presented the effect of the aspect ratio H_0/L_0 in the excess of maximal dimensionless temperature, $\theta_{\max}$, in the domain showed in Fig. 2. Initially it was fixed the values ϕ = 0.1, $\tilde{k}_p = 400$ and H/L = 1 for several values of the parameter λ. Then it was varied the values of H_0/L_0 until the minimum maximal dimensionless excess of temperature was achieved, $\theta_{max,min}$. It was defined the thermal conductivity of the body with heat generation, $\tilde{k} = 1$. Figure 3 shows the dimensionless optimization of the dimensionless maximal excess of temperature, $\theta_{\max}$, for different values of λ. Figure 3 indicates that there is an optimal value of $(H_0/L_0)_{opt}$ which minimizes the maximal excess of temperature as function of the dimensionless group λ. Figure 3 also shows that $(H_0/L_0)_{opt}$ decreases as $\theta_{max,min}$ increases.

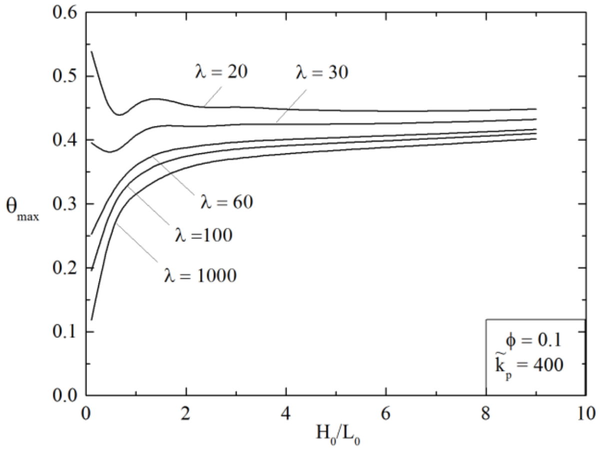


Figure 3. Minimization of the dimensionless global thermal resistance in function of H_0/L_0 for different values of λ .

The optimal values of Fig. 3 are summarized in Fig. 4, as it presents the dimensionless minimum maximal excess of temperature minimized once, $\theta_{max,min}$, and the optimal value of $(H_0/L_0)_{opt}$ as a function of the dimensionless group λ . Figure 4 indicates that $\theta_{max,min}$ decreases and the same happens for the aspect ratio $(H_0/L_0)_{opt}$ while the value of the λ increases.

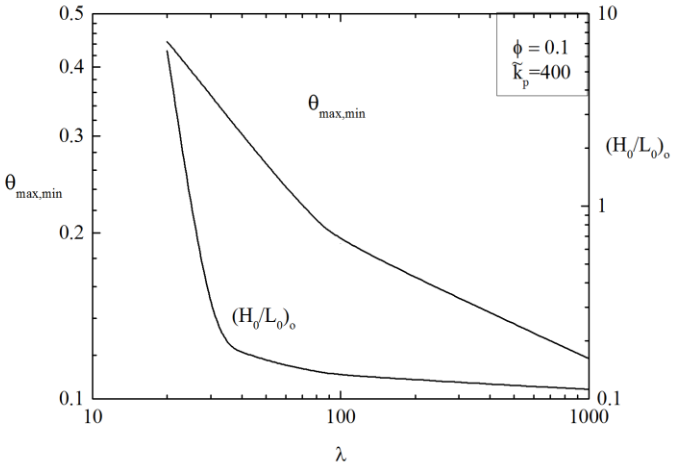


Figure 4. Best configurations obtained and the minimal dimensionless global thermal resistance evaluated in Fig. 3 as function of dimensionless group λ .

Figure 5 shows the behavior of the minimum maximal excess of temperature, $\theta_{max,min}$, as a function of dimensionless group, λ and thermal conductivity material, $\tilde{k}_p$, when the solid body is squared, $(H/L = 1)$. The Fig. 5 indicates that the minimum maximal excess of temperature decreases as the values of the thermal conductivity material, $\tilde{k}_p$, and dimensionless group, λ , increase. Thus, the minimum thermal resistance occurs for $\tilde{k}_p = 400$ and $\lambda = 1000$, resulting in $\theta_{max,min} = 0.1183$.

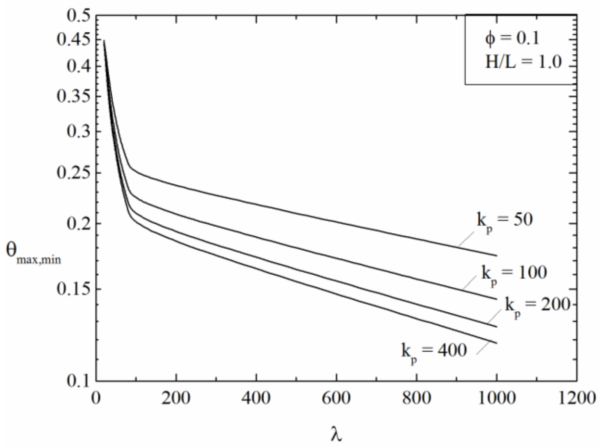


Figure 5. Minimization of the dimensionless global thermal resistance as function of λ for different values of $\tilde{k}_p$.

Figure 6 condensates the optimal value of $(H_0/L_0)_{opt}$ for different values of $\tilde{k}_p$ and the dimensionless group, λ , for the area fraction, $\phi = 0.1$ corresponding the values calculated in Fig. 5.

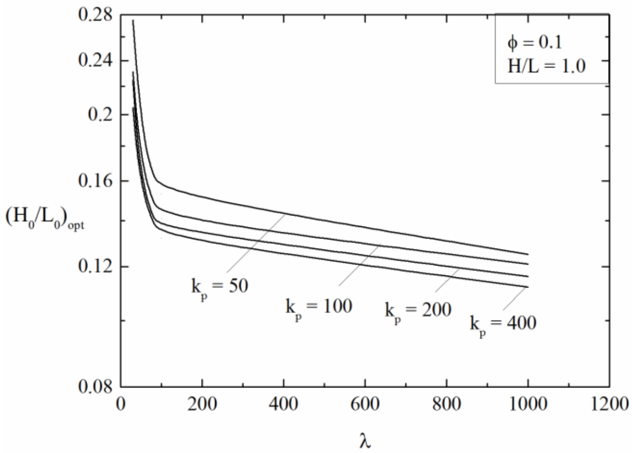


Figure 6. Optimization of the $(H_0/L_0)_{opt}$ as function of dimensionless group, λ and thermal conductivity material, $\tilde{k}_p$.

The Fig. 7 indicates that $\theta_{max,min}$ also decreases as the area fraction value, ϕ , and the parameter λ increases. The optimal value $(H_0/L_0)_{opt}$ corresponding to the minimal values calculated in Fig. 7 are shown in Fig. 8.

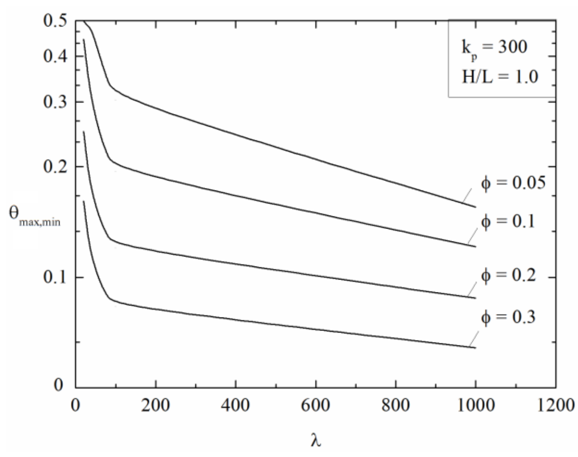


Figure 7. Minimization of the dimensionless maximal excess of temperature as function of dimensionless group, λ for different values of area fraction, ϕ .

This Figure 8 shows that the value of $(H_0/L_0)_{opt}$ decreases as the value of λ increases and the area fraction ϕ decreases.

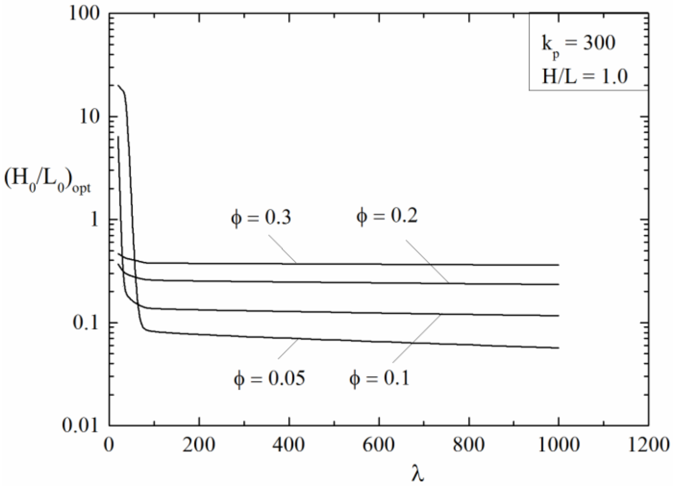
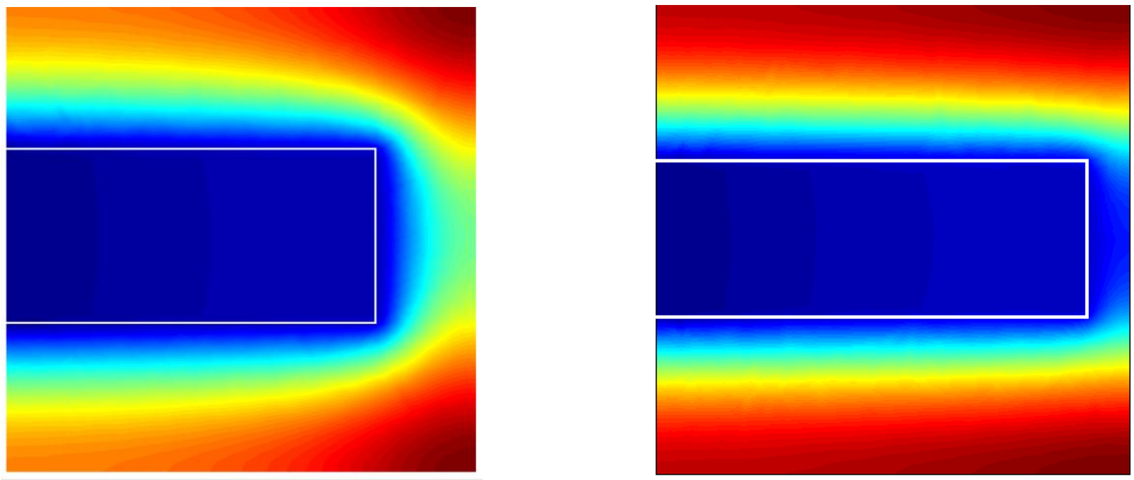


Figure 8. Optimization of the $(H_0/L_0)_{opt}$ as function of λ for different values of area fraction, ϕ.

Figure 9 shows the temperature field and some of the optimal configurations calculated during the simulations. The largest maximal dimensionless excess of temperature is located in upper and lower parts of the domain in the right corner where the hot spots are located. Noteworthy is the great influence of the dimensionless group, λ, on the minimization of non-dimensional maximal excess of temperature. When compared the value of $\theta_{max.min}$ for $\lambda = 1000$ with the value of $\theta_{\max,\min}$ for $\lambda = 20$ showed in Figs. 9 (a) and (b) and keeping fixed the value $\phi = 0.3$ and $\tilde{k}_p = 300$ the optimal aspect ratio has been reduced by 30%.



(a) (b)

Figure 9. Optimal configuration that minimizes the overall dimensionless thermal resistance for $\phi = 0.3$ and $\tilde{k}_p = 300$: a) $(H_0/L_0)_{opt} = 0.468$ and $\lambda = 20$, b) $(H_0/L_0)_{opt} = 0.361$ and $\lambda = 1000$.

5. CONCLUSIONS

This study applies the Constructal Design method for minimizing the maximal excess of temperature of a solid body with a uniform heat generation by inserting an I-shaped high conductive pathway. The outer surfaces of the solid were assumed perfectly insulated and the generated heat current was removed by convection heat transfer mechanism located in the rim of the body with lower temperature.

The results showed the minimization of the dimensionless maximal excess of temperature. For example, considering $\phi = 0.3$, $(H_0/L_0)_o = 0.361$, $\tilde{k}_p = 300$ and $\lambda = 1000$, the minimum maximal excess of temperature is

$\theta_{max,min} = 0.06434$. The results also show that dimensionless minimum maximal excess of temperature, $\theta_{\max,\min}$, decreases as the value of the area fraction, ϕ, and the non-dimensional group, λ, increases.

The results also show that Constructal Design is a reliable method to discover better and better shapes that facilitate the heat flow. The method shows to be effective to find the optimal geometry, because it reaches the best combination of the degrees of freedom in a deterministic way. The optimal geometries successfully removed the generated heat in the solid, and the optimal settings are those that best distribute the hot spots of the studied system.

6. ACKNOWLEDGEMENTS

L.A.O. Rocha, L. A. Isoldi, E. D. dos Santos, and C. C. Beckel's work are supported by CNPq, Brasília, DF. E.X. Barreto's work is supported by CAPES, Brasília, DF.

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