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MODELING ROTORDYNAMIC SYSTEMS SUPPORTED BY BEARINGS WITH VARIABLE STIFFNESS BASED ON SHAPE MEMORY ALLOYS

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Abstract. *Shape memory alloys have great potential in applications involved in the control of vibration in many structures. In rotating machines, the critical operating speeds are functions of the system equivalent stiffness. This work focus in the thermoelastic temperature induced martensitic transformation in SMA, capable of altering between two crystallographically reversible stages, martensite and austenite. The phase change of shape memory alloy can result in very significant modifications in the material's modulus of elasticity. Hence, it seek to develop a concept of adaptive bearing with SMA elements embedded in bars format, with the intention of affecting the equivalent stiffness of the bearing and thus change the intrinsic critical speed of the rotating machine, resulting in a configuration with various elements of a conventional bearing (rolling element or fluid film bearing, pedestal) arranged in series with the bars. Phase transformation is induced by Joule heating, requiring thermal and electrical isolation of the bars made of SMA. Uses different amounts, sizes and geometries of bearing elements and models mathematically a rotating system with flexible shaft, stiff disc, and bearings with changeable stiffness, in order to investigate the dynamic response of the system, as well as changes in the amplitude and natural frequency.*

Keywords: *Adaptive Vibration Control, Rotordynamics, Shape Memory Alloys, Smart Materials*

1. INTRODUCTION

It is possible to predict unwanted vibrations in rotors through mathematical models, extensively studied in the context of rotordynamics, which include: analytical models using simple discrete systems; calculation of critical speeds using Dunkerley equation or the Rayleigh-Ritz method; modeling as elastic beams like Euler-Bernoulli or Timoshenko; numerical methods, including the Finite Element Method; besides some other models (Muszynska, 2005). Typically, the selection of bearings and rotating elements of machine is done estimating the critical speed of the system, particularly the speed related to the machine's shaft. After checking the critical speed, the typical design of rotating machine ensures that the critical speed remains at least twice the speed at which the machine operates.

In rotating machines, the critical operating speeds are functions of the system equivalent stiffness. One of the initial ideas was to change the stiffness of the supports of a rotating machine using SMA wires to control and avoid the critical resonant speeds. This idea is emphasized by the fact that the elastic modulus of the SMA material, changing it from the martensitic phase to austenitic phase or vice versa, can even triple, proved experimentally by heating or cooling of Ni-Ti alloys (Cross *et al.*, 1970). In Zak *et al.* (2003), maximum amplitude peaks at different frequencies are evaluated using a composite sleeve with SMA strips for support to the rotating engine, aided by Finite Element Method. An experimental arrangement is proposed by Lees *et al.* (2007) composed of a rolling bearing, an elastomeric ring and SMA wires arranged vertically. All of these bearing elements are arranged in series to calculate the equivalent vertical stiffness. An adaptive bearing with SMA springs is proposed, where the stiffness of the springs is estimated by conventional calculations of a helical spring in machine design (He *et al.*, 2007a; He *et al.*, 2007b). An analytical method of dynamic responses of the system is examined and an experimental setup is designed later.

The adaptive control differs from a purely active control. An active control device works using actuators placed in parallel to other passive elements of the system, since the adaptive device is a naturally passive element, but that may have changed their physical parameters to improve equipment performance. The SMA, for example, can change its equivalent stiffness as it changes its crystalline state induced by stress or heat and thus change the dynamic behavior of the system as a whole, characterizing it as adaptive control.

The aim herein was to develop a concept of an adaptive bearing for vibration control, evaluating how its stiffness can be altered by the thermomechanical behavior of shape memory alloys to be embedded in the bearing-in-bars format. The thermoelastic temperature-induced martensitic transformation in SMA is capable of altering between two crystallographically reversible phases, martensite and austenite. The phase change of shape memory alloy can result in very significant modifications in the material's modulus of elasticity, aiming to change the stiffness of the whole structure. Phase transformation is induced by Joule heating, requiring thermal and electrical isolation of the bars made of SMA.

2. BEARING DESIGN AND MATHEMATICAL MODELING

To better understand the full bearing frame and prepare a convenient mathematical model for the design, it must be recognized that the adaptive bearing functions as a common bearing, except to its inserted SMA bars that change the stiffness of the entire bearing. The proposed bearing is composed of the following elements:

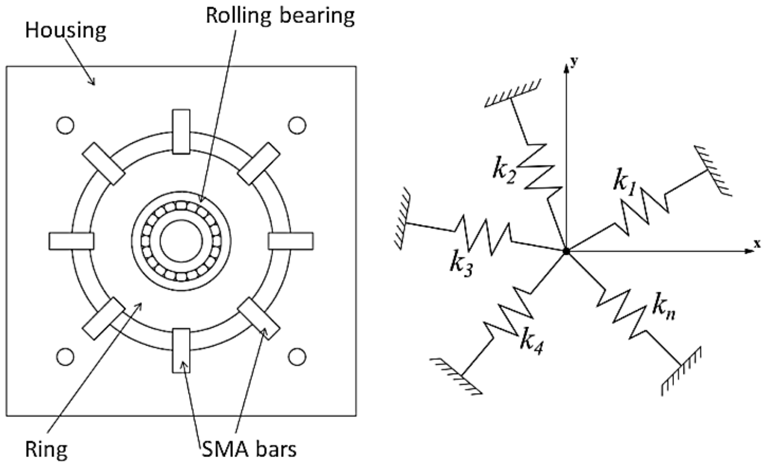

Figure 1. Bearing design and model.

The bearing is composed of a rolling-element bearing, a central steel ring, the SMA bars and the bearing housing. It is considered a set of bars arranged as shown in Fig. (1), with a centralized point element, the shaft, to be forced to move in a radial direction (x-direction). The machine shaft moves from its position of static equilibrium during vibration, deforming X, represented by a spring. Each bar has its contribution to X.

A rotor with simple features is adopted as representative and sufficient to verify the system's response to the bearing's stiffness adaptation. A model that keeps many of the features of a complex system, in spite of apparent simplicity, is the Jeffcott rotor, also known as De Laval rotor. The model consists of a flexible shaft with single central and stiff disc supported by two stiff bearings. The disc has an unbalance described by its eccentricity and, in circumstances such as this, the center of mass does not coincide with the geometric center of the disc. The Jeffcott rotor is sufficient to analyze the appearing of critical speeds on rotors with an unbalanced disc, especially the critical speeds of less value. Seeking improvements in the mathematical model of the problem, it modifies the Jeffcott rotor adding flexibility to the bearings that supports the rotor, in other words, assuming a stiffness element of finite value. The association between the bearings and the shaft results into a mass-spring model with one degree of freedom. The simplified model assumes insignificant damping and isotropic bearings (Adams, 2010).

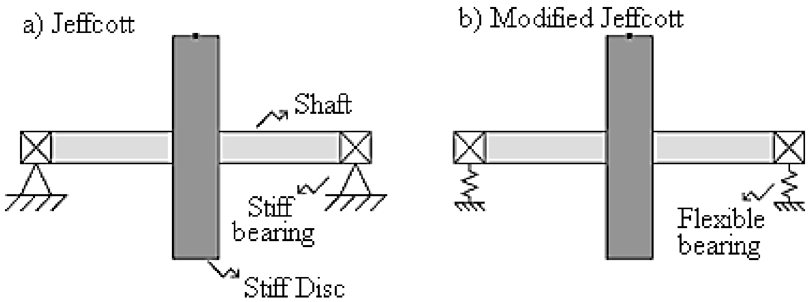

Figure 2. Jeffcott Rotor and the Modified Jeffcott Rotor

It is assumed that the equivalent mass of the system is:

$$M_{eq} = M_{disc} + 0{,}5\ m_{shaft} \tag{1}$$

And other shaft parameters, where E means the elastic modulus of a shaft made of steel.

$$k_{shaft} = \frac{48\, E_{steel} I}{L^3} \quad (2)$$

$$I = \frac{\pi d^4}{64} \quad (3)$$

Normally, it is considered that shaft's supports are perfectly stiff in Jeffcott approach. Such consideration can set equivalent stiffness for the rotation system that escapes from reality; however, while not representing an exact numerical value, the consideration may be enough to solve practical problems of vibration in rotating machinery, as it seeks to manufacture bearings hard enough. In other words, the magnitude of the stiffness of a conventional bearing is usually much greater than the magnitude of the bearing's bending stiffness.

Finally, as show in Fig. (3), we have the concentrated parameters model of the problem.

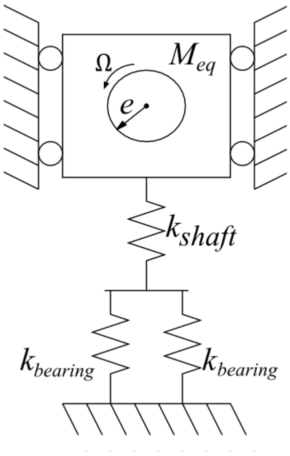

Figure 3. Mass-spring model considered in the bearing design.

Equation (4) describes the problem, where m_0 is the unbalanced mass, e is the eccentricity and $\mathbf{\Omega}$ is the operating speed of the rotating shaft. The equivalent stiffness can be calculated as show in Eq. (5).

$$M_{eq}\ddot{x} + K_{eq}x = m_0 e\Omega^2 sen(\Omega t) \quad (4)$$

$$\frac{1}{K_{eq}} = \frac{1}{2\times k_{bearing}} + \frac{1}{k_{shaft}} \rightarrow K_{eq} = \frac{(2k_{bearing})(k_{shaft})}{2k_{bearing}+k_{shaft}} \quad (5)$$

Stiffness associations in parallel means that springs perform the same deflection to be combined, and result in equivalent stiffness determined by the sum of stiffness. Stiffness associations in series add up all deflections when a traction or compressive force is applied to the springs, it means that the less stiff spring tends to have a greater deflection in this type of association. This explains why there is an association in series between the radial stiffness of the bearings with the flexural stiffness of the shaft, and both bearings are associated in parallel.

The bending stiffness of rotating shafts is usually less than the stiffness of the rolling bearing, explaining the fact that we consider stiff bearing in other common methods. Equation 6 will show how to estimate the bearing stiffness, assuming that only the SMA bars and the rolling element contribute to its deflection.

$$\frac{1}{k_{bearing}} = \frac{1}{k_{rolling\ element}} + \frac{1}{k_{SMA\ bars}} \rightarrow k_{bearing} = \frac{k_{rolling\ element} k_{SMA\ bars}}{k_{rolling\ element}+k_{SMA\ bars}} \quad (6)$$

A single SMA bar has a prismatic shape and generates a mechanical prism with constant cross section, the value of stiffness is given in Eq. (7), which is a function of the elastic modulus, cross sectional area and its length. The difference between the elastic modulus in each process of temperature-induced of shape memory alloys justify a change in the stiffness of the bars, so martensite and austenite phase has different stiffness.

$$k_{SMA\ bar} = \frac{EA}{L} \quad (7)$$

As explained above, each SMA bar have their contribution to the overall deflection of the bearing. The bar with k_1 stiffness is displaced $X\cos\theta_1$, it represents its contribution to the bearing displacement. Then, the force component of this bar is $k_1(X\cos\theta_1)\cos\theta_1$. Similarly, the force components for each of the n present in the adaptive bearing pads are: $k_1(X\cos\theta_1)\cos\theta_1, k_2(X\cos\theta_2)\cos\theta_2, \ldots, k_n(X\cos\theta_n)\cos\theta_n$. In the end, we have the expression that gives $k_{SMA\ bars}$.

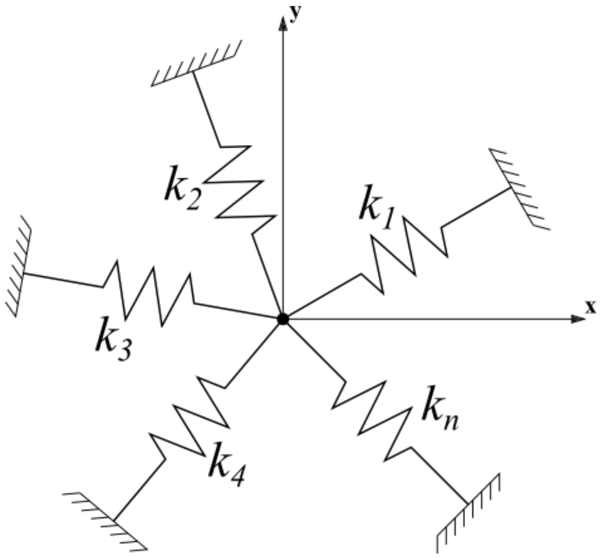

Figure 4. Orientation of n SMA bars embedded in the bearing

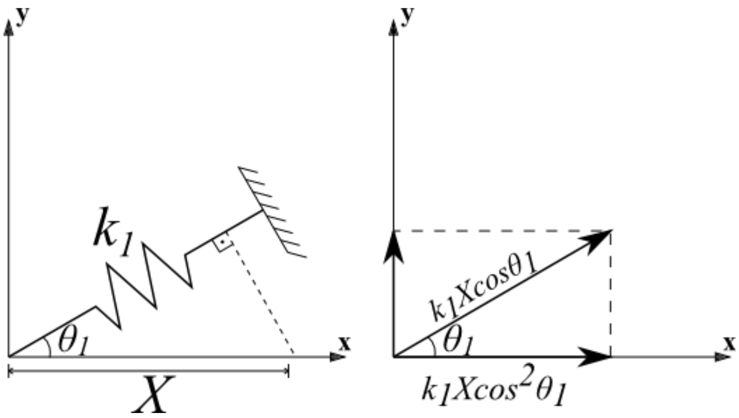

Figura 5. Deflection X of the shaft and contribution of a SMA bar to the resultant force.

$$(k_1 \cos^2 \theta_1 + \ k_2 \cos^2 \theta_2 + \cdots + \ k_n \cos^2 \theta_n)X = (k_{SMA\ bars})X \tag{8}$$

Expanding modeling for more degrees of freedom, we seek to develop a model that incorporates the equations of motion of stiff discs supported by flexible bearings, for cases where the disc is not centralized, as show in Fig. (6).

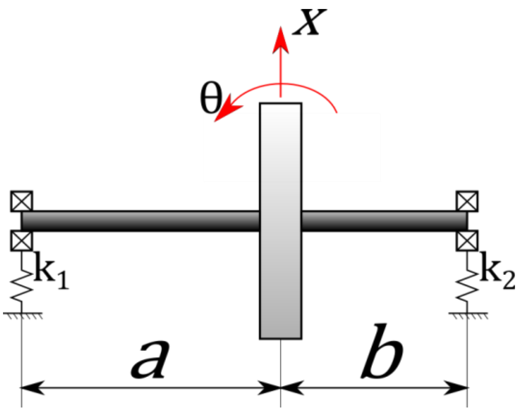

Figure 6. Shaft with stiff discs supported by adaptive bearings.

With the stiffness value calculated by the adaptive bearings, the equations of motion to the disc without damping and homogeneous solution, i.e. without considering the exciting force from the unbalance:

$$M_{disc}\ddot{x} + k_T x + k_c \theta = 0, \tag{9}$$

$$M_{disc}\ddot{y} + k_T y + k_c \varphi = 0, \tag{10}$$

$$I_d\ddot{\varphi} + I_p \Omega \dot{\theta} - k_c y + k_R \varphi = 0, \tag{11}$$

$$I_d\ddot{\theta} - I_p \Omega \dot{\varphi} + k_c x + k_R \theta = 0. \tag{12}$$

Where I_d defines the diametral inertia around x and y axes in kgm^2, radial direction. I_p is the polar inertia around the z axis in kgm^2, axial direction.

$$I_d = \frac{M_{disc}D^2}{16} + \frac{M_{disc}h^2}{12} \tag{13}$$

$$I_p = \frac{M_{disc}D^2}{8}. \tag{14}$$

Therefore, with the values of transversal, rotational and coupling stiffness (k_T, k_R and k_c) is possible to calculate the natural frequencies of the system in four degrees of freedom. In fact, the preliminary analysis by just one degree of freedom is well able to verify system response during vibration control with SMA bars, since checks the first mode, consistent with the first and lower natural frequency system. Except where the disc has moments of inertia (polar and diametrical) of very high values, the lowest values of frequency can be detected in the degrees of freedom of rotation but are unusual situations (Muszynska, 2005; Adams, 2010).

$$k_T = 3EI[3EI(k_1 + k_2) + (\mathrm{a}^3 + \mathrm{b}^3)k_1 k_2]/Den \tag{15}$$

$$k_c = 3EI[3EI(-\mathrm{a}k_1 + \mathrm{b}k_2) + \mathrm{ab}(\mathrm{a}^2 - \mathrm{b}^2)k_1 k_2]/Den \tag{16}$$

$$k_R = 3EI[3EI(\mathrm{a}^2 k_1 + \mathrm{b}^2 k_2) + \mathrm{a}^2\mathrm{b}^2(\mathrm{a} + \mathrm{b})k_1 k_2]/Den \tag{17}$$

Where:

$$Den = (3EI + \mathrm{a}^3 k_1)(3EI + \mathrm{b}^3 k_2) \tag{18}$$

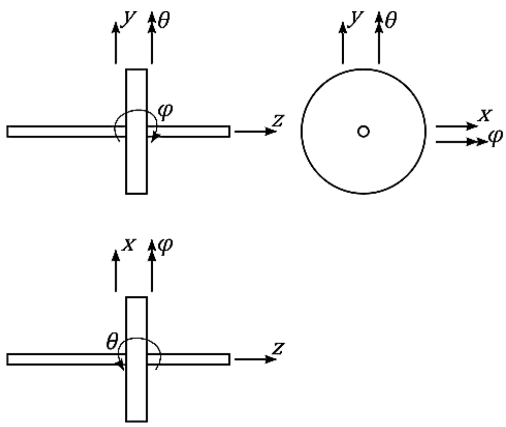

Figure 7. Convention of coordinates adopted.

According to Friswell *et al.* (2010), the pair of the positive roots of polynomial of Eq. (4:44) below can provide the undamped natural frequencies of the system. Note the dependence on the rotor speed of operation to calculate the natural frequencies.

$$\omega_n{}^4 \mp \left(\frac{I_p}{I_d}\right)\Omega\omega_n^3 - \left(\frac{k_R}{I_d} + \frac{k_T}{m}\right)\omega_n^2 \pm \left(\frac{k_T I_p}{m I_d}\right)\Omega\omega_n + \frac{k_R k_T - k_c^2}{m I_d} = 0 \tag{19}$$

3. MODEL ANALYSIS

The validation and bearing functionality occurs when the supports have a moderate stiffness, so as to represent deflections next to the shaft deflections. In other words, the bearing stiffness order of magnitude must be compatible with the axle bending order of magnitude, as shown in Fig. (8). You can check that the intrinsic stiffness of the bearings will only be functional to the system, or modify the equivalent stiffness of the rotary engine, when presenting stiffness similar to shaft stiffness.

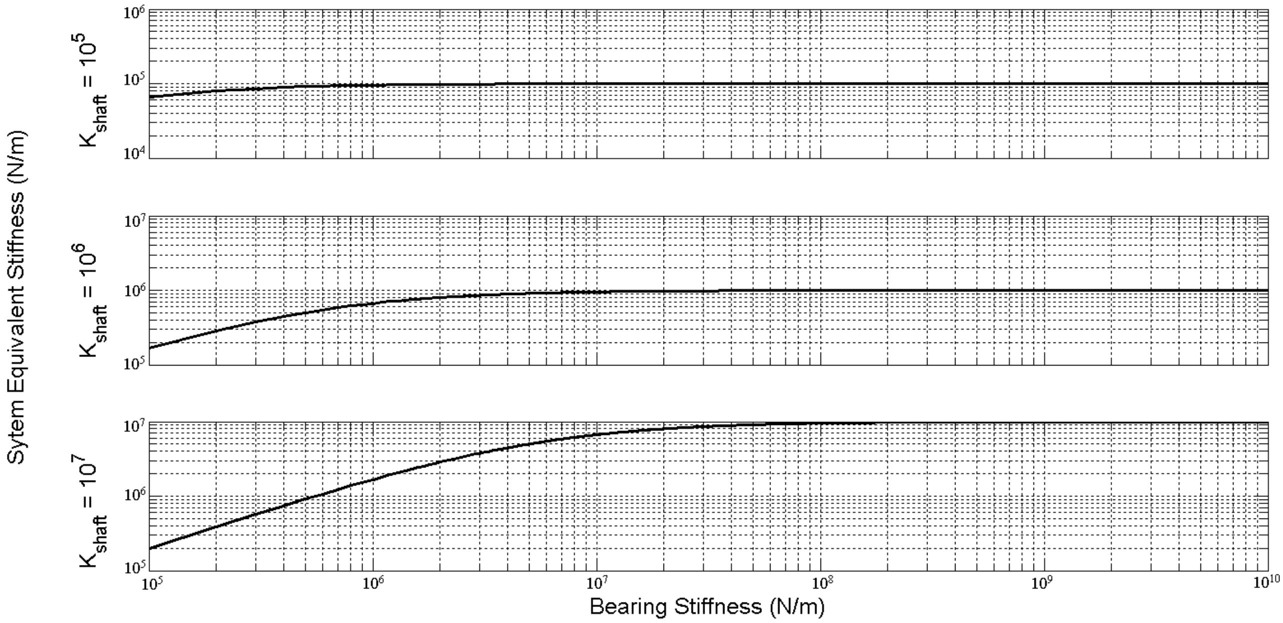

Figure 8. Equivalent stiffness.

Figure (9) shows useful results from a MATLAB simulation where the first vibration mode was shifted by about 500 rpm over phase transformation in the SMA bearing bars. The bearing bars with SMA presents a limitation of rigidity, being more effective on rotating shafts with higher stiffness (more robust shafts with short length and large diameter) or shafts supporting heavy inertia. The bars must be long to ensure a low stiffness, which can affect the overall size of the project.

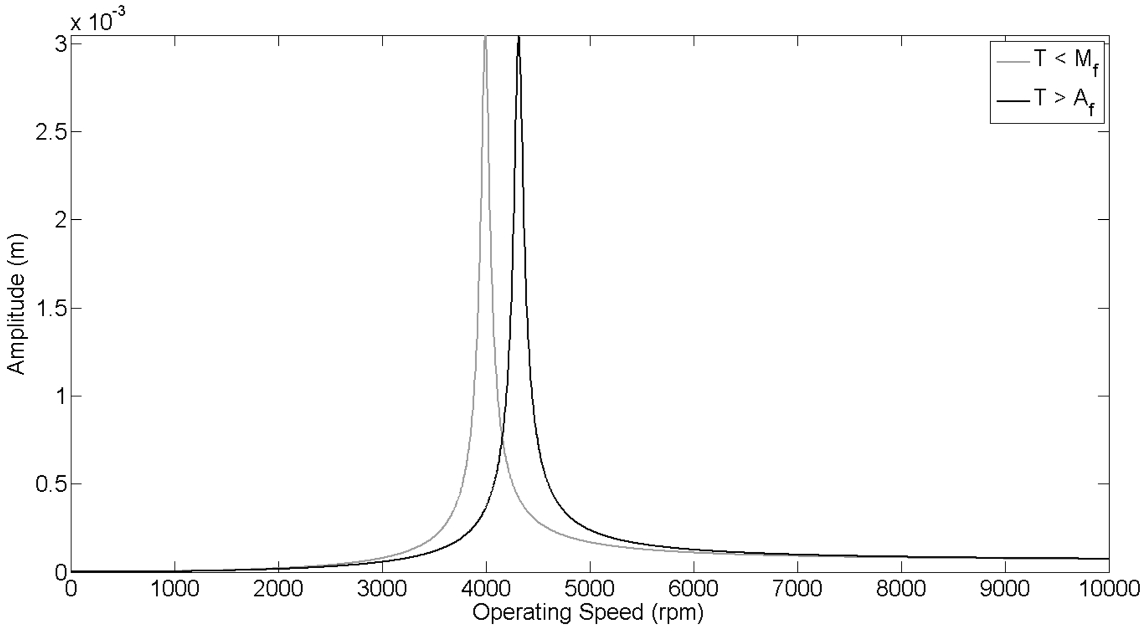

Figure 9. Resonance shift in a casual vibration problem.

The addition of gyroscopic effect in the analysis in four degrees of freedom is justifying the phenomena observed on the map of natural frequencies described in Fig. (10) and Fig. (11). Plotting the same map of frequencies with a bisect which equals the natural frequency to shaft rotation, we can see the critical rotor system speeds. This map is known as Campbell diagram and it's possible to see the change in the diagram when the bars are activated.

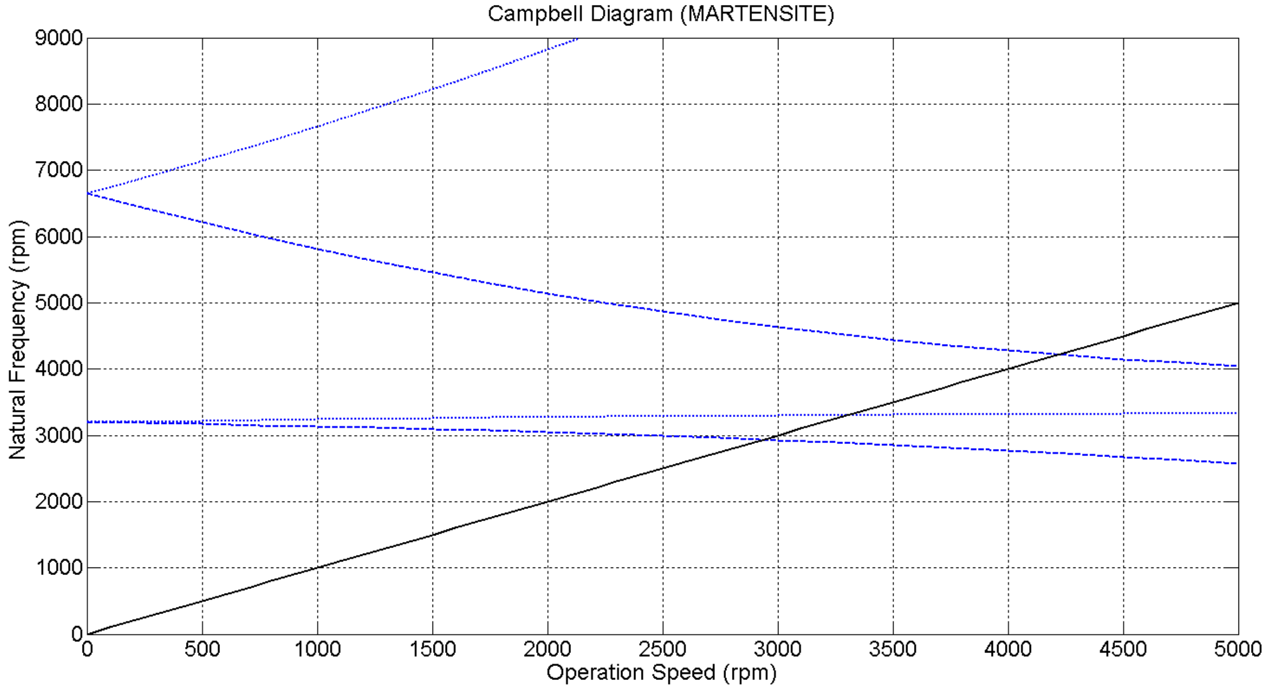

Figure 10. Campbell diagram for non-activated SMA bars.

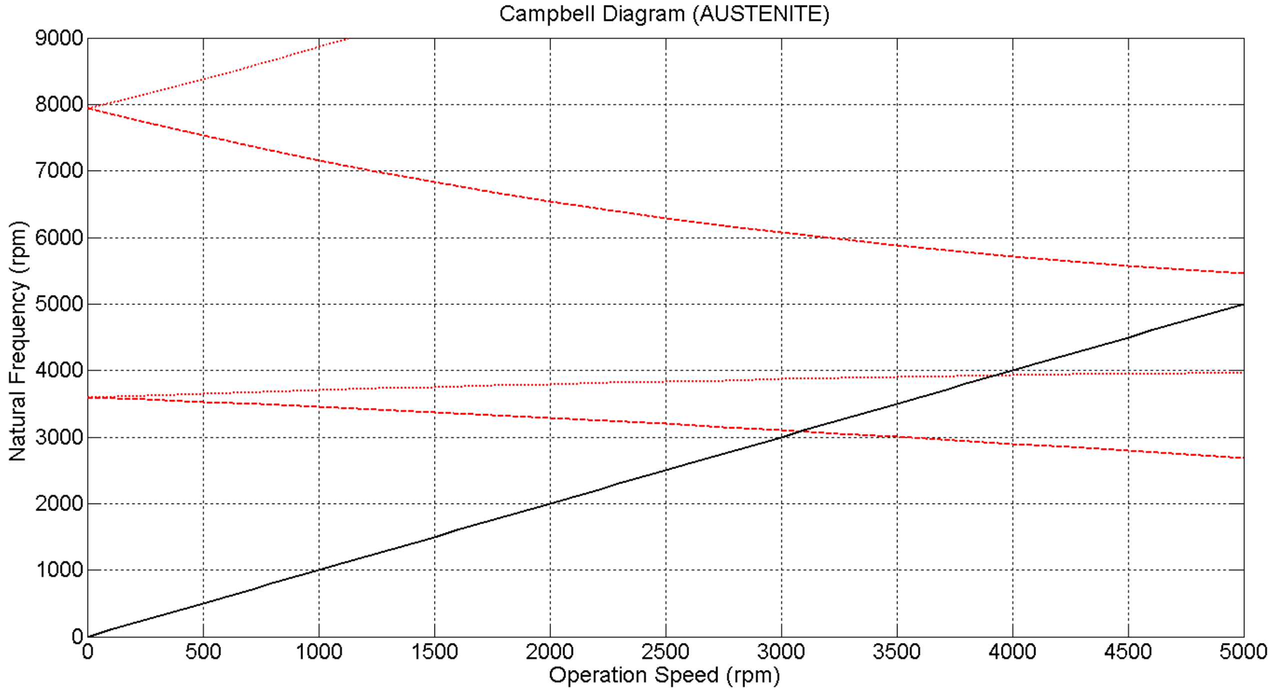

Figure 11. Campbell diagram for activated SMA bars.

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