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EVOLUTIONARY STRUCTURAL OPTIMIZATION APPLIED FOR PROBLEMS WITH NONLINEAR BEHAVIOR

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Abstract. *Structural optimization has received ever increasing attention in all engineering areas and it has been identified as the most challenging and the most economically rewarding task in the structural design field. As a result of high speed computers and parallel processing, several topology optimization methods have been developed, including the ones that apply heuristic procedures in the analysis and act on an evolutionary manner. In this context, it is presented here a methodology based on the Evolutionary Structural Optimization (ESO) that corresponds to an evolutionary procedure applied for topology optimization in which the finite elements with the lowest stress levels are progressively removed from the structure. The proposed paper presents optimization studies applied for structures subjected to multiple load cases, where through the implemented technique, the nonlinear behavior for both statics and dynamics conditions can be considered in the analysis in order to generate a reliable design by structural optimization. The nonlinear response is extended to elastoplasticity where a von Mises material with linear, isotropic work-hardening for small strains is used. The implementation was accomplished through an algorithm developed in Python programming language, due to its interface with the commercial software Abaqus®. The effectiveness of this new approach by evolutionary procedure optimization will be demonstrated through the comparison with classical examples from literature.*

Keywords: *Topology Optimization, Evolutionary Structural Optimization (ESO), Nonlinear Analysis, Multiple Load Cases*

1. INTRODUCTION

At the 1990s decade, the ESO method was proposed by Xie and Steven (1993), and since then significant development has been made to improve the algorithms of ESO. This evolutionary approach is based on the gradual removal of material considered less efficient from the design domain, which must be discretized by using a suitable polynomial approximation in a chosen finite element mesh, where the maximum von Mises equivalent stress is used as a parameter for the element removal through a rejection criterion.

Unlike some traditional techniques, this method does not require an excessive amount of parameters for the beginning of evaluations, which easily enables its use, since due to increased competitiveness caused by the short deadlines for engineering solutions, calculation procedures that require a smaller amount of parameters has become a more suitable alternative. Moreover, once several of real optimization problems are non-differentiable and some of them do not even have explicit equations, these evolutionary methods became more practical because they allowed the employment of their formulations in the cases treated as non-convex (Huang and Arora, 1997).

Although this methodology has already provided many studies involving linear static analyses as shown in the book of Huang and Xie (2010), little progress has been made with respect to the other more complex application conditions. Thus, the implemented algorithm used in the current paper analyses is able to perform optimization studies for structures under multiple load cases, considering the non-linear behavior effects for either statics or dynamics conditions, with the purpose of achieving a more reliable structural optimization design.

The positive points of the evolutionary procedure are the possibility of knowing every stage of the shape and the layout path towards the true optimum, the simplicity of implementation and execution and the easy integration with any commercial program, which provides an excellent alternative of using.

2. EVOLUTIONARY STRUCTURAL OPTMIZATION (ESO)

The evolutionary structural optimization methodology is based on the Finite Element Method and the basic idea is to analyze the complete area where there may be structure, i.e. design domain, and following, based on a given objective function it is evaluated the effectiveness of each element in the structure and the least efficient elements are removed or penalized gradually.

As main advantage, this method does not require the gradient calculation of the objective function, which enables the iterative calculation process. The gradient calculation is computationally expensive and for cases where the objective function presents discontinuity or complex functions, this calculation is not always easy to achieve (Das, *et al.*, 2011). Another important feature of the heuristic methods is the tendency to find the global optimum, avoiding then the convergence towards a local optimum, even when the initial solution is far from the global optimum. On the other hand, the disadvantage of this optimization category method is the fact of it is not able to ensure that the final design will converge exactly to the correct optimal solution due to some possible instability problems. However, with the application of some control techniques of these instabilities, this method tends to converge quickly to an optimum design or at least close to it. More about numerical instabilities can be found in the paper of Sigmund and Petersson (1998).

For each optimization problem can exist one or more types of material removal criteria, which may be the stiffness criterion, displacement, pressure, stress level, natural frequency, heat conduction, buckling, among others. In this paper it was used the stress level criterion for the topology optimization of structural systems through a computer code developed in Python programming language.

2.1 Evolutionary Structural Optimization based on stress criterion

The optimization of a structure based on its stress level is a process applied in many areas of structural mechanics, once the stress level can be used as an indicator of the efficiency of each element within the structure, where by the Finite Element Method the stress distribution throughout the structure is obtained and then, it is possible to establish a rejection criterion based on the maximum stress level, in which the material under low stress is considered under-utilized and thus it can be removed from the structure. Since the structure has been discretized into an acceptable mesh of finite elements, the removal of material from the structure can be conveniently represented by excluding the elements from the mesh. The stress level at each point can be measured by some sort of average of all the stress components. For this purpose the von Mises equivalent stress has been one of the most frequently used criteria for isotropic ductile materials (Xie and Steven, 1997; Tanskanen, 2002).

Equation (1) represents the von Mises equivalent stress σ^{vm} in terms of arbitrary components of the stress tensor in three-dimensional state.

$$\sigma^{vm} = \frac{\sqrt{2}}{2}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6.(\tau_{xy}{}^2 + \tau_{yz}{}^2 + \tau_{xz}{}^2)} \quad (1)$$

Simplifying, Eq. (2) shows the von Mises equivalent stress for plane stress state problems,

$$\sigma^{vm} = \sqrt{\sigma_x{}^2 + \sigma_y{}^2 - \sigma_x.\sigma_y + 3.\tau_{xy}{}^2} \quad (2)$$

where, $\sigma_x, \sigma_y, \sigma_z$ are the normal stress components at x, y and z directions, respectively, and $\tau_{xy}, \tau_{yz}, \tau_{xz}$ are the shear stress components.

According to Xie and Steven (1997), after the structural analysis obtained by Finite Element Method, the stress level of each element is determined by comparing the von Mises equivalent stress of the element σ_e^{vm} with the maximum von Mises equivalent stress of the whole structure σ_{max}^{vm}.

Thus, all the elements that satisfy the condition presented in Eq. (3) are excluded from the model:

$$\frac{\sigma_e^{vm}}{\sigma_{max}^{vm}} < RR_i \quad (3)$$

where, RR_i is the current rejection ratio (RR) at iteration "i".

Such a finite element analysis and element removal cycle is repeated using the same value of RR_i until a steady state is reached, which means that there are no more elements to be removed by the current RR (Xie and Steven 1997). This implies that the number of removed elements for each analysis is not necessarily the same for all iterations. Reaching up this state of equilibrium, but not reaching the stopping criterion of the iterative process (optimal configuration by ESO method), the evolutionary process is restarted by adding an evolution ratio ER to the RR_i. Thus, a new evolutionary cycle begins, until there are no more elements to be eliminated with this new rejection ratio.

Equation (4) shows that whenever the equilibrium is reached, the ER will be added to RR_i:

$$RR_{i+1} = RR_i + ER \quad (4)$$

The initial rejection ratio is usually subject to prescribed values in a range of 0< RR_i <1%, but there are cases where depending on the type of analysis and the finite element mesh discretization used, it may be necessary to consider values above 1% due to non-removal of the elements. The initial value of RR_i is defined empirically as user experience for each type of problem.

According to Querin (1997) to ensure a better convergence, it should be used small amounts of ER, being about no more than 1%, and one of two stopping criteria of the evolutionary process can be established, in which they are: the adoption of a prescribed final volume (V_f) or the establishment of a final rejection ratio (RR_f). This implies that an optimized structure will have a more homogeneous stress distribution than the initial one, since the stress variation rate of one element to another is reduced.

This evolutionary procedure can be summarized as follows (Huang e Xie, 2010):

Step (1): Discretize the structure using a fine mesh of finite elements;
Step (2): Carry out finite element analysis for the structure;
Step (3): Remove elements which satisfy the condition in Eq. (3);
Step (4): Increase the rejection ratio according to Eq. (4) if a steady is reached;
Step (5): Repeat Steps 2 to 4 until a desired optimum is obtained.

2.2 ESO Procedure applied for nonlinear analyses

According to Huang e Xie (2010), the evolutionary procedure for nonlinear structures is similar to that for linear structures except that nonlinear finite element analysis must be used. If the optimization problem is solved using linear finite element analysis, the optimal topology would not depend on the magnitude of the applied load, but if the nonlinear behavior is considered in the analysis the opposite occurs, and for this reason the topologies using linear and nonlinear analysis could be quite different.

2.2.1 Nonlinear material behavior

The elastoplastic behavior is characterized by a material response, initially elastic and from a specified stress level, it is characterized by an essentially plastic behavior (Natal Jorge and Dinis, 2005).

The current paper will simulate an elastoplastic material with linear, isotropic work-hardening, where its representation is described according to Fig. 1.

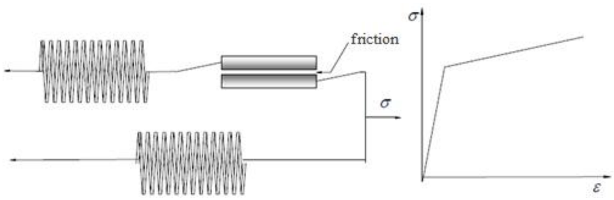


Figure 1. Linear isotropic work-hardening elastoplastic model (Natal Jorge and Dinis, 2005).

The material nonlinear behavior will be described by the von Mises yield criterion according to Eq. (5), once it is suitable for metallic materials with ductility. On this criterion, the yield function $\emptyset(S,k)$ includes the deviatoric stresses tensor $|S|$ and the yield stress with isotropic hardening σ_k which is described by Eq. (6) as a function of the internal variable k, which is related to plastic deformation as shown in Eq. (7) (Maute, *et al.*, 1998).

$$\emptyset(S,k) = |S| - \sqrt{\frac{2}{3}}.\sigma_k \tag{5}$$

$$\sigma_k = \sigma_y + E^h.k \tag{6}$$

$$k = \sqrt{\frac{2}{3}}.\gamma \tag{7}$$

where, E^h denotes the plastic hardening modulus, γ the plastic multiplier and σ_y the material yield stress.

Assuming small strains, the strain increments can be partitioned into an elastic and a plastic part according to Eq. (8). Thus, an associative flow rule can be adopted by considering the elastic and plastic increments described by Eq. (9) and Eq. (10) respectively.

$$d\varepsilon = d\varepsilon^{el} + d\varepsilon^{pl} \tag{8}$$

$$d\varepsilon^{el} = D^{-1}.d\sigma \tag{9}$$

$$d\varepsilon^{pl} = d\gamma \left| \frac{\partial \emptyset}{\partial \sigma} \right. \tag{10}$$

where, $d\varepsilon^{el}$ and $d\varepsilon^{pl}$ are respectively the elastic and plastic increments and D is the elastic material tensor.

Equation (11) shows the gradient $\frac{\partial \emptyset}{\partial \sigma}$ which is evaluated at the end of the time step for a fully implicit Euler backward algorithm and the stresses S must satisfy the yield function $\emptyset(S, k)$.

$$\frac{\partial \phi}{\partial \sigma} = \frac{S}{|S|} \tag{11}$$

To occur plastic deformation the yield function must be null $\emptyset(S, k) = 0$, if $\emptyset(S, k) < 0$ there is only the elastic regime and therefore $d\varepsilon^{pl} = 0$, but when $\emptyset(S, k) > 0$ the stress state is inadmissible and a response for $\emptyset(S, k) = 0$ should be found by using an iterative algorithm. Several algorithms have been developed in order to represent a return mapping of the stress to the yield surface and integrate the elastoplastic equations. Further details on return-mapping algorithms can be found in Simo and Hughes (1998).

Through the Newton-Raphson iterative method the elastoplastic constitutive tensor must be derived by a consistent linearization considering the plane stress conditions (Ramm and Matzenmiller, 1988).

Equation (12) represents the sum of the elastic compliance tensor and the Hessian matrix of the yield condition $\emptyset$.

$$H = D^{-1} + d\gamma \frac{\partial^2 \emptyset}{\partial \sigma^2} \tag{12}$$

For the obtaining process of the Hessian matrix there are two implicit algorithms based on tangent formulations, the standard Newton-Raphson method and the modified Newton-Raphson method (Greco, 2004). In the standard Newton-Raphson method the Hessian matrix is updated at every each iteration and therefore it presents fewer iterations to converge. In the modified Newton-Raphson method the Hessian matrix remains constant, which is an advantage for advanced plastification problems once the Hessian matrix may become singular at some point of the analysis, but it needs more iterations to converge.

The relationship between the changes of stresses and strains in Eq. (13) defines the consistent elastoplastic tangent tensor D_{ep}

$$d\sigma = \left(H^{-1} - \frac{H^{-1} \frac{\partial \emptyset}{\partial \sigma} \frac{\partial \emptyset}{\partial \sigma}^T H^{-1}}{\frac{2}{3} A + \frac{\partial \emptyset}{\partial \sigma}^T H^{-1} \frac{\partial \emptyset}{\partial \sigma}} \right). d\varepsilon = D_{ep}.d\varepsilon \quad \text{with} \quad A = \frac{E^h}{1 - \frac{2}{3} E^h . d\gamma} \tag{13}$$

This assumption is permissible for the present material model with linear work-hardening and it will be verified by numerical examples.

2.3 Numerical instabilities

Due to design domain discretization to perform the finite element analysis in the optimization process, some numerical instabilities may arise to the model. According to Sigmund and Petersson (1998) these instabilities can be classified into three categories: checkerboard irregularities, mesh dependence and local minima problems.

2.3.1 Checkerboard irregularities

The irregularities of the checkerboard are characterized by alternating structural configuration between solid and void elements which can overestimate the structural stiffness and diverge from the optimal solution. Díaz and Sigmund (1995) and Jog and Haber (1996) state that the source of the problem is associated with characteristic numerical errors of the finite element approximation process, not representing, therefore, the feature of an optimal design. The use of elements with higher order interpolating functions is suggested to avoid such instability, since these elements may simulate more accurately the displacement field, reducing the occurrence of regions in chess (Bendsøe and Sigmund, 2003).

Figure 1 shows an example of a region with such a numerical instability.

Figure 1. Representation of regions with checkerboard instability (Sigmund and Petersson, 1998 – adapted)

Another technique that can be used as an attenuating of these irregularities is a constraint called Nibbling ESO, which is an adaptation of the ESO method with shape optimization features. This technique evaluates the stress field and verifies the possibility of opening a cavity inside the domain, so it just acts on the elimination of the elements that are present in the boundaries of the structure, avoiding unnecessary void spaces during the optimization process (Querin, 1997).

This can be done by including the following statement to Step (6) in section 2.1:

Step (6): If an element satisfies Eq. (3), this element can only be removed if at least one of its edges or sides is not connected to any other elements in the structure.

2.3.2 Mesh-dependence

Mesh-dependence is an anomaly inherent to the domain division, where the solution obtained is not qualitatively the same for different discretizations. Intuitively, it is expected that a more refined mesh should result in a better finite element modelling for the same optimal structure and a better description of its boundaries, but what is obtained in many problems is a more detailed and qualitatively different structure. Another negative aspect regarding the excessive mesh refinement, relates to the analyses of dynamic problems, in which very refined mesh generates contributions of spurious vibration modes in the response.

It is observed that with the increasing number of elements, there is an increase trend in void spaces. To reduce this dependence on evolutionary optimization process, some studies about boundary constraint schemes have been performed. Kim, *et al.* (2002) presented a study demonstrating mesh-dependence reduction when the mean von Mises equivalent stress of the finite element was used as a removal criterion in the stress level analysis. The maximum von Mises equivalent stress calculation depends greatly on the mesh density due to the uniqueness that occurs near the boundary conditions, while the mean von Mises equivalent stress does not vary greatly as the mesh density increases. Sigmund and Petersson (1998) identified that the approaches taken to reduce mesh-dependence, also reduced checkerboard effects.

Figure 2 shows some results obtained from the work of Kim, *et al.* (2002). It is observed that with the use of the maximum instead of the mean von Mises equivalent stress, the final result becomes more dependent on the finite element mesh for the same structural analysis problem.

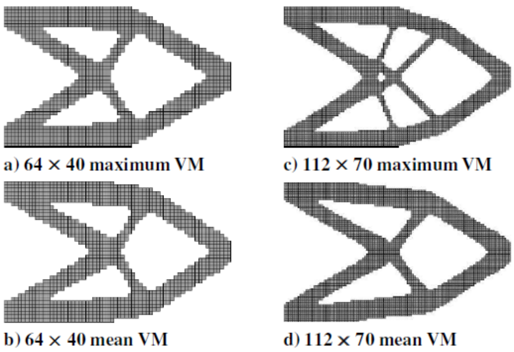


Figure 2. Qualitative representation of mesh-dependence for varying optimization driving criteria (Kim, *et al.*, 2002)

2.3.3 Local minima

The problem of local minima is related to the non-convex nature of topological optimization projects, due to a large number of results with localized solutions (Arora, 1989; Coutinho, 2006). For the same problem, many optimal solutions can be found depending on the choice of initial parameters, such as number of elements, geometry of design domain, iterative or stochastic coefficients of approximation, among others (Sigmund and Petersson, 1998). For Simonetti (2009), this is an important problem, since the solution is extremely sensitive to changes in its parameters. However, the control schemes applied for the other numerical instabilities tend to convexify the problems and produce reproducible designs (Sigmund and Petersson, 1998).

The difference between a local and global minimum for a bounded domain is shown in Fig. 3.

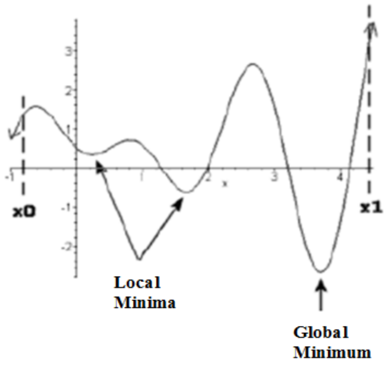


Figure 3. Representation of optimum points for a bounded domain (Arora, 1989 – adapted)

To eliminate or reduce the probability of a local optimum, it might be necessary that ESO should start from the full design domain, so that all elements are involved in the finite element analysis at least once (Huang and Xie, 2010).

3. NUMERICAL EXAMPLES

For the numerical examples resolution presented in this paper, it was considered a rectangular design domain with dimensions of 1.60 m x 0.20 m and thickness of 0.10 m. The material properties were assumed as Young's modulus $E = 30$ GPa and Poisson's ratio $\upsilon = 0.3$. To initiate the evolutionary process, it was used an initial rejection ratio $RR_0 = 10\%$ and an evolutionary ratio $ER = 0.5\%$. The high initial rejection ratio was adopted due to non-removal of elements for lower values, and for this reason a good element removal process could be obtained.

The design domain was discretized into a finite element mesh with 371 quadrilateral elements of type M3D4 from Abaqus® and its representation is shown according Fig. 4 where a long slender beam with its boundary conditions is related to two load cases, one static and one dynamic. The beam is fixed along both ends and the magnitude load and its application point will be the same for both load cases, so it is possible to evaluate the optimization effect for different types of analyses.

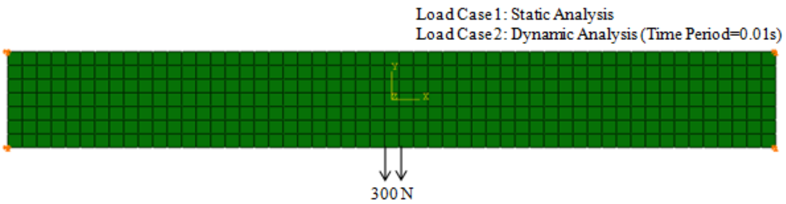
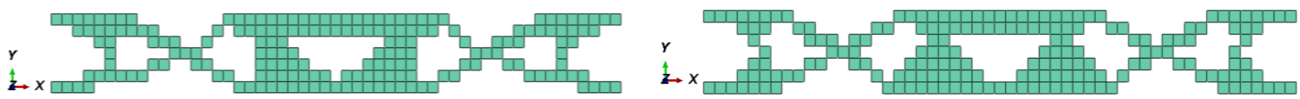


Figure 4. Initial design domain with boundary conditions (Author)

For both load cases it will be analyzed the linear and nonlinear effect which is characterized by a material response, initially elastic and from a specified stress level, it is characterized by an essentially plastic behavior (Natal Jorge and Dinis, 2005). The proposed optimization problem corresponds to an example studied in the paper of Jung and Gea. (2004), where 20% of the total design domain was used as the volume constraint. It will simulate an elastoplastic von Mises material with linear, isotropic work-hardening. The yield stress $\sigma_y = 280$ MPa and the hardening modulus $E^h = 500$ MPa were assumed.

The optimization problem studied in this paper was formulated as minimizing the mean compliance with a volume constraint of 50%.

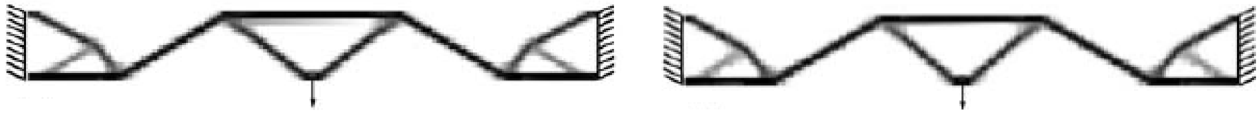
Figure 6 shows the optimal topology obtained for the linear and nonlinear static condition.



(a) (b)

Figure 6. Slender beam optimal design for static condition: (a) Linear Analysis and (b) Material Nonlinear Analysis (Author)

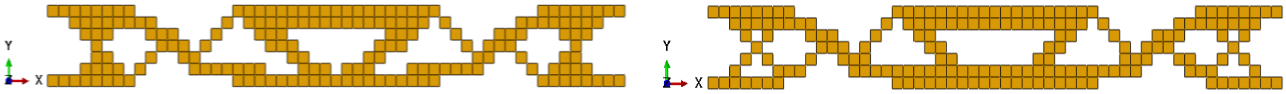
To verify the effectiveness of the proposed approach by the ESO methodology, the solution proposed by Jung and Gea (2004) is illustrated in Fig. 7. It is worth noting that the results from linear analysis and materially nonlinear analysis are almost identical for static condition.



(a) (b)

Figure 7. Solution of Jung and Gea (2004) for the static condition: (a) Linear Analysis and (b) Material Nonlinear Analysis (Jung and Gea, 2004 – adapted)

For the dynamic condition Fig. 8 shows the optimal topology which presented little difference between linear and nonlinear behavior.

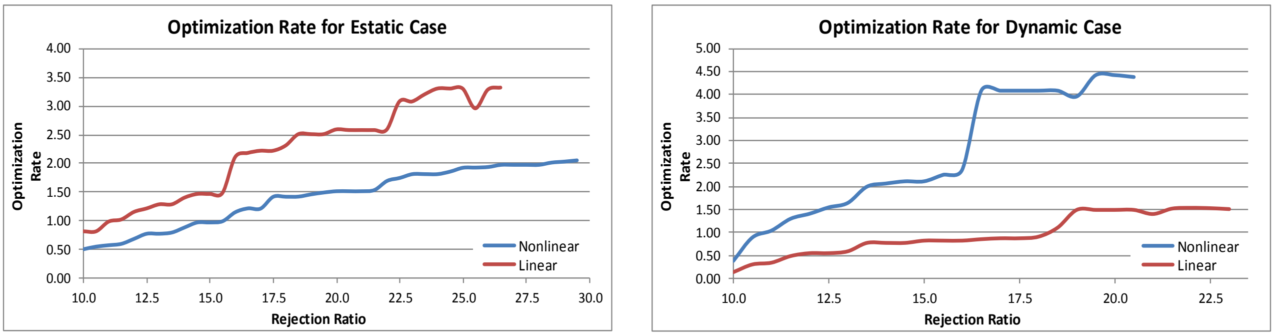


(a) (b)

Figure 8. Slender beam optimal design for dynamic condition: (a) Linear Analysis and (b) Material Nonlinear Analysis (Author)

To ensure that this evolutionary process is optimizing the structure, an optimization rate was created. It consists in a relation between removed volume fraction per maximum von Mises equivalent stress obtained by considering all finite elements involved in the analysis. This maximum von Mises equivalent stress can occur at a different finite element during the evolutionary process, giving a global performance for the structural optimization.

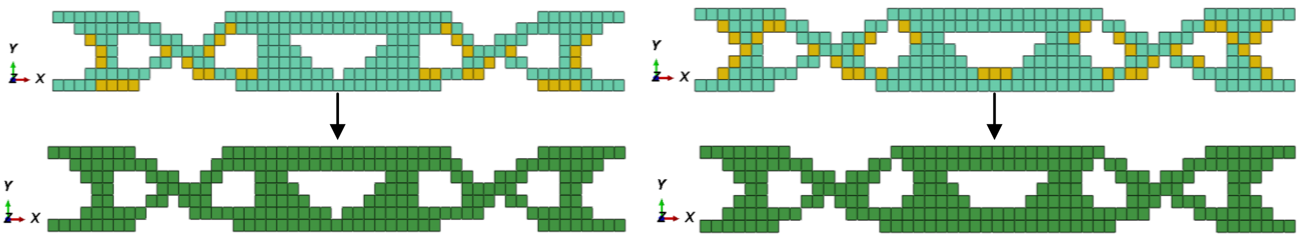
Figure 8 represents the optimization rate per rejection ratio RR_i for the static and dynamic cases.



(a) (b)

Figure 8. Optimization rate of evolutionary process for (a) Static and (b) Dynamic (Author)

Figure 9 shows the final optimal design by combining the performance of each load case for linear and non linear response.



(a) (b)

Figure 9. Slender beam optimal design considering the combination of static and dynamic load cases: (a) Linear Analysis and (b) Nonlinear Analysis (Author)

4. CONCLUSIONS

The evolutionary method presented in this paper has proved to be able to execute a routine in which it is possible to obtain a good direction of the optimal final shape of a structure under certain loading and boundary conditions.

The results instability has not taken into significant changes when compared with the classical example presented in the literature, once filtering techniques such as Nibbling ESO was implemented.

For better efficiency of the algorithm, it was used the resources available in the commercial software Abaqus®, which allows the execution of structural analyses requested by either static or dynamic actions, with or without nonlinearity.

Once the nonlinear analysis is more computationally expensive than the linear one, the applications that do not involve in such a high nonlinearity can produce satisfactory topology results with linear analysis too.

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6. REFERENCES

Arora, J. S., 1989. *Introduction to Optimum Design*. Elsevier, Iowa, 3rd edition.

Bendsøe, M.P. and Sigmund, O., 2003. *Topology optimization: Theory, Methods and Application*. Springer, Berlin, 2nd edition.

Coutinho, K.D., 2006. *Método de otimização topológica em estruturas tridimensionais*. MSc. dissertation, Universidade Federal do Rio Grande do Norte, Natal.

Das, R., Jones, R. and Xie, Y.M., 2011. *Optimal topology design of industrial structures using an evolutionary algorithm*. Optimization and Engineering, Vol. 12, p.681-717.

Díaz, A.R. and Sigmund O., 1995. *Checkboard patterns in layout optimization*. Structural Optimization, Vol. 10, p.40-45.

Greco, M., 2004. *Análise de Problemas de Contato/Impacto em Estruturas de Comportamento Não Linear pelo Método dos Elementos Finitos*. Ph.D. thesis, Escola de Engenharia de São Carlos, São Carlos.

Huang, M. W. and Arora, J.S., 1997. *Optimal design with discrete variables: Some numerical experiements*. International Journal for Numerical Methods in Engineering, Vol. 40, p. 165-188.

Huang, X. and Xie, Y.M., 2010. *Evolutionary Topology Optimization of Continuum Structures: Methods and Applications*. John Wiley & Sons, United Kingdom, 1st edition.

Jog, C.S. and Haber R.B., 1996. *Stability of finite element models for distributed-parameter optimization and topology design*. Computer Methods in Applied Mechanics and Engineering, Vol. 130, p. 203-226.

Jung, D. and Gea, H.C., 2004.*Topology optimization of nonlinear structures.* Finite Elements in Analysis and Design, Vol.40, p. 1417-1427.

Kim, H., Querin, O.M., Steven, G.P. and Xie, Y.M., 2002. *Determination of an optimal topology with a predefined number of cavities*. AIAA Journal, Vol. 40, p. 739-744.

Maute, K., Schwarz S. and Ramm E., 1998. *Adaptive topology optimization of elastoplastic structures*. Structural Optimization, Vol. 15, p. 81-91.

Natal Jorge, R.M. and Dinis, L.M.J.S., 2005. *Teoria da Plasticidade*. Universidade do Porto, Porto.

Querin, O.M., 1997. *Evolutionary Structural Optimisation: Stress Based Formulation and Implementation*. Ph.D. thesis, University of Sydney, Sydney.

Ramm, E and Matzenmiller, A., 1988. *Consistent Linearization in elastoplastic shell analysis.* Engineering Computations, Vol. 5, p. 289-299.

Sigmund, O. and Petersson, J., 1998. *Numerical instabilities in topology optimization: a survey on procedures dealing with checkerboards, mesh dependencies and local minima*. Structural Optimization, Vol. 16, p. 68-75.

Simo, J.C. and Hughes, T.J.R., 1998. *Computational Inelasticity.* Springer-Verlag, New York, Vol. 7.

Tanskanen, P., 2002. *The evolutionary structural optimization method: theoretical aspects*. Computer Methods in Applied Mechanics and Engineering, Vol. 191, p. 5485-5498.

Xie, M. Y. and Steven, G. P., 1993. *A simple evolutionary procedure for structural optimization*. Computer & Structures, Vol. 49, p. 885-896.

Xie, M. Y. and Steven, G. P., 1997. *Evolutionary Structural Optimization*. Springer-Verlag, London, 1st edition.

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