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APPLICATION OF INVERSE ANALYSIS TO OBTAIN THE PARAMETERS OF THE WEIGHTED-MULTI-POINT-SOURCE MODEL FOR FREE FLAMES

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Abstract. *Thermal radiation is responsible for a substantial portion of the heat release in a flame. Radiative characteristics of jet fires are usually expressed through the use of the fraction of heat radiated. Detailed flame simulations provide useful information but can be prohibitive for practical applications due to the significant computational resources required. The weighted-multi-point-source model use point sources with different weights to simulate the contribution of each discretized portion of flame. While previous studies used the trial-and-error approach to determine the sources weight, this paper proposes a method to obtain the weight of each source by inverse analysis. In the analysis, the experimental measured radiation heat flux distribution is the input data, while the weights of the sources and the fraction of heat radiated are the output data. The inverse problem is formulated as an optimization problem, which is solved by the generalized extremal optimization. The present study analyzes the influence of the number of sources and the length that the sources should be distributed on model results. Using seven sources uniformly distributed over 1.75 times the flame visible length, the maximum deviation between experimental radiation heat flux and results for all points of twelve flames were kept less than 5%. As will be show in the paper, the inverse method is capable of recovering the sources weight for the model that generates satisfactory results.*

Keywords: *Inverse analysis, weighted-multi-point-source model, open flame, optimization methods*

1. INTRODUCTION

Hankinson and Lowesmith (2012) stated that the radiative characteristics of jet fires can be expressed through the use of the fraction of radiated heat. The simplest model to predict the radiation heat flux around a flame is to consider that the flame is a source point. This approach gives good results in the far field, i. e., for distances larger than two flame lengths, but fails in the near field. According to Ekoto et al. (2014), detailed flame simulations have provided useful information on the interplay between flow dynamics and combustion chemistry, but frequently are prohibitive for practical applications due to the significant computational effort that is required. Sivathanu and Gore (1993) showed that predictions from a weighted multi-ray calculation of radiative heat transfer could replicate the radiation heat flux distribution from jet flames. The weighted-multi-point-source model (WMP), presented in Hankinson and Lowesmith (2012), use point sources with different weights to simulate the contribution of each discretized portion of flame.

In the present study, the WMP model is adopted to show that the radiation heat flux can be predicted satisfactorily at both the near-field as well as far-field radiation. In the solution, an inverse analysis is carried out, in which the radiation heat fluxes, measured at different heights at a fixed distance from the flame, are the input data, while the source weights and the fraction of heat radiated are left as the sought parameters. Two parameters will be analyzed. The first one is the flame length coefficient, which is used to multiply the predicted flame length and determine the distance of the sources distribution. The second parameter is the number of sources, based on the analysis of different number of sources and on the comparison of the model results with experimental data. The inverse problem is formulated as an optimization problem, which is solved by the generalized extremal optimization (GEO), as proposed in Souza et al (2003).

2. MATHEMATICAL MODEL

For this current study, a modelling approach is sought to predict the near-field as well as far-field radiation from a methane-air, laminar flame. The adopted model is a weighted-multi-point-source (WMP) model, whereby the radiation emanates from a number of point sources distributed along the flame axis, and the received incident radiation is determined as the sum of the radiation from each individual point source.

Incident radiation at a location, outside the fire, as shown in Figure 1, is given by the following relation:

$$q''_{i,f} = \sum_{j=1}^{N} \overrightarrow{q''_j} = \sum_{j=1}^{N} \frac{w_j X_R \dot{m} LHV \tau_j}{4\pi \overrightarrow{S_j^2}} cos\varphi_j \quad (1)$$

where w_j is the weight of the j_{th} point source, X_R is the fraction of heat radiated, $\dot{m}$ is the mass flow rate of fuel, LHV is the fuel lower heating value, τ_j is the transmissivity over the distance S_j, from the j_{th} point source to the receiver, and φ_j is the angle between the normal of the receiver and the line of sight to the j_{th} point source.

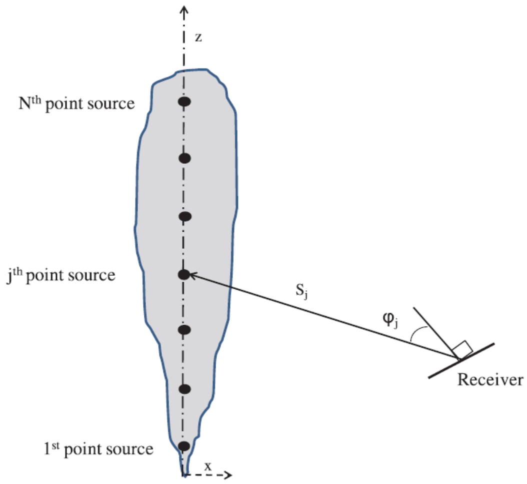

Figure 1. Multi-point source model adapted from Hankinson and Lowesmith (2012)

For this model, the length of the flame must be predicted in order to locate the sources point. Among the available relations, the correlation presented in Lowesmith et al. (2007), based on extensive large scale data, can be used:

$$L_f = \alpha \times Q_f^{0.3728} \quad (2)$$

Where the constant α is equal to $0.22 \frac{m}{kW^{0.3728}}$, Q_f is the flame power ($Q_f = \dot{m}LHV$). The sources are equally distributed over the length L_S, which can be related to the flame visible length by a constant c:

$$L_S = c \times L_f \quad (3)$$

According to Hankinson and Lowesmith (2012), this model approximates of the single point model in the far-field, and can give reliable predictions of incident thermal radiation (within ±20%). However, unlike the single point model, the WMP model can also provide reliable predictions in the near-field, providing therefore a more complete description of the radiation emitted from a flame. At the same time predictions of incident radiation very close are not meaningful, since the flame envelope itself would very likely extend to a diameter of 0.17 L_f (Turns and Myhr, 1991).

3. GENERALIZED EXTREMAL OPTIMIZATION ALGORITHM (GEO)

The inverse problem will be set as an optimization problem, which will be solved by the generalized extremal optimization (GEO) algorithm (Souza et al, 2003). This is an evolutionary algorithm devised to improve the extremal optimization method (Boettcher and Percus, 2001) so that it could be easily applicable to virtually any kind of optimization problem. Both algorithms were inspired by the evolutionary model of Bak and Sneppen (1993). Following the Bak and Sneppen (1993) model, in GEO L species are aligned and for each species it is assigned a fitness value that will determine the species that are more prone to mutate. One can think of these species as bits that can assume the values of 0 or 1. Hence, the entire population would consist of a single binary string. The design variables of the optimization problem are encoded in this string that is similar to a chromosome in a genetic algorithm (GA) with binary representation. Each design variable is represented by m bits, so the binary string has m bits times the number of design variables (N), L bits or species. Figure 2 show a generic string with N design variables, and L bits.

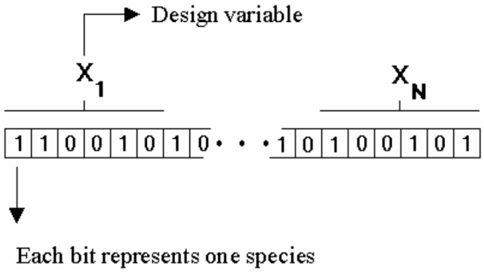

Figure 2. Design variables encoded in a binary string adapted from Souza et al (2003)

To each species (bit) it is assigned a fitness number that is proportional to the gain (or loss) the objective function value has in mutating (flipping) the bit. All bits are then ranked from rank 1, for the least adapted bit, to L for the best adapted. A bit is then mutated (flipped) according to the probability distribution of the k ranks, given by:

$$P(k) = k^{-\tau} \tag{4}$$

In which $(1 \leq k \leq L)$. In eq. (4), τ is a positive adjustable parameter (for $\tau \rightarrow 0$, the algorithm becomes a random walk, whereas for $\tau \rightarrow \infty$, we have a deterministic search). This process is repeated until a given stopping criteria is reached and the best configuration of bits found through the process is returned.

In a practical application of the GEO algorithm, the first decision to be made is on the definition of the number of bits that will represent each design variable. This can be done simply setting for each variable the number of bits necessary to assure a given desirable precision for each of them. For continuous variables the minimum number (m) of bits necessary to achieve a certain precision is given by:

$$2^m \geq \left[\frac{x_j^u - x_j^l}{p} + 1\right] \tag{5}$$

Where x_j^l and x_j^u are the lower and upper bounds, respectively, of the variable x_j (with $j = 1, N$), and p is the desired precision. The physical value of each design variable is obtained through the equation:

$$x_j = x_j^l + \left(x_j^u - x_j^l\right)\frac{I_j}{(2^m - 1)} \tag{6}$$

In which I_j is the integer number obtained in the transformation of the variable x_j from its binary form to a decimal representation. The complete implementation of the GEO algorithm was presented in De Sousa et al. (2003).

4. SOLUTION PROCEDURE

In this inverse problem, the weight of each source and the fraction of heat radiated are the GEO design variables. To define the precision of weights, it is chosen a precision that could represent the linear distribution of weights proposed by Hankinson and Lowesmith (2012) to twenty sources. Based on this precision, the number of bits that represent each weight and fraction of heat radiated is calculated with Eq. (5). In the particular problem, each variable was represented by 8 bits. As the solution of the inverse problem consists of optimization, one must set the objective function to be minimized or maximized. In the equation below, the objective function is given by the sum of the differences between the measured heat fluxes and obtained by the model at i_{th} point of measurements of f_{th} flame.

$$F_o = \sum_f^F \sum_i^I \sqrt{\left[q''_{\mathrm{meas,i,f}} - q''_{i,f}\right]^2} \tag{7}$$

in which $q''_{\mathrm{meas,i,f}}$ is the experimental radiation heat flux measured in the i_{th} measurement point of the f_{th} flame and $q''_{i,f}$ is the radiation heat flux estimated by the WMP model at the same point; I is the number of measurement points; and F is the number of flames, each one at a different power magnitude.

To minimize the objective function, the following procedure is followed:

1. A set of parameters are chosen randomly.
2. Each bit is flipped by the GEO and the model is solve to each set of design variables.
3. The objective function is calculated, and the bits are classified by least adapted to best adapted.
4. A bit is chosen randomly, and by Eq. (4) it is decided if it could be changed. This step is repeated until a bit mutation takes place.
5. Repeat from step 2 until a stop criterion is reached.
6. The value of F_O is stored, and the algorithm is restarted at step 1 with the other set of random parameters until a stop criteria.
7. The mean value of F_O is stored and the algorithm is restarted again in step 1 with another value of the parameter τ.
8. It is chosen the design variable of lowest F_O of τ that provides the lowest mean F_O.

In an effort to measure the accuracy of the model, it is proposed a mean deviation and a maximum deviation given by the following equations:

$$\gamma_{i,f} = \left| \frac{q''_{i,f} - q''_{\mathrm{meas,i,f}}}{\max\limits_{\mathrm{i}} q''_{\mathrm{meas,i,f}}} \right| \times 100\% \quad (8)$$

$$\gamma_{f,mean} = \frac{\sum_{i=1}^{I} \gamma_{i,f}}{I} \quad (9)$$

$$\gamma_{f,max} = \max_{i} \gamma_{i,f} \quad (10)$$

5. RESULTS AND DISCUSION

To feed the inverse problem and obtain the weights of each source, it is needed radiation heat flux data. The experimental data were measured in laminar flames of methane stabilized in a Santoro burner. This burner has a central tube with 11,1 mm i.d. for the fuel injection and a 101,6 mm i.d. annulus for the coflow. The values of radiation heat fluxes at each position are represented by the difference between the main and the background signal for a sample of 50 acquisitions. The uncertainty in the fluxes is given by the root-sum-square (RSS) of the uncertainty of each acquisition of the sample, accounting the variance of the sample, the uncertainty of the sensor and the uncertainty of the acquisition system. The sensor utilized was a MEDTHERM 64-0.5-20/ZnSeW, which has transmissivity almost constant from 0.7 µm to 17 µm and is factory calibrated with a blackbody simulator. The uncertainty for the sensor is 3% of the reading signal for a confiability of 95%. The acquisition system is an Agilent 34972A with uncertainty of 0.0030% of the reading plus 0.0035% of the range used. Details of the experimental procedure can be found in Machado (2014).

In the first analyzed case, the WMP model was supplied with radiation heat flux from 39 measurements point of a 0.25 kW methane flame. By the application of the inverse analyses, it was obtained the weight of each point source, and the fraction of heat radiated. It was considered seven sources placed at the flame axis, with the weight obtained by inverses analyses, the radiation heat flux was calculated at the same coordinates of the experimental measurements. First, two GEO parameters need to be set. The first one was τ, which was tested in a range of 0.25 to 2.5 in steps of 0.25 and plotted on Figure 3(a).

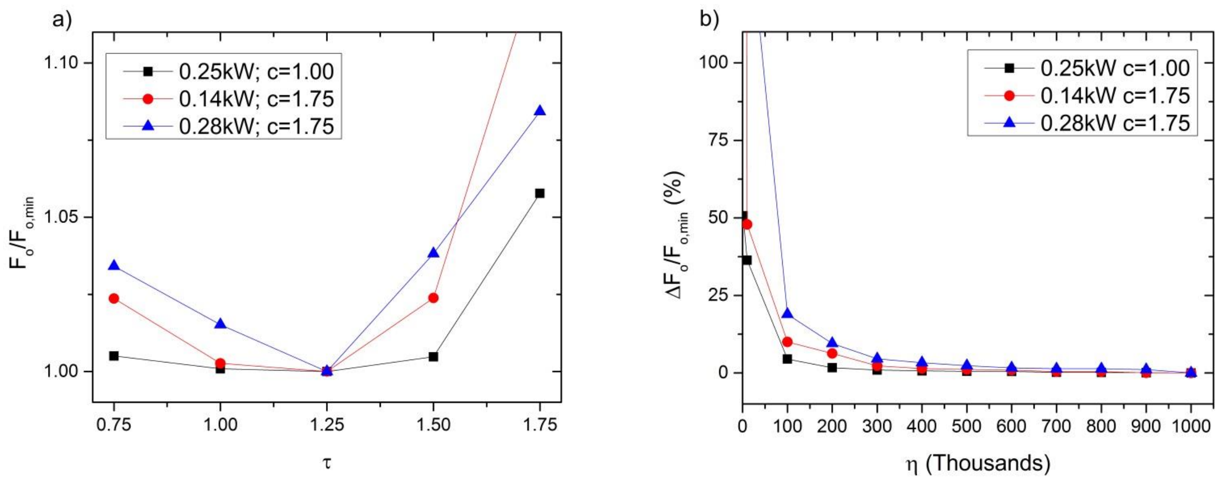

Figure 3: (a) GEO's parameter τ; (b) Number of objective function evaluation

It is possible see in Fig. 3(a) that τ equal to 1.25 generate the lowest mean F_o, that is, the used parameter. Secondly, it is necessary to guarantee that the number of objective function evaluations (η) is sufficient to achieve the needed accuracy. Figure 3(b) shows the mean objective function over the number of objective functions evaluations (η). The mean objective function has no substantial changes after 500 thousands evaluations, less than 0.5%. Figure 4 a) shows the heat flux distribution obtained by WMP model and experimental data.

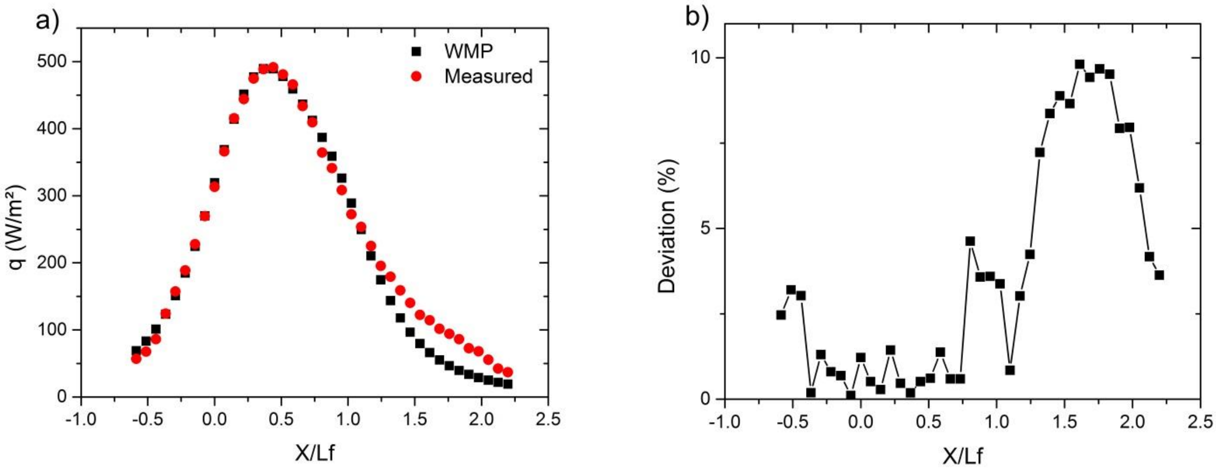

Figure 4: (a) Radiation heat flux at 0.0536 m of flame axis, (b) Deviation between WMP model and experimental data

The graphic show that the radiation heat flux distribution, obtained by the WMP model, has good results between zero and one time the flame length. Although the most radiation heat flux occurs at the visible portion of the flame, the high temperature combustion products, a participating medium, contribute to the radiation heat transfer. This occurs because the utilized fuel, methane, has little soot production. According to Nathan et al. (2012) the presence of soot increases substantially the heat transfer by radiation. Figure 4(b) shows the deviation between the WMP model and experimental data. The deviation increase, at a axial distance greater then a flame length, is due to the sources are allocated only at the visible portion of the flame, because of that the radiation heat flux are more sensitive to the changes of sources weights at that portion of measurements points. To account the high temperature gases radiative heat contribution the seven sources were distributed differing the coefficient that multiply the flame visible length.

5.1 Sources Distribution

The analyzed coefficients are in the range of 1.0 to 2.0 with increments of 0.25. The flame length utilized was calculated by Eq. (2), for this flame is 0.131 m. Despite the experimental measurements of flame length was of 0.088 m, and others equation have more precise results, the Eq. (2) is utilized because of its simplicity, since it could predict the flame length with only the flame power. Figure 5(a) shows the deviation between the radiation heat flux obtained by the WMP model and the experimental data over the flames lengths coefficients range.

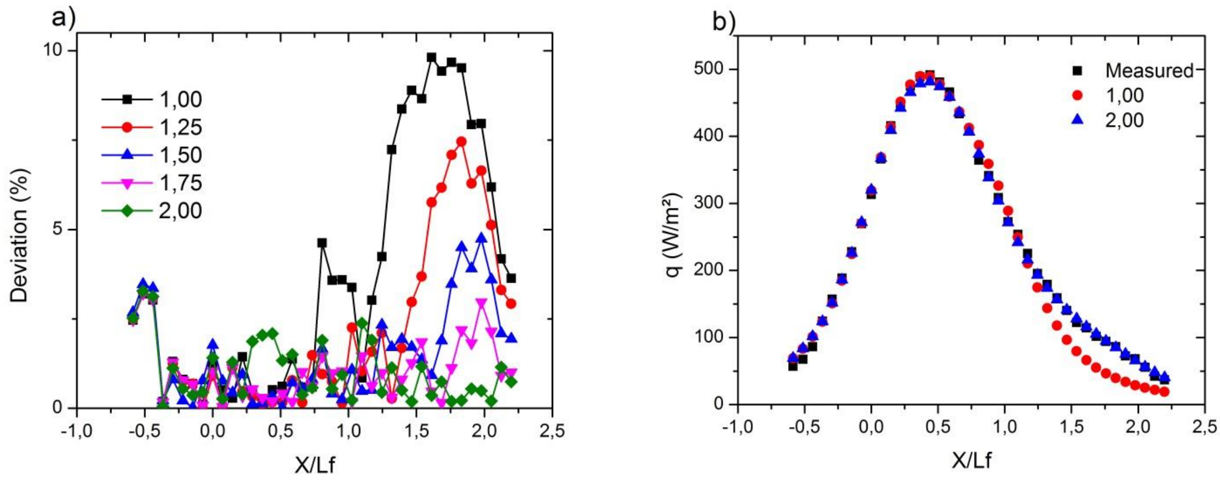

Figure 5: (a) Deviation of WMP model and experimental, (b) Radiation heat flux c equals to 1.0, 2.0 and experimental

The deviation decrease when the length where the sources are placed are higher than the flame length, showing that the gases portion of radiation heat flux have an important contribution in that location. The results with the sources placed at two times the flame length have the minor mean deviation, 1.04 %, despite the deviation increase at distances less than one flame length. Figure 5(b) shows the results of the calculated radiation heat flux for sources placed at one and two times the flame length in comparison with experimental data.

In the figure, it can be observed that in the second case the results are closer of the experimental data. The results rely on the allocated of the sources. So to define the flame length coefficient, the WMP model was fed with experimental measurements of a twelve-flame set. The set has the volumetric flow rate of methane ranging from 0.25 l/min and 1.00 l/min. Through inverse analyses the weight of each of seven sources, of each of twelve flames, for five different flame length coefficients in a range of 1.0 and 2.0, a total of 60 trials. All trials followed the same procedure that was shown for case one to determine parameter τ. Table 1 shows the results summary.

Table 1. Deviation between WMP model results and experimental data.

c	Mean Deviation [%]	Maximum Deviation [%]	Fraction of Heat Radiated Deviation [%]
1.00	2.47	12.26	5.43
1.25	1.87	8.61	5.15
1.50	1.54	6.38	6.21
1.75	1.38	4.99	6.33
2.00	1.58	6.87	6.81

The results shows that the WMP model could predict the experimental measurements with deviation less than 5 % for 1.75 times the flame length, for all measurements point of twelve flames. Another parameter that can affect the results is the number of sources, more sources might discretize better the flame. On the other hand with more sources the solution is more difficult for the inverse algorithm, i. e., there are more variables to be sought, with considerable increase in the solution.

5.2 Number of Sources

To analyze the source's number on the results, the WMP model was fed with experimental data of two flames, 0.14 kW and 0.28 kW. These flames are chosen because the first one has the greatest difference between the measured visible length and predicted by Eq. (2). If this disturbs the results, a solution with more sources could be solved since the inverse solution might consider the weights over a specific distance to zero. i. e. this flame may need more sources to reach satisfactory results. The second flame power is in the middle of the set flames and is a representative flame.

By mean of the inverse analysis, it was calculated the weight of each source and the fraction of radiated heat for the configurations of seven to fifteen sources. Figure 6(a) shows the mean deviations between the WMP model results and the experimental data for two analyzed flames. The GEO parameter τ is shown in Figure 3 for both flames, with length coefficient equal to 1.75 times the flame length and for the case of seven sources.

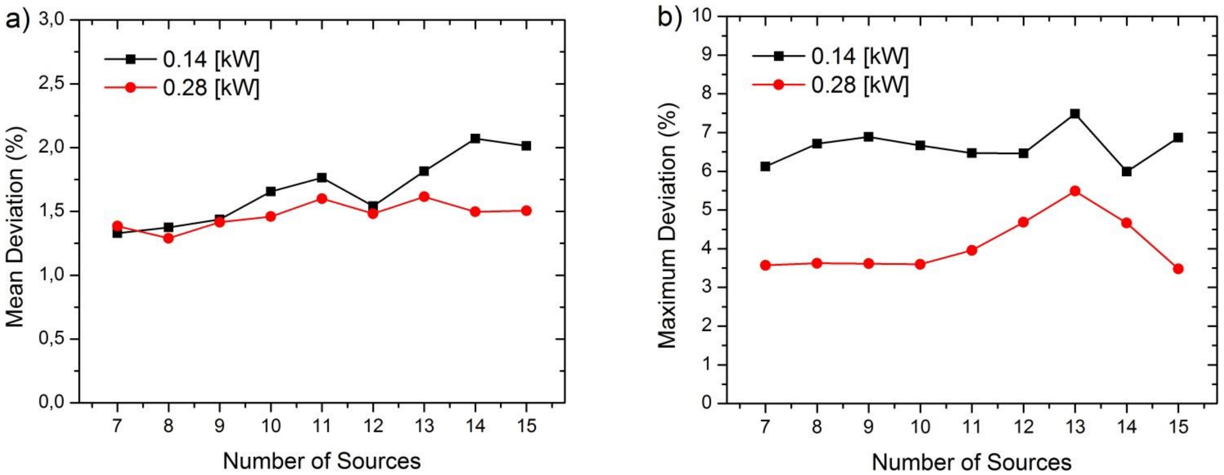

Figure 6: (a) Mean deviation (b) Maximum deviation between WMP results and experimental data

As seen, the mean deviation lightly increases with the increase in the number of sources, which shows that a higher order discretization of flame in source points does not guarantee that the results will be better. This can be explained by the fact that more sources introduces more unknowns to the inverse problem, making it more difficult to find the global minimum. Figure 6(b) shows the maximum deviation between WMP results and experimental data for the two analyzed flames.

Thus, the increase in the number of source points did not generate more accurate results. Actually the fluctuation of the maximum deviation seemed more related to the characteristics of the inverse method than to the number of sources. As the results were not dependent on the number of sources, the computational cost can be used to determine the number of sources. Figure 7 shows the computational cost normalized by the time of solution with seven sources.

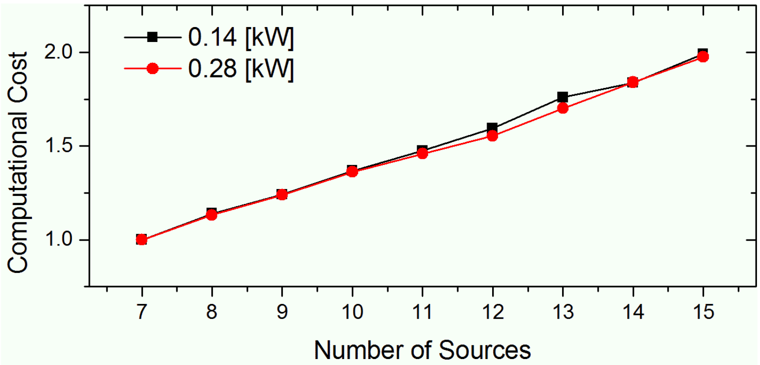

Figure 7. Computational cost

As shown in the figure, the computational costs increase linearly with the increase of the number of sources. The best choice here is the lowest computational solution, in this case seven sources. Although the results suggest testing with fewer sources, the computational cost is low for direct calculation with seven sources that have not been tested.

6. CONCLUSION

The weighted multi point source model was capable of making satisfactory prediction of the radiation heat flux in the near field of the considered set of flames. The proposed inverse approach proved capable of obtaining the sources weights, which proved to be a better alternative than the method of trial and error. Although the results did not have correlations with the tested number of sources, it was found that only seven sources required relatively low computational cost, but was enough to guarantee accurate results. As seen, the products of combustion for the analyzed set of flames had a considerable contribution to total radiation heat; the flame length coefficient that showed the best results was 1.75. The methodology was applied to a set of laminar methane flames, but it could be used to others fuels, flow regime and different magnitudes of flame powers.

7. ACKNOWLEDGEMENTS

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