COBEM2015

ABCM Associação Brasileira de Engenharia e Ciências Mecânicas

23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

A RELIABILITY-BASED TOPOLOGY OPTIMIZATION APPROACH USING BESO

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Abstract. *Topology optimization has been given a significant amount of research due to its importance in several fields of engineering. Nowadays many studies take into account the present randomness and uncertainties in the variables of the problem. The objective of this work is to study reliability-based topology optimization (RBTO) problems in order to obtain optimal structural topology where constraints are satisfied while uncertainties in design variables are considered. As the suggested topology optimization method it is adopted the bi-directional evolutionary structural optimization (BESO) where material can be removed and added simultaneously. The reliability methodology used in this paper to find the most probable point (MPP) of failure is the first order reliability method (FORM). The resulting topology of the deterministic topology optimization (DTO) is compared with that obtained from the RBTO. The examples include 3D structures and a comparison on different parameters for the filter scheme in the BESO method.*

Keywords: *reliability, optimization, RBTO, BESO, FORM*

1. INTRODUCTION

The topology optimization of continuum structures consists of determining the shape, connectivity and voids in a given domain satisfying some constraints. That type of design provides more freedom than other optimization methods but technically, it is more challenging. The research in this topic have been growing considerably in the past two decades in academia as well as in industrial applications, being used from large-scale structures (Stromberg *et al.*, 2011) to microstructures (Zhou and Li, 2008). In the survey conducted by Deaton and Grandhi (2014) it is presented several modern works using different methods of topologic optimization such as the density-based, boundary variation, “hard-kill” and others.

Design variables or parameters might have some degree of uncertainty in real life problems. That variability should be taken into account, and there are several approaches to quantify it. In practical optimization studies, reliability based techniques are becoming popular, due its capability to deal with that uncertainty. In the last few years there are already some works that research the topology optimization with reliability constraints like in Kharmanda *et al.* (2004), Rozvany and Maute (2011) and Eom *et al.* (2011).

In this work, it is used the bi-directional evolutionary structural optimization (BESO) as the topology optimization method integrated with the first order reliability method (FORM). BESO research started initially by Yang *et al.* (1999) but today the literature on both ESO and BESO is extensive (Huang and Xie, 2010). The BESO methodology allows material to be removed and added simultaneously. In case of reliability-based design, FORM is choose, as previously said, due to less computational cost when compared to other methods, for instance Monte Carlo Simulation, but it might have low accuracy. Examples for 3D structures comparing different parameters for the BESO filter scheme and final topology solutions in deterministic optimization and RBTO are performed.

2. RELIABILITY-BASED TOPOLOGY OPTIMIZATION (RBTO)

In this section it will be discussed the methodology for the BESO topology optimization and the reliability analysis based on FORM.

2.1 Bi-directional Evolutionary Structural Optimization (BESO)

The BESO method is based on the removal and addition of elements to maximize stiffness, which is the same as minimize compliance, for a given volume. In that case, each element is treated as a design variable. The problem is usually stated as the minimization of compliance with a volume constraint, Eq. (1).

$$\begin{aligned} &\mathit{Minimize} \quad C = \frac{1}{2}\boldsymbol{f}^T\boldsymbol{u} \\ &\mathit{Subject\ to}: \quad V^* - \sum_{i=1}^{N} V_i x_i = 0 \quad x_i \in [0,1] \end{aligned} \tag{1}$$

where $\boldsymbol{f}$ is the applied load, $\boldsymbol{u}$ is the displacement vector, C is the mean compliance. V^* is the objective volume, V_i is the *i*-th element volume and x_i is either the existence, 1, or absence, 0, of that element.

When an element is removed or added to the structure, the change of the mean compliance is the same as the elemental strain energy and defined as elemental sensitivity number:

$$\alpha_i^e = \Delta C_i = \frac{1}{2}\boldsymbol{u}_i^T\boldsymbol{K}_i\boldsymbol{u}_i \tag{2}$$

where $\boldsymbol{u}_i$ is the nodal displacement vector of the *i*-th element and $\boldsymbol{K}_i$ is the element stiffness matrix.

2.1.1 Filter scheme

Depending on the methodology, checkboard patterns might harm topology optimization and result in structures that are harder or infeasible to be produced. To bypass that difficulty there are techniques such as filters schemes. Herein it is used the well described filter proposed by Huang and Xie (2010). The filter also helps with another problem: the mesh-dependency, which is defined as the different topology resulting from different mesh discretization.

This filter works in the following way: the elemental sensitivity number, Eq. (2), is updated based on the elemental sensitivity number of its neighborhood elements, considering those elements ranging a defined distance r_{min}.

$$\alpha_i^e = \frac{\sum_{j=1}^{M} \omega(r_{ij})\alpha_j^e}{\sum_{j=1}^{M} \omega(r_{ij})} \tag{3}$$

where M is the number of elements in the range r_{min} from the *i*-th element, r_{ij} is the distance between the center of the *i*-th element and the *j*-th element, and finally, $\omega(r_{ij})$ is the linear weight factor defined as

$$\omega(r_{ij}) = r_{min} - r_{ij} > 0 \qquad (j = 1,2, \dots, M) \tag{4}$$

Figure 1 illustrates the neighborhood elements in a mesh that will be taken into account for the sensitivity number of the central element. It is considered all those that its center is inside the r_{min} range, and then they will be weighted according to their distance, Eq. (4).

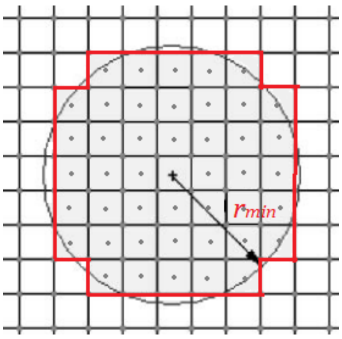


Figure 1. Filter scheme to smooth the elemental sensitivity number.

That technique smooths the sensitivity numbers in the design domain and is defined heuristically. In Section 3 it is shown an example and results for a structure, varying the value of r_{min}.

To help the convergence for the final topology, Huang and Xie (2007) also propose a temporal filter, averaging the elemental sensitivity number with the corresponding previous information, as follows:

$$\alpha_i = \frac{\alpha_i^k + \alpha_i^{k-1}}{2} \tag{5}$$

where *k* is the current iteration number.

2.1.2 Element removal and addition

At each iteration it is defined a new target volume, until the final objective volume, V*, is reached. This can be expressed as

$$V_{k+1} = V_k(1 \pm E_r) \quad (6)$$

where E_r is an evolutionary volume ratio. Once $V_{k+1} = \mathrm{V}^*$, the volume will be keep constant.

To define which elements will be present in the next iteration, it is defined a threshold sensitivity number, α^{th}. This can be easily determined by sorting all elemental sensitivity numbers and defining the threshold α^{th} according to the percentage of the volume for the next iteration. For example, if V_{k+1} is 70% of the initial volume, 30% of the elements will be voids (0), and it will be those with the lower sensitivity number $\alpha_i < \alpha^{th}_{70\%}$.

The convergence criterion to stop the topology optimization is based on the compliance stability in the last 10 successive iterations and is calculated as

$$error = \frac{|\sum_{i=1}^{5} C_{k-i+1} + \sum_{i=1}^{5} C_{k-5-i+1}|}{\sum_{i=1}^{5} C_{k-i+1}} \leq 0.001 \quad (7)$$

where k is the current iteration number and C the mean complete volume compliance. The value of E_r is divided by half if $error < 0.01$, so less material changes at each iteration as the error approaches convergence.

2.2 First Order Reliability Method (FORM)

The reliability analysis is performed on the problem constraints where uncertain parameters are present. Thus, there is an associated failure probability that can be stated as the following equation:

$$P(g(\boldsymbol{y}) < 0) < P_f^{target} \quad (8)$$

where $P()$ is the probability calculation, $\boldsymbol{y}$ is a vector of the random variables, g is the limit state function and P_f^{target} is the target failure probability. The limit state function is a surface constraint that represents failure when its value is negative.

Within FORM framework, the random variables $\boldsymbol{y}$ are transformed in standard normal and independent variables $\boldsymbol{z}$. The most probable point of failure (MPP) is calculated by finding the nearest distance (β) between the linearized failure surface, $g(\boldsymbol{z}) = 0$, and the origin. Figure 2 exemplifies the FORM method in a two-dimensional case.

The failure probability can be calculated by Eq. (9)

$$P_f = P(g(\boldsymbol{z}) < 0) = \int_{g(\boldsymbol{z})<0} f(\boldsymbol{z}) d\boldsymbol{z} \cong \Phi(-\beta) \quad (9)$$

where Φ is the standard normal cumulative distribution function (CDF) and β the reliability index.

As can be observed, there is a minimization problem with one equality constraint in order to define the MPP

$$Minimize\ ||\boldsymbol{z}||,\ \ subject\ to\ g(\boldsymbol{z}) = 0 \quad (10)$$

One very popular algorithm for structural reliability analysis to find $min||\boldsymbol{z}||$ is that proposed by Hasofer and Lind (1974) and improved by Rackwitz and Fiessler (1978). Eq (11) defines the used formulation:

$$z^{k+1} = \frac{1}{||\nabla g(z^k)||^2}[\nabla g(z^k)^{\mathrm{T}} z^k - g(z^k)]\nabla g(z^k) \quad (11)$$

Utilizing the Reliability Index Approach (RIA), the Eq. (8) can be rewritten as

$$\beta_{target} - \beta(g(\mathbf{y}) = 0) < 0 \quad (12)$$

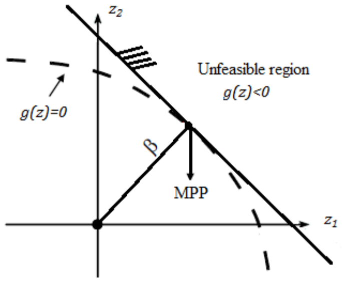


Figure 2. The FORM approach.

3. NUMERICAL EXAMPLES

In this work, it used a regular cubic element with unitary size/volume. The Poisson's ratio is 0.3 and the applied loads and Young's modulus are unitary as well. The analysis was performed for a 3D cantilever beam with a load in the free tip, as shown in Fig. 3. All the examples started the optimization with full design domain assigned with non-void elements with an evolutionary volume ratio of $E_r = 0.05$.

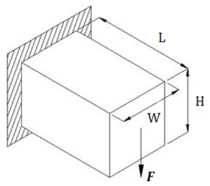


Figure 3. Design domain of the cantilever beam.

3.1 Filter scheme analysis

In this example, it is performed a regular BESO topology optimization, where the objective is to find a structure with minimum compliance for a specified volume, V^*. Here, the objective is to compare different values for the filter parameter r_{min} and its consequences on the final topology. The dimensions are, as Fig. 3, length $L = 64$, height $H = 40$ and thickness $W = 40$. The design domain was discretized in 102400 (64×40×40) cubic elements. It is performed a deterministic topology optimization (DTO) without constraints but that from Eq. (1) where the target volume V^* being 8% of the initial full volume.

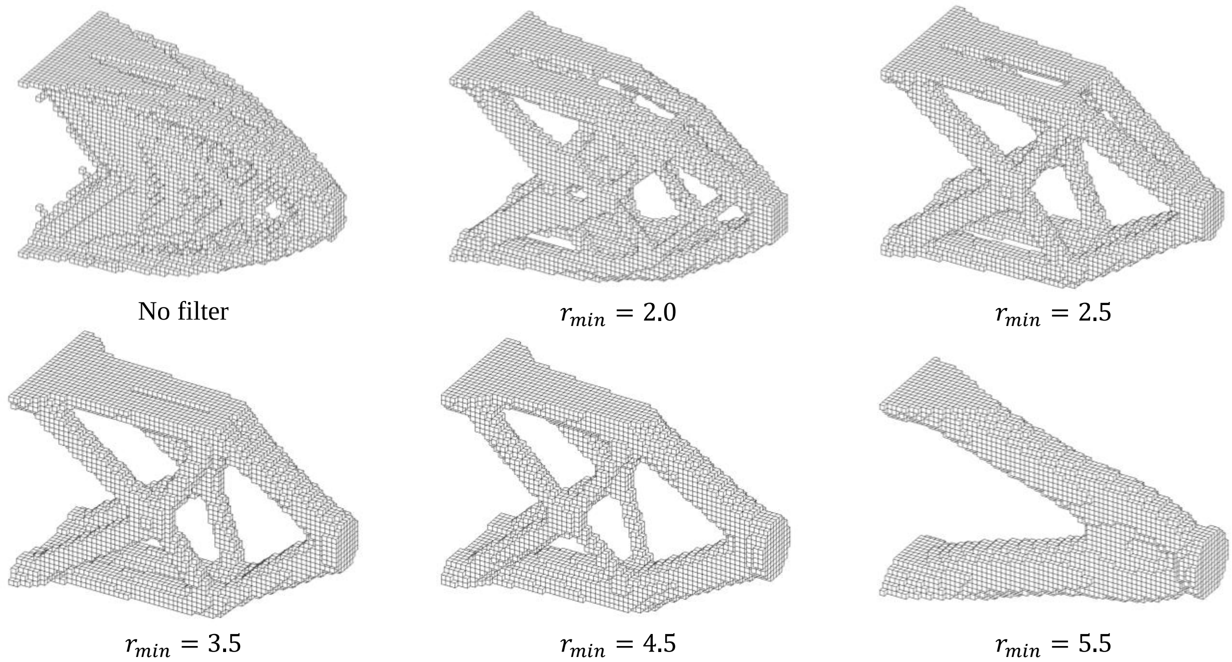


Figure 4. Filter scheme for different r_{min} parameter setting.

In Fig. 4 is shown the importance and variability on the final topology depending on the filter parameter. As a result of this process, the higher r_{min} gets, the lower the stiffness goes (or higher is the mean compliance), and that's the compensation for a smoother structure. The value of the mean compliance for the structure without filter is 9.03% lower than the one with $r_{min} = 5.5$. All the others compliances range in-between.

3.2 Deterministic Topology Optimization (DTO)

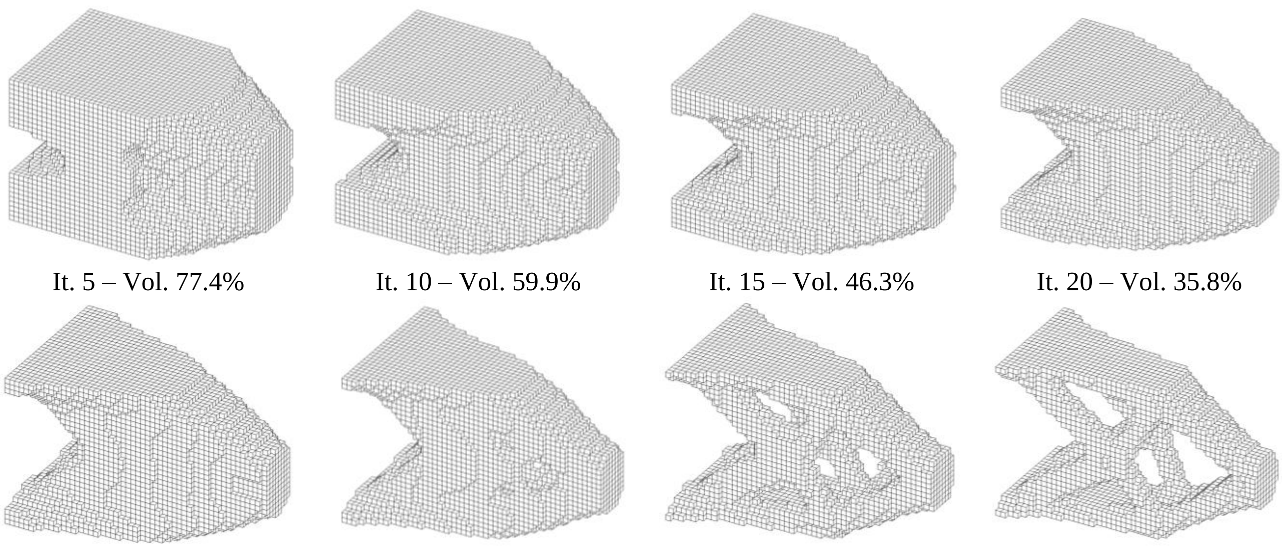
For the DTO it is imposed a displacement constraint. The problem can be stated as:

$$\begin{aligned} &Minimize && V \\ &Subject\ to\text{:} && V^* - \sum_{i=1}^{N} V_i x_i < 0 \quad x_i \in [0,1] \\ & && \delta_{max} \leq 3.6 \end{aligned} \tag{13}$$

where δ_{max} is the maximum absolute value in the displacement vector $\boldsymbol{u}$ and V^* being 8% of the initial full volume.

In this case, there is no safety factor, so the optimization will converge to one of those two constraints. The dimensions are $L = 48$, height $H = 30$ and thickness $W = 30$, (43200 elements) and a unitary load in the free tip of the cantilever beam. It is set $r_{min} = 3.0$ as the filter parameter.

Figure 5 shows the optimization evolution through the iterative progress and Fig. 6 shows the final solution for this problem, with a volume of 9.8% of the initial volume, but above V^*, with 4231 elements. The constraint for δ_{max} was reached and this stopped the volume minimization.



It. 5 – Vol. 77.4% | It. 10 – Vol. 59.9% | It. 15 – Vol. 46.3% | It. 20 – Vol. 35.8%

It. 25 – Vol. 27.7% | It. 30 – Vol. 21.5% | It. 35 – Vol. 16.6% | It. 40 – Vol. 12.8%

Figure 5. Topology optimization evolution.

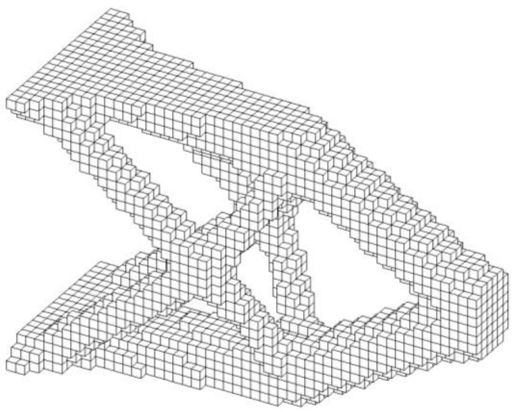


Figure 6. Final topology for DTO (It. 53 – Vol. 9.8%).

3.3 Reliability-Based Topology Optimization (RBTO)

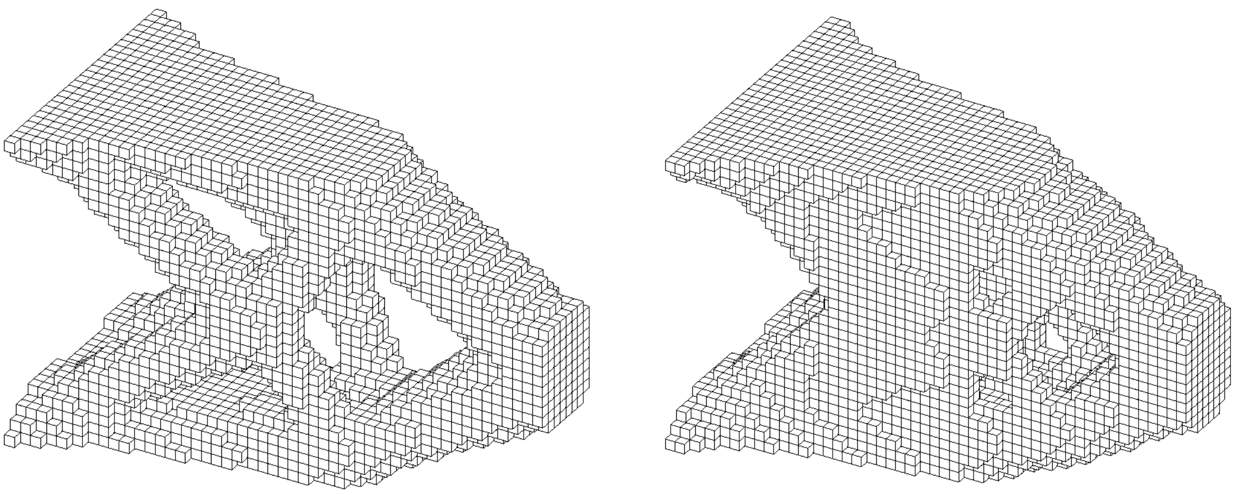
For the RBTO example, the load $\boldsymbol{f}$ is considered a random variable with unitary arithmetic mean, 0.3 of standard deviation and a normal distribution (~**N**(1.0,0.3)). The dimensions are the same as the previous example (3.2). The displacement deterministic constraint now is replaced with a probability of failure constraint. Two cases are executed here, one with $\beta_{target} = 2.0$ $(P_f^{target} = 2.28\%)$ and $\beta_{target} = 3.0$ $(P_f^{target} = 0.13\%)$.

$$
\begin{aligned}
&Minimize \quad V \\
&Subject\ to\text{:} \quad V^* - \sum_{i=1}^{N} V_i x_i < 0 \quad x_i \in [0,1] \\
&\qquad P(g(\boldsymbol{f}) = \delta_{max}(\boldsymbol{f}) - 3.6 < 0) < P_f^{target}
\end{aligned}
\tag{14}
$$

The results comparing the DTO case with those RBTO examples are exposed on Tab. 1. The resulting topologies for RBTO are shown in Fig. 7.

Table 1. Results comparison between DTO and RBTO examples.

	DTO	RBTO (β=2.0)	RBTO (β=3.0)
δ_{max}	3.60	2.26	1.91
Volume % (Elements)	9.8% (4231)	18.0% (7783)	22.2% (9582)
C	1.8549	1.1285	0.9561



β=2.0 β=3.0

Figure 7. Final topology for RBTO with both reliability constraints.

4. CONCLUSIONS

In this paper it was performed a study on topology optimization using BESO method associated with reliability constraints analyzed by FORM. A 3D cantilever beam was used as the main example. Firstly, it was shown the importance to of the filter parameter r_{min} on the final results, where a smoother structures leads to higher mean compliance. Later, a comparison between DTO and RBTO results was performed and it is shown that even if the topology seems similar, there is a larger amount of elements in the RBTO's final result. Since 3D structures needs a large amount of elements in an acceptable refined mesh, it is necessary to have a relatively large computational memory available to deal with huge matrices. The computational cost of RBTO is considerably higher than DTO since the finite element function is called several times to find the reliability index at each iteration, so methods that accelerate this process might be useful. It should be also emphasized the importance in performing 3D studies (instead of simple 2D ones), since this allows some complexity to show up in the final results, what is not possible in 2D problems.

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