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PATH ANGLE AND VELOCITY CONTROL OF AIRCRAFT USING KREISSELMEIER'S STRUCTURE AND THE LQG-LTR METHOD

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Abstract. *The closed loop control of aircraft velocity and flight path angle is addressed, to enable their use in 4D trajectory managers. The solution proposed is based on the two-degree-of-freedom topology of Kreisselmeier, which is employed along with concepts from robust control. More specifically, control design is carried out using the LQG-LTR method and its generalization TFL-LTR. To validate the proposed solution, a case study is used which bases on the model of a commercial passenger aircraft. The specifications were defined based on control law design best practices of the aviation industry. The case study showed that the controller based on the proposed topology meets the specifications and yields a closed loop system with interesting robustness properties.*
Keywords: *Kreisselmeier Topology. LQG/LTR. 4D trajectory.*

1. INTRODUCTION

The air transportation system is a key element in the world economy. The expansion of this sector is necessary to support the increased demand generated by economic growth. The air traffic control centers of large airports have found problems with the increase in demand for flights, resulting in delays and increased likelihood of accidents. Current Air Traffic Control (ATC) procedures and technologies will not be able to accommodate the increased traffic demand while maintaining the same levels of safety. In view of this, it is desirable to have automated systems that can reduce the workload of air traffic controllers. Two projects have studied the use of the 4D trajectory concept as a means of achieving that objective: NextGen and SESAR. NextGen project defines that concept as an accurate description of the trajectory of an aircraft in space and time, which is defined with reference to the latitude, longitude and altitude, as well as the time when the path should be performed (ENEA & PORRETTA, 2012).

4D flight management can be implemented through closed loop control performed by the FMS (Flight Management System), using the autopilot/auto-throttle system. The aircraft follows a path previously calculated by an optimization algorithm, taking into account mainly the fuel consumption and the total flight time. In this context, Bousson & Machado, 2010, propose an optimal trajectory generation algorithm, which delivers velocity and flight path angle references at predefined points along the trajectory of the aircraft, considering the longitudinal axis.

In this study, the design of velocity and flight path angle control for commercial aircraft is presented, aiming at their use in a 4D trajectory manager. The control topology used is the two degrees of freedom strategy proposed by Kreisselmeier, so that the regulation part of which was designed by using the LQG / LTR method (Linear Quadratic Gaussian / Loop Transfer Recovery).

This paper consists of five sections, including this introduction. Section 2 addresses context-relevant aspects of the longitudinal model of an aircraft. Section 3 deals with the system control topology and shows the control theory used. Section 4 shows the results obtained by using this topology in a commercial aircraft model. Section 5 presents the final considerations and proposals for future work.

2. AIRCRAFT MODEL

The dynamics equations for the aircraft longitudinal movement can be summarized as shown below (ROSKAM, 2001):

$$\dot{V} = \left(F_p\cos(\alpha + \alpha_F) - D - mg\sin(\gamma)\right)/m \tag{1}$$
$$\dot{\gamma} = \left(F_p\sin(\alpha + \alpha_F) + L - mg\cos(\gamma)\right)/mV \tag{2}$$

$$\dot{\alpha} = q - \dot{\gamma} \quad (3)$$
$$\dot{q} = (M + M_F)/I_Y \quad (4)$$
$$\dot{H}_a = V\sin(\gamma) \quad (5)$$
$$\dot{x} = V\cos(\gamma) \quad (6)$$

The above equations represent the state equations for the longitudinal dynamics. The state variables are velocity, (V), altitude, (H_a), longitudinal distance, (x), flight path angle, (γ), angle of attack, (α), and pitch rate (q).

In the above equations, F_p is the propulsive force and, α_F the angle between the propulsive force and the velocity vector of the aircraft. The expressions defining lift, (L), and drag, (D), are given by:

$$L = \frac{1}{2}\rho_a V^2 S C_L \quad (7)$$
$$D = \frac{1}{2}\rho_a V^2 S C_D \quad (8)$$

where, C_L and C_D are the lift and drag coefficients, S is the equivalent area considered, V is the velocity of the aircraft and ρ_a is the air density at considered altitude. The aerodynamic pitch moment, M, and the moment due to propulsive forces, M_F , are given by:

$$M = \frac{1}{2}\rho_a V^2 S C_m \quad (9)$$
$$M_F = F_p z_F \quad (10)$$

where, C_m, is the coefficient of pitch moment and z_F, the distance from the consolidated application point of propulsive force to the center of gravity.

Considering a typical midsize commercial aircraft in cruise flight condition and that the equations 1 to 6, were linearized, one obtains the state equations in the form shown below (see (VASCONCELLOS,2013) for additional details):

$$\dot{X} = AX + BU \quad (11)$$
$$Y = CX + DU \quad (12)$$

Disregarding equations 5 and 6, the matrices A, B, C and D are given by:

$$A = \begin{bmatrix} -0.0159 & -4.5730 & 0.1303 & -9.8060 \\ -0.0004 & -1.7220 & 0.9636 & 0.000 \\ -0.0017 & -11.7600 & -2.5520 & 0.000 \\ 0.0004 & 1.7220 & 0.0364 & 0.000 \end{bmatrix}; \quad B = \begin{bmatrix} -0.2303 & 0.0688 \\ -0.1211 & 0 \\ -10.1000 & 0.0021 \\ 0.1211 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \qquad D = 0$$

3. CONTROL TOPOLOGY

3.1 Kreisselmeier's Topolgy

Kreisselmeier proposed a control topology with two degrees of freedom as shown in Figure 1, and demonstrated that there is always a pre-filter F, and a compensator K, for the plant G, so that the overall system has a transfer function equal to T , being stable and causal (KREISSELMEIER, 1999).

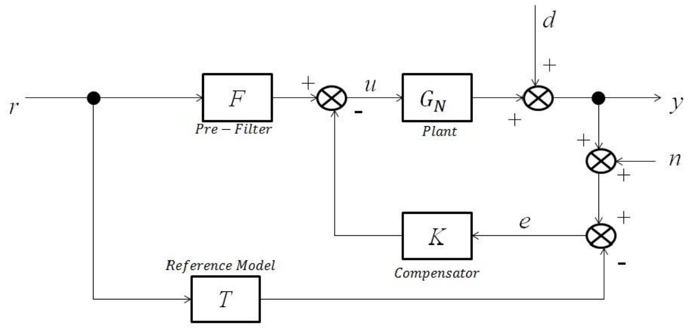

Figure 1: Two degree of freedom Kreisselmeier's Topology (KREISSELMEIER, 1999)

In figure 1, r, is the reference signal, d is the disturbance, n is the measurement noise and y is the output.

Considering the system without disturbances and noise, and by choosing $F = G^{-1}T$ (KREISSELMEIER, 1999), the transfer function from y to r is given by:

$$G_{yr} = T \tag{13}$$

This result shows that the plant output is governed by dynamics defined in T and that it is independent from the design choice of K. However, for realisability and stability reasons, the transfer matrix, T, should have equal or greater number of poles of G and it shall contain the non-minimum phase zeros of G.

Considering the disturbance occurring in the plant output, Figure 2, the transfer function from y to d is given by:

$$G_{yd} = (GK + I)^{-1} \tag{14}$$

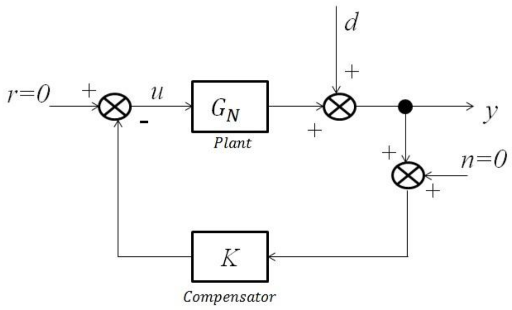

Figure 2: Kreisselmeier's Topology, r=0 and n=0

Considering that noise is present, Figure 3, the transfer function from y to n is given by:

$$G_{yn} = -(GK + I)^{-1}GK \tag{15}$$

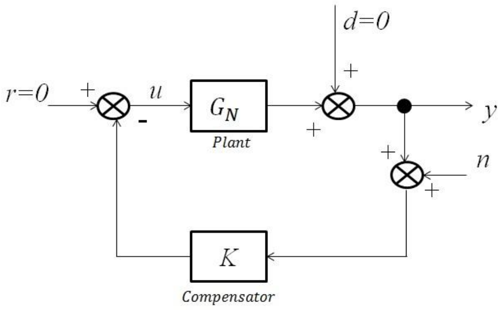

Figure 3: Kreisselmeier's Topology, r=0 and d=0

Provided that the previous conditions on T, hold and by analyzing $G_{yd}(s)$ and $G_{yn}(s)$, it can be concluded that a compensator K, can be properly designed to disturbance and noise rejection.

According Cruz (2008), the use of Kreisselmeier's topology is useful when there are complex requirements such as the control law requirements of an aircraft, because the choice of the matrix F is used to ensure the desired dynamic closed loop, while the compensator K is designed in order to meet the additional requirements of disturbance rejection, stability and robustness.

3.2 LQG/LTR Method

Doyle and Stein (1981) demonstrated the way the concept of "loop shaping" could be extended to multivariable systems so that the controller is designed with good stability margins, by using techniques recognized with such a feature. The LQG controller could be used in which the regulator part is designed using the sensitivity recovery procedure proposed by Kwakernaak (1969), thus imposing the robustness characteristics desired to loop gain. This results in the terminology "LQG/LTR"(Linear Quadratic Gaussian/Loop Transfer Recovery) (SKOGESTAD & POSTLETHWAITE, 2005).

According to Prakash (1990), LQG/LTR is a particular case of the robust controller design method TFL / LTR (Target Loop Feedback / Loop Transfer Recovery). The design procedure is divided into two stages: the first one, TFL, corresponds to the choice of the desired dynamic with specified robustness characteristics of performance and stability; the second stage, LTR, corresponds to the loop gain design such that when opening the closed loop system at a given point, it is approximately equal to the target loop gain set in the first step.

The LQG controller and closed loop system block diagrams are shown in the Figures 4 and 5:

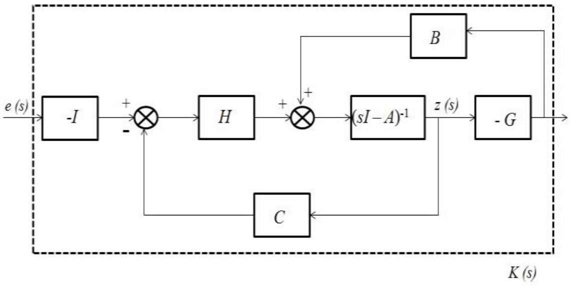

Figure 4: Controller Block Diagram

. The state equations which define the compensator K (s), shown in Figure 4, are:

$$\dot{z} = (A - BG - HC)z(t) - He(t) \tag{16}$$
$$u(t) = -Gz(t) \tag{17}$$

where, $z(t)$ is the state vector, $e(t)$ is the input vector and $u(t)$ is the output vector.

The transfer function $K(s)$ is given by:

$$K(s) = G(sI - A + BG + HC)^{-1}H \tag{18}$$

where the gain H is obtained by calculating the solution of continuous time Kalman Filter and gain G is the solution of LQR (Linear Quadratic Regulator).
Based on the basic LTR theorem, it is possible achieve gains G and H so that the LQG controller belongs to the class of TFL / LTR controllers (Cruz (1996) and Athans (1986)).

The open loop transfer matrix of the system with the LQG controller, represented in figure 5, is given by:

$$G_{(i)}(s) = G_N(s)K(s) \tag{19}$$

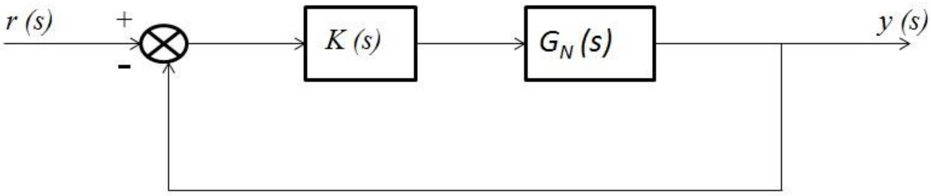

Figure 5: Closed Loop System (CRUZ, 1996)

The basic LTR theorem states that by choosing appropriately G as a solution of a LQR cheap control problem, with $R = \rho I$, where I is the identity matrix and ρ is a scalar, the controller K(s) is given by:

$$\lim K(S)_{\rho \to 0^+} = [C(SI - A)^{-1}B]^{-1}C(SI - A)^{-1}H \tag{20}$$

Thus, by considering that the nominal plant transfer matrix, $G_N(s)$ is $C(SI - A)^{-1}B$, and that the gain H was designed as a solution of a fictitious Kalman filter problem, the transfer function $C(SI - A)^{-1}H$, with good robustness properties guaranteed by the Kalman filter can be "restored". The singular values of $G_N(S)K(S)$ approach of $C(SI - A)^{-1}H$ from low to high frequencies, as ρ tends to zero (CRUZ, 1996).

However, when the plant has one or more non-minimum phase zeros, the LTR method does not recover the desired transfer function perfectly (ATHANS, 1986). If these zeros are located at frequencies above the crossover, the recovery procedure generally gives satisfactory results (CRUZ, 1996). Athans (1986) also states that for all practical purposes, the spectral density of reference signal and disturbances are concentrated at low frequencies, so the presence of distant zeros in the right half plane does not degrade the performance characteristics. Thus, imperfect recovery generates problems only when zeros are located in the same frequency region of the reference and disturbance signals.

According to Athans (1986), the use of this technique in non-minimum phase plants is strongly recommended as "non-minimum phase zeros result in performance limitations independently of the design method" (FREUDENBERG & LOOZE, 1985 cited by ATHANS, 1986, p. 1295).

3.3 Requirements

It is known, from best design practices of aviation industry, that good velocity response shall have a first order behavior while for the trajectory angle the response shall have second order behavior. During aircraft development campaign the settling time is defined based on test pilot opinion and differs from each aircraft. Therefore it is reasonable to arbitrate a value consistent with the size of the aircraft. Herein this results in a requirement that the velocity and flight path angle shall have settling time less than 40s. As the aircraft should follow the reference defined by an optimal trajectory previously calculated, according the concept of 4D trajectory, it is necessary that the system has zero tracking error to a step input.

It is desirable that the effects of disturbances are attenuated. Attenuation of -20dB was defined for the effect of a wind step of 10 knots.

Thus the control requirements can be summarized as follows:

- Velocity response:
  a) First order response
  b) Settling time 40s
  c) No steady state error for step reference
  d) Attenuation for gust disturbances: 20dB
  e) Noise attenuation: 40dB, for frequencies above 1000rad/s.

- Trajectory angle response
  a) Second order response
  b) Settling time: 40s
  c) No steady state error for step reference
  d) Attenuation for gust disturbances: 20dB
  e) Noise attenuation: 40dB, for frequencies above 1000rad/s.

Regarding robustness, it is assumed that the plant model has modeling errors at high frequencies due to neglected dynamics of actuators and sensors. Thus, requirements for performance robustness in regard to disturbance rejection and measurement noise are:

1) Disturbance rejection with error not exceeding 10% for frequencies lower than 0.5 rad/s;
2) Attenuation of noise measurement greater than or equal to 1% for frequencies above 1000 rad/s;
3) Stability robustness

Specifications as the above can be verified as a form of stability and performance robustness barriers, if the largest time constant of the neglected dynamics is 0.0159 s, which is to say that the uncertainty is located in frequencies higher than 10Hz.

The control requirements are represented in the Figure 6.

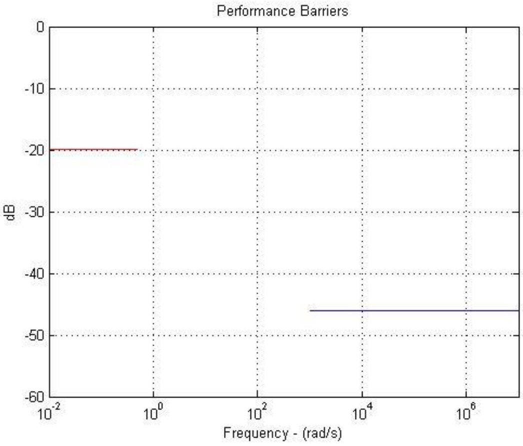

Figure 6: Loop gain specification for GK

Referring to Figure 6, it can be concluded that the open loop minimum singular values should be above -20dB for frequencies lower than 0.5rad/s. With respect to the measurement noise, the maximum singular values should be below -45dB for frequencies above 1000 rad / s.

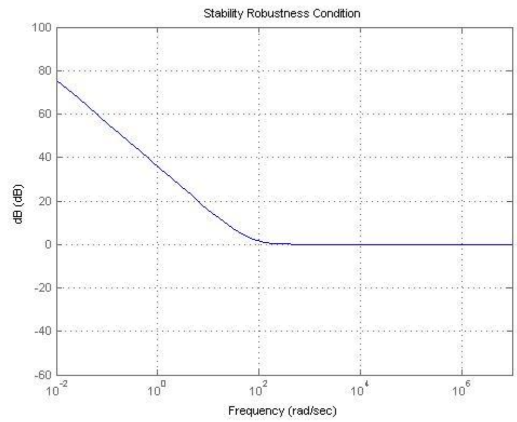

Figure 7: Stability Robustness barrier

The specifications for robust stability, refer to the closed loop transfer function. Figure 7 shows the robustness barrier with respect to stability. The maximum singular values of the closed loop transfer function shall be below the barrier (CRUZ, 1996).

3.4 The Control Design

The Kreisselmeier topology contains two degrees of freedom, so that two compensators are designed. The transfer function F is chosen to ensure the desired specifications with regard to closed loop dynamics and reference tracking. The LQG/LTR method was employed to achieve K. The controller K is employed in order to ensure stability robustness, disturbance and noise rejection, with respect to modeling errors due to high order neglected dynamics. An integrator is added to the plant model in order to design the controller, however this integrator is implemented together with the controller (CRUZ, 1996).

The controlled system was simulated using Simulink software as can be seen in the figure 8:

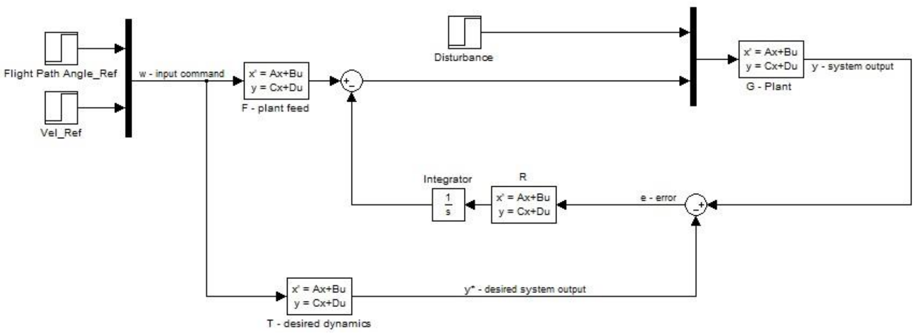

Figure 8: Kreisselmeier’s stucture simulink model

The transfer function G is the linearized model of the aircraft shown in section 2. As the F matrix contains the inverse of G, and G contains a non-minimum phase zero in +11.72, as previously discussed, that zero must be included in the transfer function T. The transfer function T contains the dynamics according the control requirements and also contains the non-minimum phase zeros of G.

As already mentioned, the design procedure using the LQG / LTR method consists primarily in to determine the desired loop gain.

Using the Kalman Identity, it is possible to derive a very useful result in the design of desired loop gain:

$$\sigma_m[G_{KF}(j\omega)] \approx \frac{1}{\sqrt{\mu}}\sigma_m[C(j\omega I - A)^{-1}L] \tag{21}$$

As discussed by Cruz (1996), the parameters μ and L can be used to shape frequency response of the loop gain GK in order to meet the specifications. After adjusting GK response, h calculation is performed. Thus, for:

$\mu = 0.015\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, and $L = \begin{bmatrix}L_L\\L_H\end{bmatrix}$, where $L_L = -[CA^{-1}B]^{-1}$ e $L_H = C^T[CC^T]^{-1}$, (ATHANS, 1986). The target loop gain is showed in Figure 9.

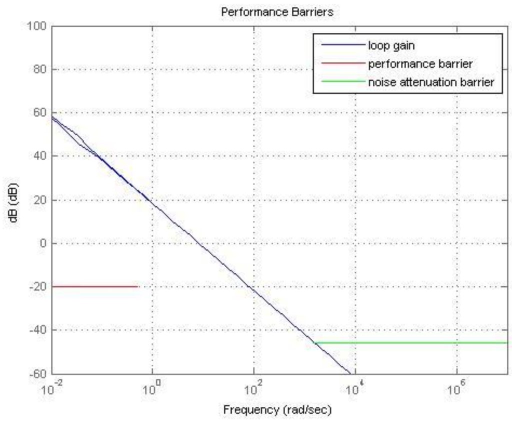

Figure 9: Target Loop Gain Singular Values

As mentioned by Cruz (1996), at frequencies below the frequency of non-minimum phase zero, it is possible to achieve good approximation of the desired response.

The value of ρ for the best recovery is 10^{-12}, which corresponds to curve in blue color.

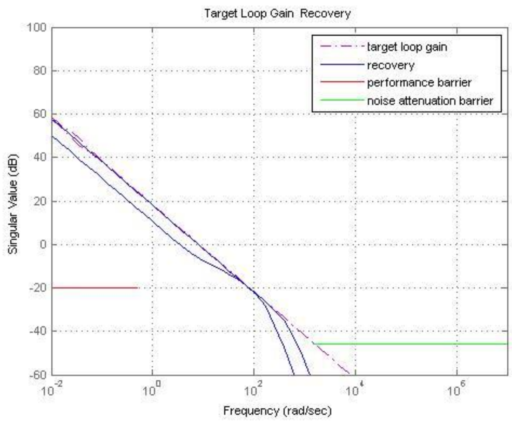

Figure 10: Transfer Loop Recovery

Observing the figure above, it can be seen that the robustness requirements for disturbances are met, up to about 0.5rad/s the minimum singular value is greater than 20dB. It can be noted that the system meets the noise requirement, the magnitude of the maximum singular value remains below -40dB for higher frequencies than 1000 rad / s.

The recovery is not perfectly performed, this fact can be attributed to the non-minimum phase zero of the system, and however the value of μ was set so that the system would meet the specifications for robustness. Thus, after applying pole/zero cancellation technique in order to reduce the controller, the controller transfer function is given by:

$$K(s) = \begin{bmatrix}K_{11} & K_{12}\\K_{21} & K_{22}\end{bmatrix},$$

where:

$$K_{11} = \left[\frac{-4406.3(s-0.1007)\,(s+7.017)\,(s^2+4.764s+9.572)}{(s+30.73)(s+11.25)\,(s^2+\ 242.6s+2.967*10^4)}\right]$$

$$K_{12} = \left[\frac{3.72*10^6(s-0.0813)\,(s+11.14)\,(s+28.97)}{(s+30.73)(s+11.25)\,(s^2+\ 242.6s+2.967*10^4)}\right]$$

$$K_{21} = \left[\frac{-2.141 * 10^6 (s - 0.0329)\,(s^2 + 4.312s + 15.73)}{(s + 30.73)(s + 11.25)\,(s^2 + 242.6s + 2.967 * 10^4)}\right]$$
$$K_{22} = \left[\frac{-7.015 * 10^6 (s - 10.69)\,(s + 19.89)\,(s + 3.262)}{(s + 30.73)(s + 11.25)\,(s^2 + 242.6s + 2.967 * 10^4)}\right]$$

The singular values for the closed loop system is shown in the Figure 11.

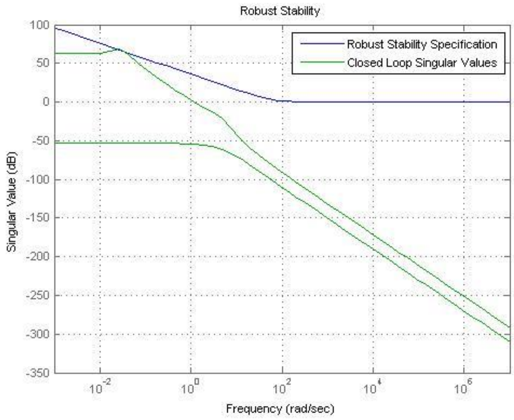

Figure 11: Closed loop singular values

The robustness of stability condition is met by the system as can be seen in Figure 11. The maximum singular values of the closed loop system remain below blue curve.

4. RESULTS

The system response without disturbances is shown in the Figures 12 to 13:

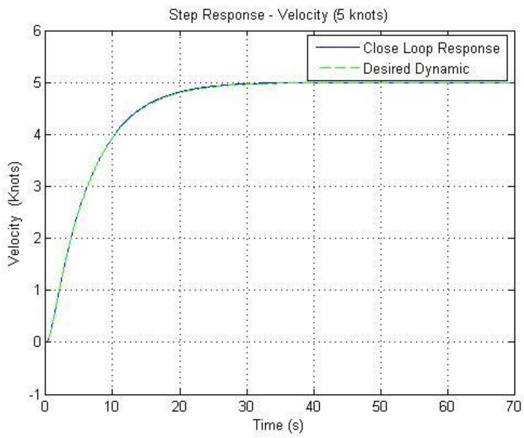

Figure 12: Step Response - Velocity

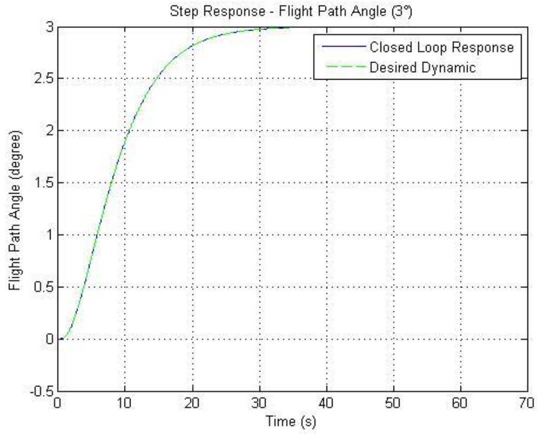

Figure 13: Step Response - Flight Path Angle

Figures 12 and 13 show that the system follows the dynamics contained in the transfer function T with zero error. This fact stresses the benefit of using the topology of Kreisselmeier, which is to impose the desired dynamics for the plant, chosen according to the defined specifications.

In order to verify the system behavior in case of a wind gust disturbation during a velocity and trajectory angle control, it was performed a simulation with wind gust starting in 10s. The system response is shown in the Figures 14 and 15.

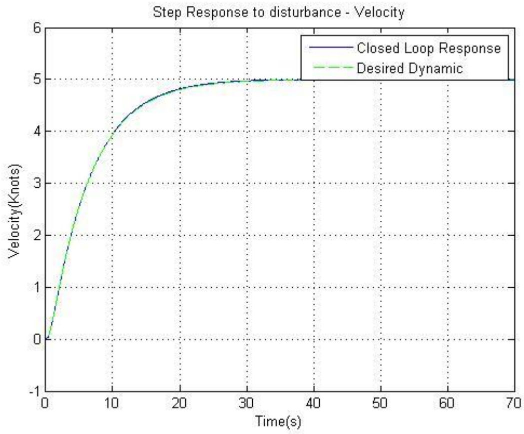

Figure 14: Response to disturbance - Velocity

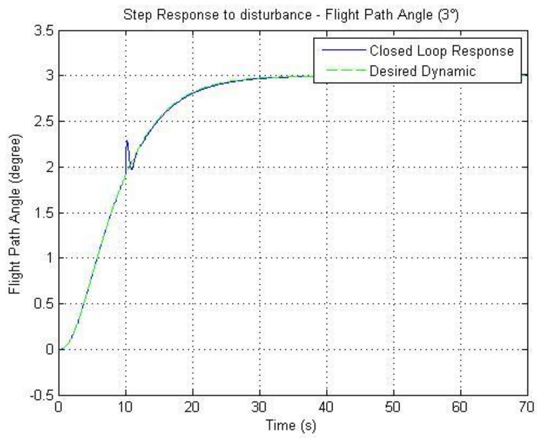

Figure 15: Response to disturbance - Flight Path Angle

It can be noted that the effect of the disturbance in velocity and trajectory angle responses have considerable attenuation.

5. FINAL CONSIDERATIONS

The objective of this work was to perform closed loop control of velocity and trajectory angle aiming its use in a 4D trajectory manager. The solution adopted is based on the topology proposed by Kreisselmeier used with the concepts of robust control, by using the LQG / LTR method.

For validation of the proposed solution was carried out a case study using the model of a commercial aircraft containing non-minimum phase zeros. The specifications were defined based on best practices control laws design in the aviation industry. The case study showed that the control topology used meets the defined specifications and the compensator designed using the LQG / LTR method was satisfactorily used in a model with non-minimum phase zeros. The system responses with the proposed control topology provide good indications that this topology can be used in practice. The Kreisselmeier topology provides characteristics of perfect reference tracking by selecting appropriately the matrix F and the compensator K provides robust stability and regulator performance against disturbance and noise.

The present study can be extended to the passenger comfort analysis, considering the aircraft vertical acceleration, and its implications in the design. One can also verify the robustness of the topology which appears as the variation of plant parameters with respect to the tracking characteristic of the reference signal. An application of this methodology to a case study involving the lateral-directional dynamics can also be a possible extension of this work.

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