ANALYTICAL DETERMINATION OF THE LEAD CORRECTION IN A GEAR MESH DUE TO BENDING-TORSION EFFECT

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Abstract. *This work focus on the study of the contact pressure intensifier between two gears, which exists due to the gearbox component deformation. This effect also occurs in gearboxes manufactured without any machining and/or assembling errors.*
The lead correction is a machining operation conducted at the gear surface to compensate the displacement of shafts and gears, due to bending and torsion, and the gear teeth displacement, due to bending and contact pressure.
Factors that are important to the lead correction in gearboxes were studied. An analytical model was proposed in order to evaluate the lead correction necessary to diminish the contact pressure intensifier as a function of the displacements described above. This analytical model was compared to a literature one and to the commercial software RIKOR ®. The results of the proposed lead correction are similar to those in the literature and RIKOR, although there are differences at the ends of the gears.
Additionally, a model of the gears and shafts with the involute profile and the lead correction evaluated analytically was developed. This model was tridimensional (3D) and was prepared using SolidWorks and Inventor software. It was simulated using finite element analysis software ANSYS and it was possible to verify that the three lead corrections - proposed, from literature and from Rikor – reduced the contact pressure intensifier. Moreover, with the finite element analysis, it was possible to verify that the lead correction proposed in this work is 3% more effective than the other ones for the case analyzed.

Keywords: *gear, lead correction, machine elements, finite element analysis*

1. INTRODUCTION

The overall concept of gearboxes has evolved in the past century, along with the materials, designs and geometries. Each design has its pros and cons, leading to optimization processes depending on the applications. However, some effects that reduce the lifetime of the gearing have not, to present date, been extensively discussed in the literature. This opens space to a deep study about the subject. One of those effects is the lead correction. When loaded, the gears, shafts, etc. deform, changing the gearing contact and concentrating the load in some regions. This load concentration diminishes the life of the gearing, since it is dimensioned considering the mean load at the tooth flank. Without the lead correction, in order to maintain the expected life, it is necessary to overestimate the stress, using stress intensity factors.

Considering a common gearing pair, since the displacement of the pinion is larger than that of the wheel, it is possible to perceive that a uniform load distribution along the teeth cannot be reached if both gears are completely straight. Thus, one should machine the pinion teeth with a lead correction that is nothing more than a compensation of this displacement, such that the load distribution is even when the components deform. The lead correction may be seen in its tridimensional form in Fig. 1a, where the tooth is represented as an opaque solid and the correction as a translucent solid. In this case, the correction is linear.

Other advantages of machining a lead correction may be found in (Guenther, 2011), (Mermoz, 2013), (Mao, 2006), (Shih, 2011), (Huang, 2008) and (Artoni 2012). These advantages include: reduction of the contact temperature, reduction of the gearing noise and reduction of the transmission error (that induces vibration).

The main characteristics of a gearing can be found in (Buckingham, 1949) and (DIN 3960, 1987). The most important equations will be presented in Section 3.1.

1.1 Lead correction

There are four main effects that can affect the lead correction and that will be studied in this work (Fig. 1b): a) Torsion of shafts and gears; b) Bending of shafts and gears; c) Bending of the gear teeth; d) Deformation of the gear teeth due to contact pressure.

These effects may be separated into two main categories: a global one, which contains the deformation due to bending-torsion of the shaft and one that focus at the gearing and covers the teeth displacement. It is important to state that all displacements analyzed in this work are elastic and reversible.

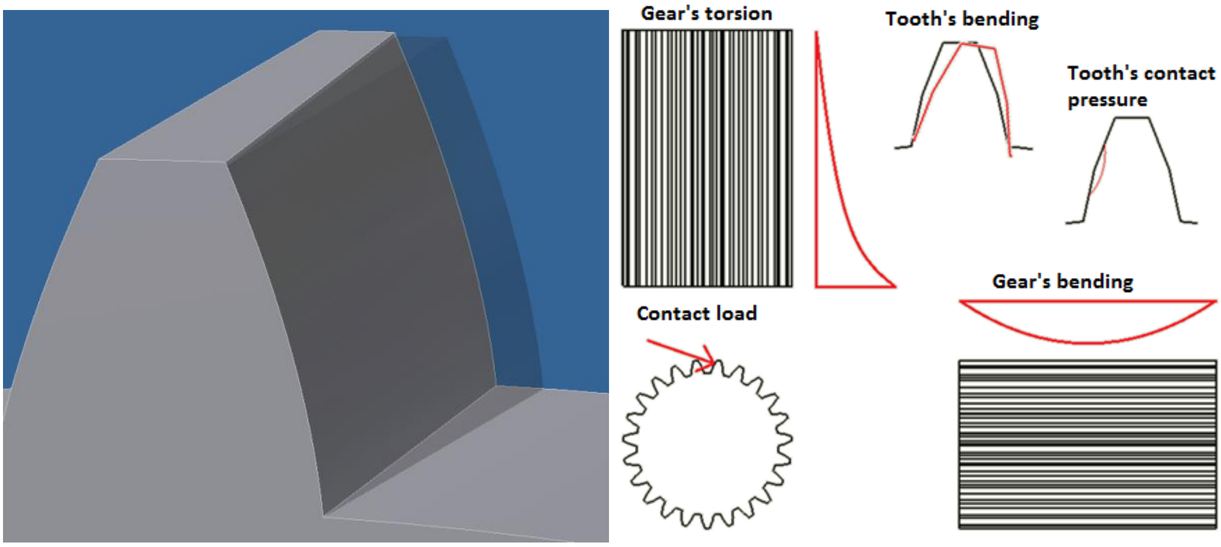


Figure 1 – a) Tridimensional form of the lead correction. b) Illustration of the four main effects that may affect the lead correction

Authors were unable to find a model that covers all of the aspects shown in Fig. 1b. Similarly, in this work each of the effects will be approached separately and combined later.

1.1.1. Shaft's bending-torsion

TDEA (2005) presents a study regarding only the bending-torsion of the shaft. That work did not study the tooth profile or the deformations of the teeth, due to bending and contact pressure. An assumption adopted by the author was that only the displacements perpendicular to the plane of contact between the gears are important and displacements parallel to this plane may be neglected. The author also defines a function relating load distribution to the distance to the gears edge and considers that the contact occurs at a fixed contact radius. However, instead of calculating the load distribution of the gears, TDEA (2005) assumes that the load distribution at the gear mesh is homogenous if the lead correction is machined. Therefore, it is possible to find the real bending and torsion of the shaft after its correction. The second factor analyzed was the shaft bending. It was also considered that the load at the gear teeth is constant and, as boundary conditions, the displacement at the bearings due to its clearance and stiffness is used. After determining the displacements at each section of the shafts with respect to the gear edge, they can be used to evaluate the lead correction value. The equivalent displacement at the gear mesh is the sum of the displacements of each gear.

Linke (2005), on the other hand, studies the lead correction in a planetary gearbox. As assumptions, it was considered that the pinion is subjected only to torsion - because the radial forces of the planet gears cancel each other - and that the planet gear is subjected only to bending - because the same load is applied at the contact between solar and planet and between planet and annulus, canceling the torsion. Other assumptions are the absence of any machining / assembling errors and that the weight of the components can be neglected. Besides that, the author considers that the annulus does not dislocate and does not need to be analyzed, due to its bigger polar moment of inertia. Thus, the lead correction at the pinion is determined only by its torsion equation and the lead correction at the planet is determined by its bending equation.

1.1.2. Gear's teeth displacement study

Different from references mentioned in the previous section, Magalhães (1990) did not focus on the calculation of the lead correction at the gear mesh, but on the evaluation of the stiffness of the gear teeth due to bending and contact pressure. For the first effect (bending) the stiffness is evaluated by approaching the tooth to an equivalent prismatic beam. An important point is that this method is only valid if the gear mesh occurs at the same diameter along the gear face (i.e. spur gears). For the second effect (torsion), a model based on the Hertz contact equations is proposed.

There are others authors that have investigated the tooth stiffness. Pandya (2013), for example, evaluates the stiffness due to bending and contact pressure, but the equations are simplified. Sanchez (2013), on the other hand, makes a complete analysis of the tooth stiffness due to contact pressure. However, that method is valid only for pressure angles between 10° and 18° and industrial gears have pressure angles larger than 20°.

2. OBJECTIVE

The objective of the present work is to define the lead correction that must be machined at the pinion in order to compensate the effects of the shaft displacement - due to its bending and torsion - and the effects of the gear tooth displacement - due to bending and contact pressure. The objective will be reached by proposing an analytical lead correction model. This model will provide results that will be compared with ones obtained by the models from the literature and from the commercial software RIKOR. Each condition will be also analyzed by finite element analysis and compared with each other.

3. ANALYTICAL METHOD

The analytical method will be divided into various steps, and in each a different variable will be studied. However, some assumptions must be made in order to elaborate the model.

The main assumptions are: a) The model will not consider the presence of the lubricant. Its effect on the gear mesh and how it affects the load distribution at the contact depend on many variables, whose modeling is not in the scope of this paper. Besides, due to the low gear speed of spur gears, the oil lubrication is reduced to its boundary lubrication thickness, such that it is possible to neglect its presence, at least for the contact at macroscopic scale. b) The sliding friction forces will be neglected. Despite assumption ‘a’, the sliding friction force at the gear mesh is low, and only perpendicular contact loads will be considered (Sainsot, 1989). c) Only spur gears will be studied. This restriction is necessary to reduce the scope of this paper and enable some analytical simplifications necessary to its conclusion. d) The contact between two gears may be approximated by a cylinder-cylinder contact. This is another important assumption necessary to use the model described in (Magalhães, 1990). e) Gear mesh occurs at the work diameter. Because of the low contact ratio of the spur gears, in some instants there is only one tooth in contact. This is a critical moment, because it is in this moment that the highest load at the tooth occurs. The lead correction will be determined for this situation. f) Static analysis. Because the gear speeds are low, the gear mesh will be studied as if it was static. Therefore, dynamic effects will be neglected.

3.1 Analytical equations

The lead correction can be evaluated and machined at both gears. However, in order to reduce machining costs, the lead correction is defined only for the pinion. Thus, it is necessary to evaluate the displacements at the wheel gear flank and, later, calculate the lead correction of the pinion in order to achieve an even pressure at the flank when the gears are subjected to its nominal torque.

Note that this method considers that the load is evenly distributed at the gear flank in order to evaluate the lead correction. If one needs to evaluate the load distribution of a specific lead correction and a specific load, DIN 3990 (1987) presents a methodology for this calculation.

3.1.1. Circle involute equation

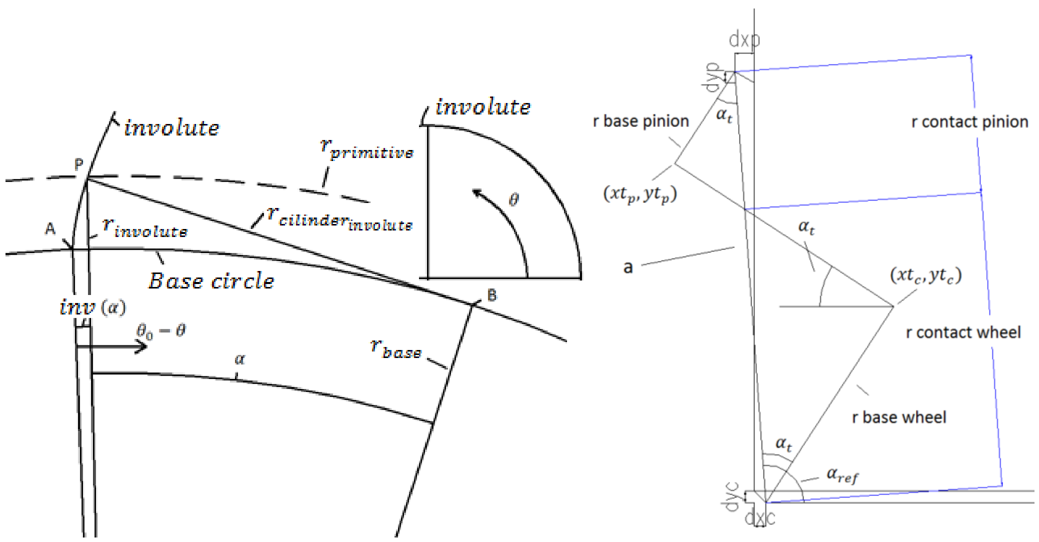


Figure 2 – a) Important parameters for the circle involute determination. b) Triangle similarity and the new gear mesh line.

The first item to be modeled is the equations of the circle involute. It is necessary to evaluate the contact radius (*rcontact*) as a function of the angle θ. Figure 2a and Fig.2b show these parameters. For convenience, it will be assumed that the center of the global coordinate system (*Oxyz*) is set at the center of the low speed shaft in its nominal position.

However, in order to ease the use of the involute equations, it is convenient to define another coordinate center (*Ox'y'z'*) with the same direction of the previous one, but centered at the center of each section of the shaft after its deformation. This auxiliary system will be used to calculate the new contact point of the gearing more easily.

After some calculations, it is possible to deduce that:

$$rinvolute(\theta)=mn.z.\cos(\alpha 0)/(2.\cos(ev\text{-}1(\pi/2+(\pi+4.xprofile.\tan(\alpha 0))/(2z)-\theta wheeldeformation-\theta))) \tag{1}$$

Where: *rinvolute* is the involute radius in mm, *mn* is the gear modulus in mm, *z* is the number of teeth of the gear (-), $\alpha 0$ is the pressure angle in radians, *xprofile* is the gear addendum modification coefficient (-), θ*wheeldeformation* is the torsion deformation angle of the wheel in radians and θ is the angle in radians according to the referential in Fig. 2a.

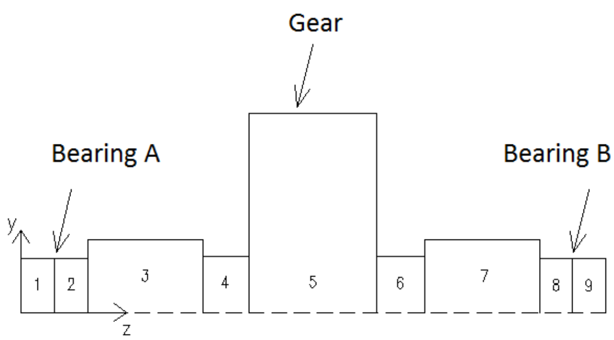
3.1.2. Gear mesh line equation

After the deformation of the gears and shafts is evaluated, it is possible to determine a new gear mesh line that is tangent to the base circle of both gears. Each point of the gear mesh is determined by this line. Consequently, in order to find the gear mesh contact point, it is necessary to find the point where this line meets the involute profile of the wheel tooth in contact.

With the θ*wheeldeformation* determined and, therefore, rcontactwheel (the radius of contact of the wheel in mm), it is possible to determine the contact point between pinion and wheel. This point indicates where the pinion should be. If one calculates where it really is, it is possible to evaluate the angular displacement necessary to establish full contact. This displacement, evaluated for each section of the gear, is the lead correction and is defined in Eq. 9.

3.1.3. Determination of the displacement due to bending

With the basic parameters of the lead correction equation defined, it is necessary to determine the new position of the shafts and gears through bending, torsion and teeth deformation. This section will focus on the determination of the bending of shafts and gears. To determine it, it is necessary to find the elastic line equation of these components. However, the bending moment and the polar moment of inertia are not constant throughout the whole shaft. Consequently, it is convenient to divide it in regions with constant diameter, as shown in Fig. 3a. Besides, in order to make the model the most generic, it is convenient to define a complete loading for each segment of the shaft, as one may see in Fig. 3b. If there is no load, it is necessary only to make it equal to zero. The loads used were: loads and moments concentrated at the beginning of each segment and loads and moments distributed along the element. As a result, after some mathematical treatment, it is possible to evaluate the elastic line equation between two generic sections '*mm*' e '*nn*', with *nn>mm*.



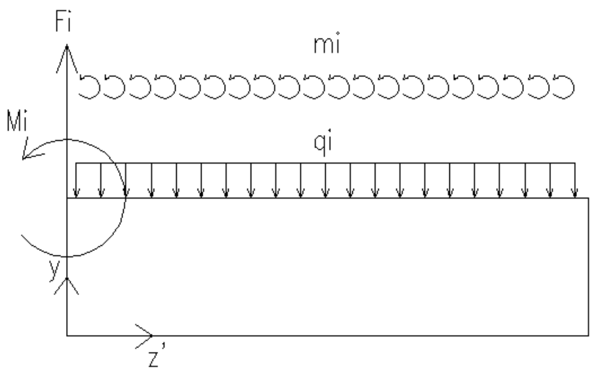


Figure 3 – a) Example of a studied shaft. b) Section of the shaft with all possible loads applied.

$$K2nn/(E.Inn)=K2mm/(E.Imm)+K1mm/(E.Imm).\sum(i=mm,nn-1,Li)+\sum(i=mm,nn-2,((Mi.Li+(Fi+mi).Li^2/2+qi.Li^2/6)/(E.Ii).\sum(j=i+1,nn-1,Lj))+\sum(i=mm,nn-1,(Mi.Li^2/2+(Fi+mi).Li^2/6+qi.Li^4/24)/(E.Ii))) \tag{2}$$

$$K1nn/(E.Inn)=K1mm/(E.Imm)+\sum(i=mm,nn-1,(Mi.Li+(Fi+mi).Li^2/2+qi.Li^2/6)(E.Ii)) \tag{3}$$

Where K1 is the angle constant for each section in radians, K2 is the displacement constant for each section in mm, E is the Young's Modulus in MPa, I is the second moment of area (for bending) in mm^4, L is the length of each section in mm, M is the concentrated moment at the beginning of each section in Nmm, m is the distributed moment along the section in Nmm/mm, F is the concentrated load at the beginning of each section in N and q is the distributed load along the section in N/mm. The boundary conditions necessary to solve this system and determine the displacements of each shaft due to bending are the displacements at the two bearings and Eq. (2) and Eq. (3).

3.1.4. Determination of the displacement due to torsion

After some treatment, and using the same assumptions as with the bending, it is possible to evaluate the torsion equation of the shafts:

$$\theta t(z)=(Ti.z+ti.z^2/2)/(G.Ji)+K3i \qquad (4)$$

Where θt is the shafts torsion angle in radians, T is the concentrated torque at the beginning of each section of the shaft in Nmm, t is the distributed torque along the section of the shaft in Nmm/mm, G is the shear modulus in MPa, J is the second moment of area in mm^4 (for torsion) and K3 is the angle constant for each section in radians.

In this case, the only consideration that must be made to ease the calculation is to consider that *K3* is zero at the beginning of each gear.

3.1.5. Determination of the tooth displacement (due to bending and contact pressure)

As said in 1.1.2, the model to describe the teeth displacements will be presented here. The equation of the tooth stiffness due to bending is described in Eq. 5. The equations relating the gear parameters to these variables may be found in (Magalhães, 1990). The contact stiffness equation is described in Eq. 6.

$$\delta f=4.F0.\cos(\alpha 0)/(E.b).(3.(L/Sd)^3.(1/2.(3-ad/L).(ad/L-1)+\ln(L/a))+(1+\upsilon)(L-ad)/(Sd.(1+ad/L))) \qquad (5)$$

$$\delta h=2.(1-\upsilon^2).F0/(\pi.E.b).(2/3+\ln((1,731.E.b.an.sen(\alpha 0))/F0)).\cos(\alpha t)/2 \qquad (6)$$

Where F0 is the contact load in N, b is the gear width in mm, υ is the Poisson coefficient of the material, αt is the work angle of the gear mesh in radians and *L*, *Sd* and *ad* are other parameters of the model proposed in (Magalhães, 1990). With these two values, it is possible to evaluate the angular displacement using Eq. 7 and Eq. 8.

$$\theta(toothbending)=\delta f/rcontact \qquad (7)$$

$$\theta(tooth(contact\ pressure))=\delta h/rcontact \qquad (8)$$

3.1.6. Synthesis of all the effects studied previously

The synthesis of the four previous effects may be translated in terms of the following equation:

$$Lead\_correction=dp/2.\theta pinion \qquad (9)$$

Where *Lead_correction* is the lead correction of the pinion in mm, *dp* is the contact diameter of the pinion in mm and $\theta pinion$ is the pinion angle that must be corrected by the lead correction in radians (determined in 3.1.2).

3.1.7. Analytical model proposed by the literature

Finally, it will be presented the model proposed by TDEA (2005). This is a simplified model if compared with the one proposed in this work. In order to determine it, all displacements must be projected at a contact perpendicular plane. Therefore, using the same coordinate system as this paper, it is possible to compare both models. Using the subscript "c" for this model, we have:

$$\delta ctorsion(z')=dcontact(z').\theta ctorsion(z')\rightarrow\theta ctorsion(z')=(Tc.z'+tc.z'^2/2)/(G.Jc) \qquad (10)$$

$$\delta cbendingx(z')=(Mcx.z'^2/2+Fcx.z'^3/6+qcx.z'^4/24)/(E.Ic)+Kcx1'.z'+Kcx2' \qquad (11)$$

$$\delta cbendingy(z')=(Mcy.z'^2/2+Fcy.z'3/6+qcy.z'4/24)/(E.Ic)+Kcy1'.z'+Kcy2' \qquad (12)$$

With the displacements defined, it is necessary only to project them at the plane:

$$\delta cliterature=\sum(i=1,6,factori.\delta componenti) \tag{13}$$

The factors used in Eq. 13 are in Tab. 1.

Table 1 – Factors of Eq. 13

Displacement	Pinion	Wheel
$\delta ctorsion$	$-cos(\alpha n)$	$-cos(\alpha n)$
$\delta cbendingx$	$-cos(\alpha n)$	$+cos(\alpha n)$
$\delta cbendingy$	$+sen(\alpha n)$	$-sen(\alpha n)$

4. RESULTS AND DISCUSSION

In this section the results obtained with both analytical models, with the software RIKOR and with the finite element analysis will be presented.

4.1 Analytical model

An industrial gearing was simulated with both models from section 3. The spur gears studied are from a sugar cane mill gearbox. The net bending and net torsion of the gears are shown in Fig. 4 (note that rigid body displacements are neglected):

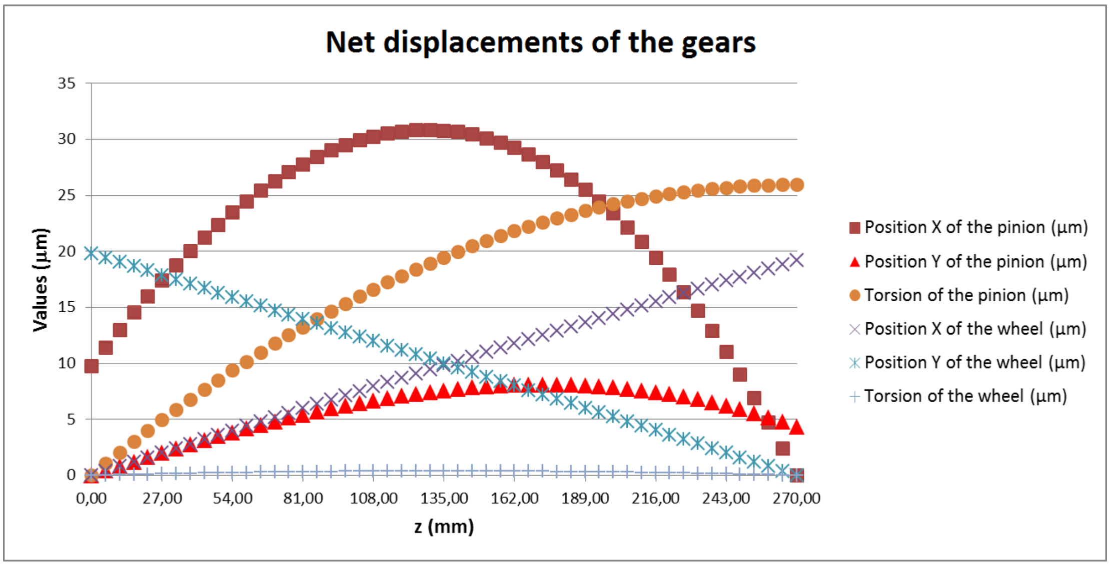


Figure 4 - Net displacement for each parameter analyzed

Analyzing Fig. 4, it is possible to see that only the pinion presents significant displacements. The wheel, due to a higher diameter, has a much bigger stiffness. Consequently, if subjected to a given contact load, it will deform less. Also, the bending of the wheel is a line. This indicates that the shaft where this gear is mounted has moved, although the displacement at the wheel is small. Therefore, the gear makes an angle with the shaft, but does not deform itself. The contribution of bending and torsion of the wheel is linear, while the pinion's is parabolic.

Besides that, a sensibility analysis of the teeth displacement was conducted. It was calculated the displacement of the gear teeth at the nominal contact diameter (with the gears at their nominal position without any kind of displacement) and at the real contact diameter (after the new position was found with the equations in chapter 3). The difference in the values obtained is two or three orders of magnitude smaller. Thus, even if the contact diameter varies along the gear face, its effect on the lead correction may be neglected.

The curves shown in Fig. 5 indicate the amount of material that must be removed from the gear tooth as a function of its width. A bigger lead correction means a bigger material removal. The "+" sign shows the lead correction obtained with the literature's model, the "x" sign shows the one obtained with RIKOR® and the "•" shows the one proposed in this paper.

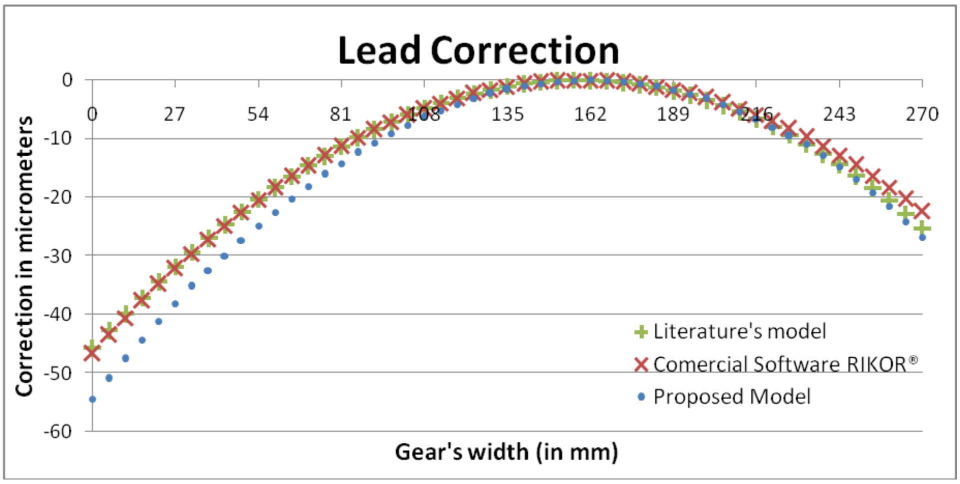


Figure 5 - Lead correction for each model

When analyzing Fig. 5 it is possible to see that the correction obtained with RIKOR® and with the literature's model are practically the same. The values at the borders are 47-48μm at the left and 22-26μm at the right and the maximum point occurs at the same gear length – 160mm.

The model proposed in this paper presents a bigger lead correction, especially at the gear ends. Because the displacements due to bending at those points are bigger, the distances between the gears increase. Therefore, displacements tangent to contact begin to be important, due to the involute profile. The difference between the models is approximately equal to 8%.

4.2 Finite element analysis

Because the lead corrections from literature and from RIKOR are almost the same, their correction will be simulated as one. So, there will be three simulations: with no correction, with the one from literature and with the one from this paper. The results were compared with each other.

Figure 6 shows the pressure distribution along the flank of the tooth for the three lead corrections simulated. It may be observed that the distribution for the model without lead correction is not constant, being bigger at the beginning of the gearing, smaller at the center and big one more time at the end. This explains the shapes of the lead corrections found by the literature and this paper.

The second model simulated was the one from RIKOR® / literature. **T**his model presents a better pressure distribution along the tooth. However, at the borders there is still a load concentration. Nonetheless, this model presents an upgrade from the one with no correction, smoothening the load and improving the life of the gear.

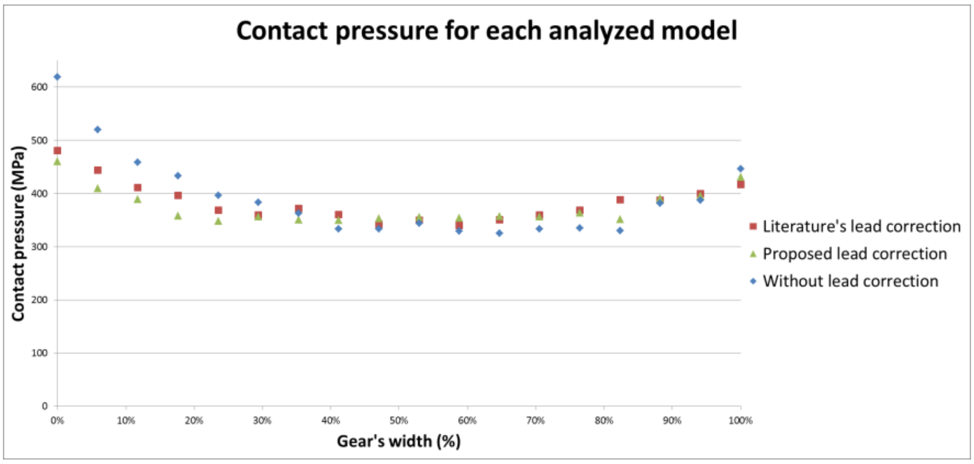


Figure 6 - Finite element analysis comparison of the models

The final lead correction simulated is the one presented by this paper. It may be viewed that there is a load concentration at the tooth's border. However, it is smaller than the one from the literature, showing an improvement at the modeling.

4.3 Relation between analytical model and FEA

Analyzing the data from the finite element analysis, it is possible to evaluate the load distribution factor from each calculation method. The values are found by dividing the highest pressure found by the mean. The load distribution factor of the first model is 1,58, from RIKOR® / literature is 1,25 and from this paper, 1,23.

5. CONCLUSIONS

The fact that the lead corrections from the literature and from RIKOR® are almost equal (Fig. 5) is strong evidence that the model used in RIKOR® is the same as the literature: only the displacements normal to contact are considered.

It is worth saying that the lead correction that may be machined by a common grinding machine has to be linear or parabolic. All lead correction dealt with in this paper may be approximated by a second degree equation. The correlation between the profile points and a parabolic curve are all bigger than 99,8%. Thus, although the effects that are governing the lead correction are not linear, the final result may be approached by a parabola, granting its machinability.

The differences found between the analytical model and the finite element analyses occur due to various factors. Although much care was taken in order to transport the boundary conditions from the analytical model to the FEA, it is bi-dimensional, while the FEA is tridimensional. Therefore, it is expected that some discrepancies exist. For example, the analytical model considers that the whole gear is subjected to torsion, while it is not the real behavior of the component when it is analyzed through finite elements. Another difference may be found at the bearings. At the analytical model their interfaces with the shaft are punctual, whilst at the FEA it is an area that tends to diminish the deformation of the shaft.

With the results found at section 4 it is possible to conclude that the model proposed in this paper, when compared with the others through FEA, shows an improvement of the load concentration by 3%. Using the gear calculation methodology presented in (DIN 3990, 1987), a reduction of 3% of the load concentration allows a reduction of 3% of the gear width. This reduces the general cost of the gearbox by 1%. Note that this reduction is achieved by only changing the methodology of the lead correction calculation, without any investment in machinery or software.

The calculation, however, is not trivial. The literature model is linear and may be solved with no knowledge of the gear geometry and properties. It is necessary to know only the gear ratio, diameter and torque.

With the proposed model, it is necessary to know the exact gear geometry. However, the gear manufacturer owns this data, justifying the use of this methodology.

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