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BINARY COAXIAL JET FLOW

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Abstract. *There is a large number of applications where mixing between different gas species is relevant, in particular applications related to combustion, where mixing between fuel and oxidizer dictates the combustion efficiency and the combustion stability. In order to study the stability and acoustic characteristics of coaxial binary jets an accurate solution of the base flow is needed. By solving the parabolic boundary layer equations numerically such accurate results may be obtained for laminar and turbulent flows. The present investigation uses a boundary layer approach to study the characteristics of binary coaxial jets. The boundary layer equations are written in a streamwise, streamline coordinate system and solve implicitly for different configurations in order to investigate the effect of inner and outer nozzle velocity ratio, temperature ratio, Mach number ratio and density ratio for two different chemical species. Variable Prandtl, Lewis and Chapman-Rubesin parameters are considered. Results for simple and coaxial jets of homogeneous and binary species are compared with analytical results to validate the code. Then, the main topological characteristics of the given results are discussed in terms of velocity distribution, aiming at the study of stability and acoustics to be performed in future studies.*

Keywords: *jets, boundary layer, hydrodynamic stability, numerical simulation*

1. INTRODUCTION

The performance of combustion systems in gas turbines and rocket engines depends strongly on the proper mixing between fuel and oxidizer. The temperature profiles at the exit of the combustion chamber, the flame stability, the thermo-acoustic coupling and pollutant emissions are some of the parameters that must be controlled in a combustor. The dynamics of the flow is related to acoustic, hydrodynamic and thermal disturbances that are inter-related.

The hydrodynamic stability processes result in the generation and propagation of vorticity that cause disturbances in the heat release through oscillations in the flame front. The unsteady heat release and vortex paring excite acoustic modes that may resonate with the natural frequencies of the combustor leading to its destruction. A stability analysis in propulsive systems furnishes the frequencies, phase velocity, group velocities, wavelengths and amplification rates of the disturbances that propagate inside these devices.

Recently Gloor *et al.* (2013) published a study of the stability and acoustic characteristics of compressible coaxial jets. That study investigated the parameters that influence the development of hydrodynamic and acoustic modes through the solution of the stability equations, allowing the identification of interaction mechanisms between the mixing layer and the acoustic modes. The present project proposes to build on the methodology used by (Gloor *et al.*, 2013) in the study of binary coaxial jets. Different from (Gloor *et al.*, 2013), the base flow will be obtained through the solution of the boundary layer equations, similar to the work presented by (Dahl and Morris, 1997a,b). The solution of the boundary layer equations are proposed since the works presented by (Kennedy and Gatski, 1994) and (Kozusko *et al.*, 1996) have shown that the resulting temperature, velocity, density and mass fraction profiles are very different from the usual canonical profiles used in other studies.

In a preliminary study this paper presents results of the development of the binary, compressible boundary layer code. Comparisons with analytical similarity solutions for a simple jet are presented followed by a study of homogeneous non-similar jets and binary jets. Both faster and slower outer jets are considered.

2. FORMULATION

2.1 Boundary Layer Equations

The two-dimensional, compressible, binary coaxial jet base flow boundary layer equations are nondimensionalized by the inner jet properties and diameter (Anderson, 2000; Dahl and Morris, 1997a). The pressure is considered constant across the layer, consistent with the boundary layer approximation.

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0, \tag{1}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\frac{1}{Re}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{2}$$

$$\frac{\partial p}{\partial y}=0, \tag{3}$$

$$\rho u\frac{\partial T}{\partial x}+\rho v\frac{\partial T}{\partial y}=\frac{1}{RePr}\frac{1}{c_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{(\gamma-1)Ma^2}{Re}\frac{\mu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{4}$$

$$\rho u\frac{\partial s_1}{\partial x}+\rho v\frac{\partial s_1}{\partial y}=\frac{1}{ReLePr}\frac{\partial}{\partial y}\left(\rho\mathcal{D}_{12}\frac{\partial s_1}{\partial y}\right), \tag{5}$$

$$1=\rho RT, \tag{6}$$

where Eq. 1 is the continuity or mass conservation equation, Eq. 2 is the momentum conservation equation in the x direction, Eq. 3 is the momentum conservation equation in the y direction, Eq. 4 is the energy conservation equation, Eq. 5 is the species conservation equation and Eq. 6 is the perfect gas equation of state. The nondimensional numbers are Reynolds, Mach, Prandtl and Lewis numbers.

$$Re=\frac{\rho_1 U_1 x_0}{\mu_1},\quad Ma=\frac{U_1}{\sqrt{\gamma R_1 T_1}},\quad Pr=\frac{\mu_1 c_{p_1}}{k_1},\quad \text{and}\quad Le=\frac{k_1}{\rho_1 c_{p_1}\mathcal{D}_{12}}, \tag{7}$$

where, ρ is the density, u and v are the velocity components in the streamwise and normal directions, p is the pressure, μ is the dynamic viscosity, k is the thermal conductivity, T the temperature, c_p the specific heat at constant pressure, R the gas constant s_1 the mass fraction of the fast stream component, γ is the specific heat ratio and $\mathcal{D}_{12}$ the mass diffusivity coefficient.

The nondimensional mixture properties vary across the mixing layer according to the species concentration as given below. The equation of state is used to calculate the density as a function of the mixture temperature T and mass fractions s_1 and s_2.

$$1=\rho R_u T\left(s_1 W_1+S_2 W_2\right). \tag{8}$$

The gas constant and specific heat at constant pressure are,

$$R=s_1+R_{\text{ratio}}(1-s_1),\qquad c_p=s_1+c_{p_{\text{ratio}}}(1-s_1). \tag{9}$$

The dynamic viscosity is,

$$\mu=\frac{X_1\mu_1}{X_1+X_2\phi_{12}}+\frac{X_2\mu_2}{X_2+X_1\phi_{21}}, \tag{10}$$

$$\phi_{12}=\frac{\left[1+(\mu_1/\mu_2)^{1/2}(\mathcal{M}_2/\mathcal{M}_1)^{1/4}\right]^2}{\left[8\left(1+\mathcal{M}_1/\mathcal{M}_2\right)\right]^{1/2}} \tag{11}$$

$$\phi_{21}=\phi_{12}\frac{\mu_1}{\mu_2}\frac{\mathcal{M}_1}{\mathcal{M}_2} \tag{12}$$

The thermal conductivity is,

$$k=\frac{X_1 k_1}{X_1+X_2\phi_{12}}+\frac{X_2 k_2}{X_2+X_1\phi_{21}}, \tag{13}$$

$$\phi_{12}=\frac{\left[1+(k_1/k_2)^{1/2}(\mathcal{M}_2/\mathcal{M}_1)^{1/4}\right]^2}{\left[8\left(1+\mathcal{M}_1/\mathcal{M}_2\right)\right]^{1/2}} \tag{14}$$

$$\phi_{21}=\phi_{12}\frac{k_1}{k_2}\frac{\mathcal{M}_1}{\mathcal{M}_2} \tag{15}$$

where $\mathcal{M}$ is the molar mass of a given species, X_i the molar fraction of species i, the underscore *'ratio'* indicates the fast to low stream properties ratio and R_u is the universal gas constant.

The velocity away from the jet is set to a small value due to the streamline coordinates transformation. The temperature, mass fraction, density and other properties away from the jet are specified and equal to the external flow values. The ratios between inner and outer jets are U_1/U_2, T_1/T_2, s_1/s_2.

2.2 Numerical Method

The above boundary layer equations are solved by marching in the streamwise direction with an implicit second order backward differencing scheme. In order to avoid the computation of the normal velocity component the equations are written in a streamline coordinate system (ξ, ψ) Dahl and Morris (1997a).

$$\rho u = \frac{\partial \psi}{\partial x}, \qquad \rho v = -\frac{\partial \psi}{\partial y}. \tag{16}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} - \rho v \frac{\partial}{\partial \psi}, \qquad \frac{\partial}{\partial y} = \rho u \frac{\partial}{\partial \psi}, \tag{17}$$

$$\rho u \frac{\partial u}{\partial \xi} = \frac{\rho^2 u^2}{Re} \frac{\partial^2 u}{\partial \psi^2} + \frac{\rho u}{Re} \frac{\partial \rho u \mu}{\partial \psi} \frac{\partial u}{\partial \psi} \tag{18}$$

$$\rho u \frac{\partial T}{\partial \xi} = \frac{\rho^2 u^2 k}{Re Pr c_p} \frac{\partial^2 T}{\partial \psi^2} + \frac{\rho u}{Re\, Pr\, c_p} \frac{\partial \rho k u}{\partial \psi} \frac{\partial T}{\partial \psi} + \frac{(\gamma-1) Ma^2}{Re} \frac{\mu \rho^2 u^2}{c_p} \left(\frac{\partial u}{\partial \psi}\right)^2 \tag{19}$$

$$\rho u \frac{\partial s_1}{\partial \xi} = \frac{\rho^2 u^2 \rho D_{12}}{Re\, Pr\, Le} \frac{\partial^2 u}{\partial \psi^2} + \frac{\rho u}{Re\, Pr\, Le} \frac{\partial \rho^2 D_{12} u \mu}{\partial \psi} \frac{\partial s_1}{\partial \psi} \tag{20}$$

The gris is uniforme in the y plane and the corresponding ψ coordinates are calculated integrating the transformation equation.

$$\rho u dy = d\psi, \qquad \psi_{j+1} = \psi_j + \frac{1}{2} dy \left[(\rho u)_{j+1} + (\rho u)_j\right] \tag{21}$$

In order to discretize the boundary layer equations in the normal direction, given that in the ψ_j points are not equally spaced, the following is used.

$$\frac{\partial}{\partial \psi}\left[d \frac{\partial a}{\partial \psi}\right] = \frac{2}{\Delta\psi_{j+1} + \Delta\psi j}\left[d_{j+1/2} \frac{a_{j+1} - a_j}{\Delta_{j+1}} - d_{j-1/2} \frac{a_j - a_{j-1}}{\Delta_j}, \tag{22}\right.$$

$$\Delta\psi_{j+1} = \psi_{j+1} - \psi_j, \quad \Delta\psi_j = \psi_j - \psi_{j-1}, \quad d_{j+1/2} = \frac{1}{2}\left(d_{j+1} + d_j\right) \tag{23}$$

Comparisons between the present formulation and the similarity solution for a simple jet in terms of centerline velocity decay and similarity velocity profile 4 diameters downstream are presented in figure 1. The initial condition for this test was place one diameter downstream of the jet nozzle where a similarity profile was imposed.

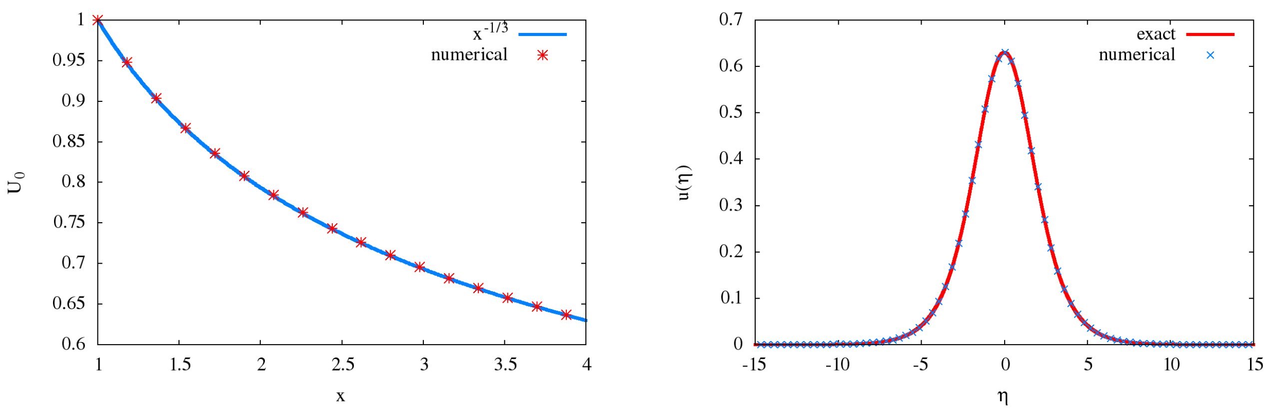

Figure 1. Centerline velocity decay (left) and velocity profile (right). Comparison between boundary layer solution with similarity solution for a simple jet.

3. RESULTS

The boundary layer code solves the parabolic equations for velocity, temperature and mass fraction not necessarily in the self-similarity region in order to capture the evolution of the coaxial jet in the near-field. All the following results consider $Re = 100$. The first test to be presented considers the evolution of a simple jet from a non-similar initial condition to investigate how fast the profile converges to a similar profile. Results for an initially thick profile are presented in Figs 2, 3 for the velocity profile and centerline velocity decay. Initially the non-similar velocity initial condition decays much faster to adjust to the similarity condition. Beyond ten diameters downstream the velocity profile is already very close to the similarity profile and the decay rate converges to the decay rate of the similarity solution of $x^{-1/3}$ as shown if Figs. 4 and 5.

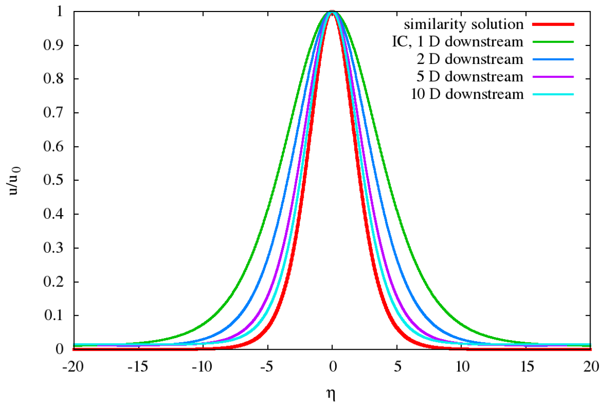

Figure 2. Velocity profiles at different positions downstream. Comparison with the similarity solution for a thick initial profile.

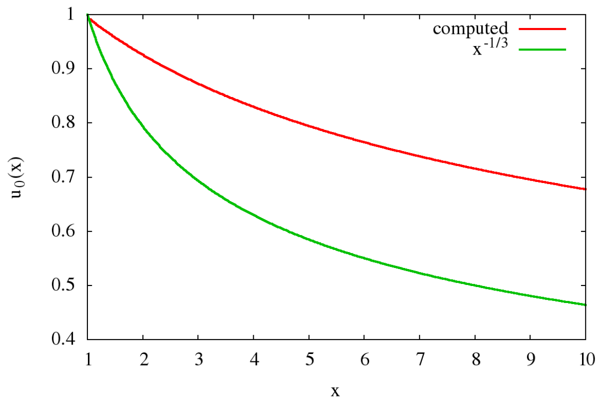

Figure 3. Centerline velocity variation along the streamwise direction. Comparison with the similarity solution for a thick initial profile.

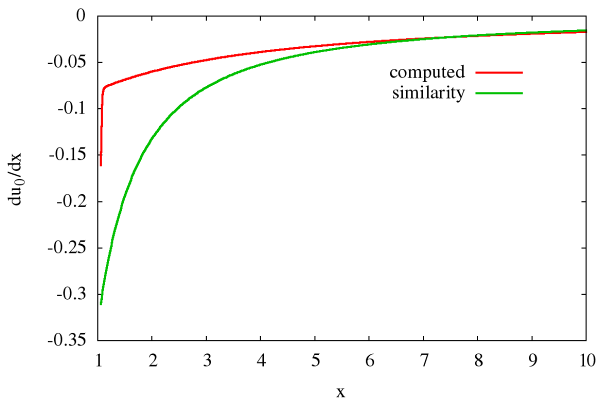

Figure 4. Velocity decay along the streamwise direction. Comparison with the similarity solution for a thick initial profile.

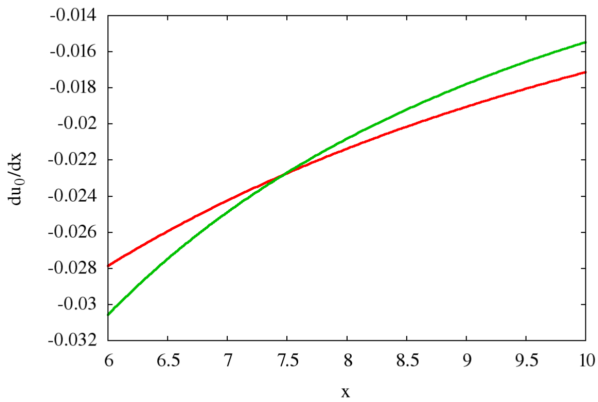

Figure 5. Velocity decay along the streamwise direction. Downstream region from 6 to 10 diameters.

When a thinner velocity profile is imposed at the initial condition position $x_0 = 1D$ the same behaviour is observed but the convergence to the similarity solution is much faster. At around five diameters downstream the velocity profile is already close to the similarity solution and the decay rate is close to the similarity -1/3 decay rate as shown in Figs. 6 and 7. Figures 8 and 9 show the convergence of the decay rate.

The following test cases compare the velocity profile decay to a similarity solution for coaxial jets. This test case considers a homogeneous flow of Oxigene first for an inner jet velocity twice as high as the outer jet velocity and then for an inner jet velocity half the outer jet velocity. The fast outer jet decay very rapidly to the similarity solution by diffusing momentum both outward and inward to the inner jet as shown in Fig. 10. The fast inner jet is somewhat shielded from the stagnant outer region by the outer jet and the resulting decay is slower as seen in Fig 11.

Next the characteristics of binary coaxial jets are explored in terms of velocity distribution for an inner jet of hydrogen surrounded by an outer jet of oxygen. The results are shown in Fig. 12 and 13. When the outer jet of oxygen is faster the jet spreads outward considerably corresponding to the behaviour of a thick jet. The jet behaviour when the hydrogen stream is faster is significantly different and the flow remains more concentrated. The mass fraction distribution reveal the same results in terms of penetration depth, as shown in Figs 14 for the outer jet velocity half the inner jet velocity.

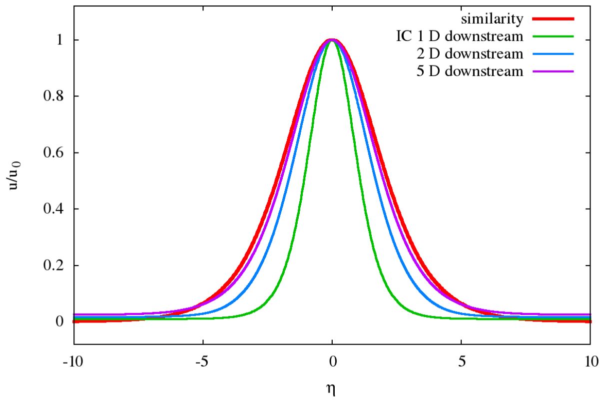

Figure 6. Velocity profiles at different positions downstream . Comparison with the similarity solution for a thin initial profile.

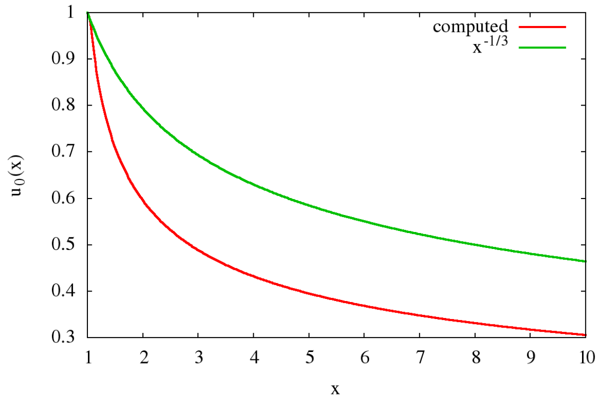

Figure 7. Centerline velocity variation along the streamwise direction. Comparison with the similarity solution for a thin initial profile.

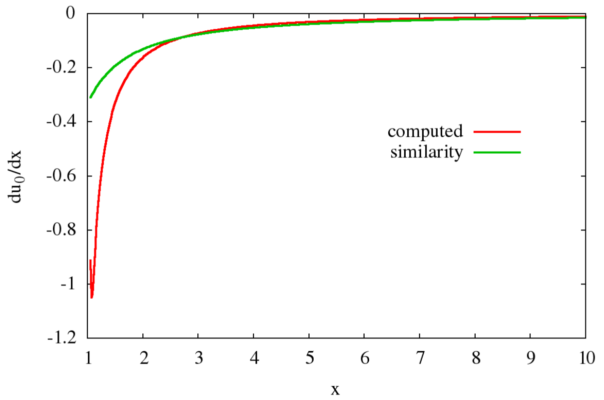

Figure 8. Decay rate along the streamwise direction. Comparison with the similarity solution for a thin initial profile.

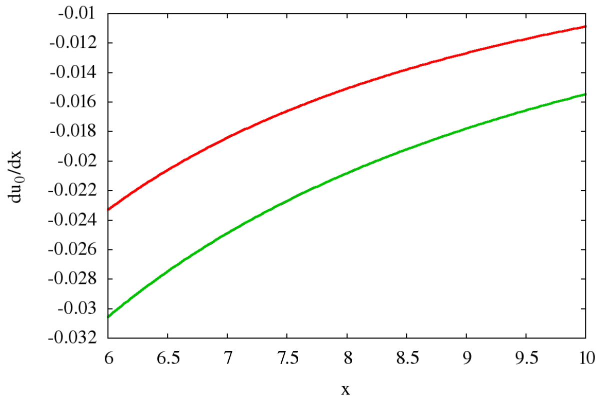

Figure 9. Decay rate along the streamwise direction. Downstream region from 6 to 10 diameters.

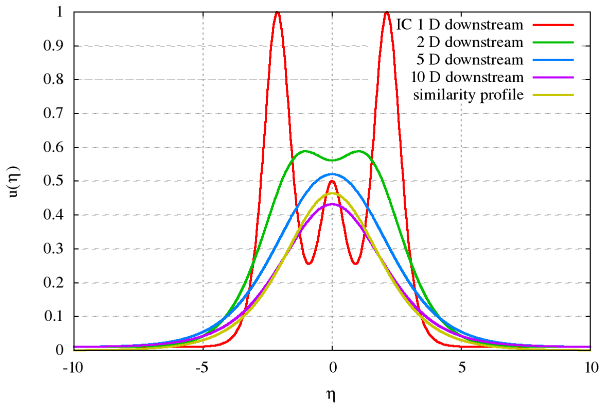

Figure 10. Velocity profile evolution in the strewise direction with outer jet faster than the inner jet.

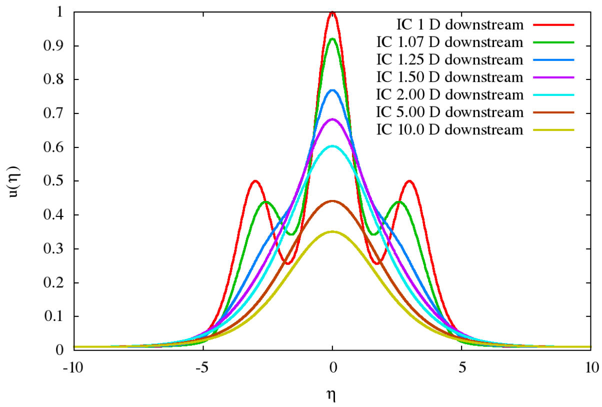

Figure 11. Velocity profile evolution in the strewise direction with outer jet slower than the inner jet.

4. CONCLUSIONS

A numerical algorithm was developed for the solution of boundary layer equations for binary coaxial jets and preliminary validation and flow configurations were studied. Other configurations will be evaluated in the following month to complement the results presented here. Preliminary results for the behaviour of binary compressible coaxial jets show considerable influence on the flow structure and the resulting flow configurations are expected to have significant influence on the stability of the flow with considerable changes on the amplification rates, gas mixture and critical frequencies and wave numbers. These stabilities analysis will be performed next, after a detailed study of the laminar flow behaviour.

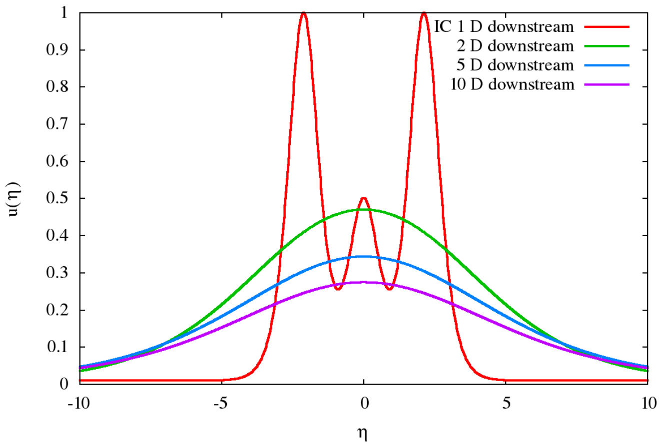

Figure 12. Velocity profile evolution in the strewise direction. Inner H2 jet with an outer O2 jet.

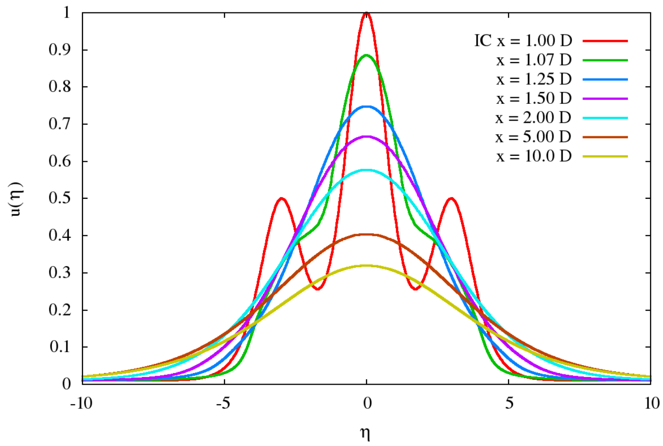

Figure 13. Velocity profile evolution in the strewise direction. Inner H2 jet with an outer O2 jet.

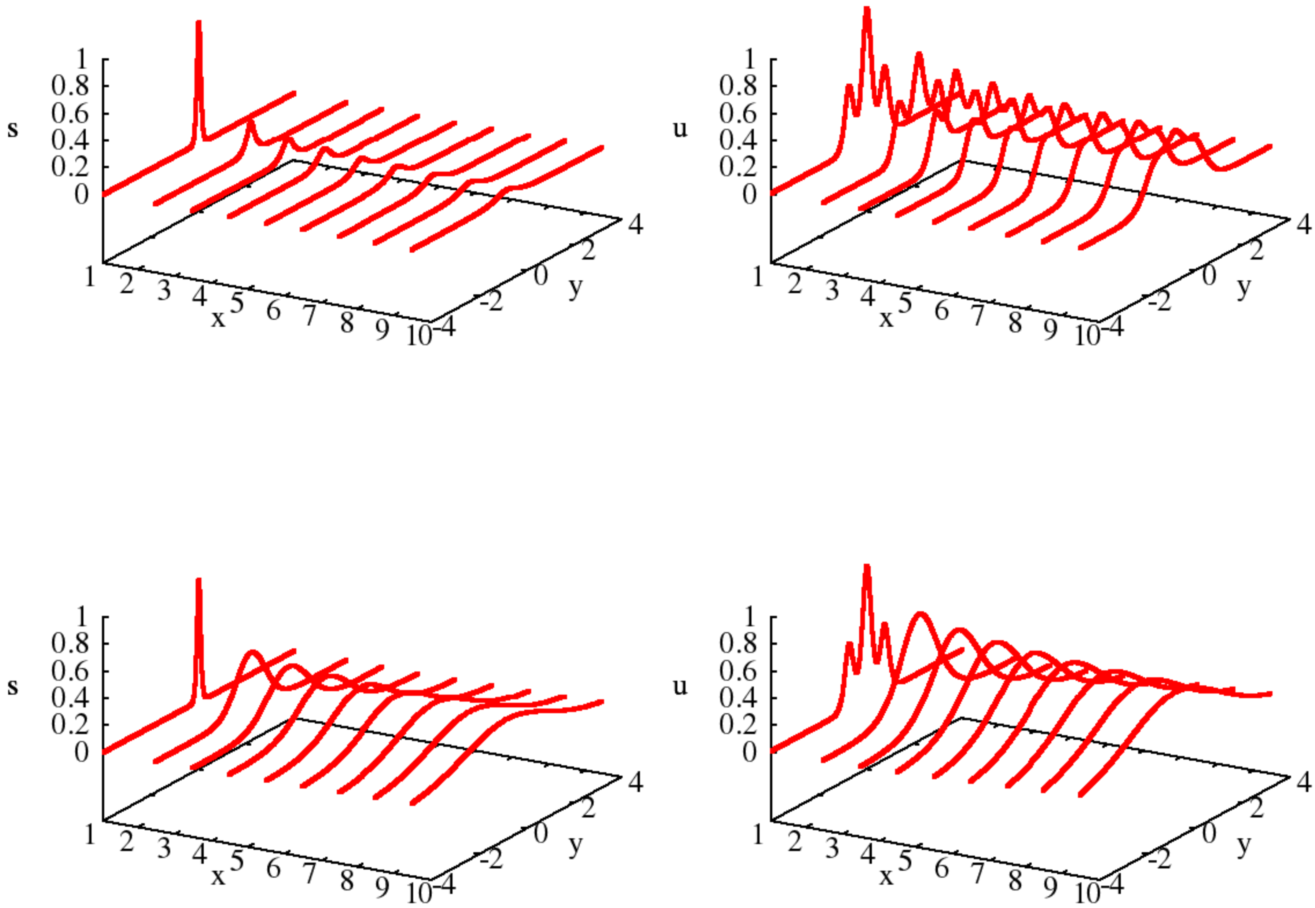

Figure 14. Mass fraction and velocity distribution, H2 (above) and O2 (below) inner jets.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

Anderson, J.D., 2000. *Hypersonic and High Temperature Gas Dynamics*. General Interest. AIAA.

Dahl, M.D. and Morris, P.J., 1997a. "Noise from supersonic coaxial jets, part 1: Mean flow predictions". *Journal of Sound and Vibration*, Vol. 200, No. 5, pp. 643–663.

Dahl, M.D. and Morris, P.J., 1997b. "Noise from supersonic coaxial jets, part 2: Normal velocity profile". *Journal of Sound and Vibration*, Vol. 200, No. 5, pp. 665–699.

Gloor, M., Obrist, D. and Kleiser, L., 2013. "Linear stability and acoustic characteristics of compressible, viscous, subsonic coaxial jet flow". *Physics of Fluids*, Vol. **25**, pp. 084102–1–084102–25.

Kennedy, C.A. and Gatski, T.B., 1994. "Self-similar supersonic variable-density shear layers in binary systems". *Physics of Fluids*, Vol. 6, No. 2, pp. 662–673.

Kozusko, F., Grosch, C.E., Jackson, T.L., Kennedy, C.A. and Gatski, T.B., 1996. "The structure of a variable property, compressible mixing layers in binary gas mixtures". *Physics of Fluids*, Vol. 8, No. 7, pp. 1945–1953.

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