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ROBUSTNESS BARRIERS IN LQG/LTR CONTROLLER VIA HYBRID MODEL GENETIC-NEURAL IN ROBOTIC MANIPULATOR

Ivanildo Silva Abreu
Alfredo Costa Oliveira Junior
Ismael Silva de Melo
José Charles Medeiros
Paula da Costa Sousa
Universidade Estadual do Maranhão, Cidade Universitária Paulo VI, s/nº, Tirrical, CEP 65055-970, São Luís- MA.
ivanildoabreu@yahoo.com.br, alfredo.coj@gmail.com, melosilva2@yahoo.com.br, medeiros_charles@yahoo.com.br,pauamizuna@hotmail.com

Abstract. *This research describes the design and analysis of a robust controller LQG/LTR in a robotic manipulator using a neuronal genetic intelligent hybrid model. It aims to recover the performance properties and stability that are lost with the inclusion of a stochastic estimator. The methodology of LQG/LTR design, where these properties of robustness are retrieved is formulated by defining the meshgrid goal and subsequently by means of an asymptotic procedure to recover the frequency response characteristics of this surface, adjusting a parameter that will generate singular values increasingly close. For this, a genetic algorithm and multivariable Bode diagram are used. Thus, the syntony is performed by parametric variations in the Riccati equation that are coordinated by a genetic algorithm and singular value decomposition. Therefore, the result of this LQG/LTR design, allowed a better performance with respect to robustness.*

Keywords: *genetic algorithm, neural network, Riccati equation, singular value decomposition, Multivariable Bode diagram.*

1. INTRODUCTION

This research presents the basic concept of robust control for linear and time-invariant system (LTI), in particular, a robotic manipulator (Chiu and Lee, 1997). The development of this research is performed in the domains of time and frequency, and for that, we use the Bode plot multivariable. Discusses the issues of stability and robust performance in closed loop, so that the technique is LQG/LTR. One of the most attention items of control scientific community is the study of the robustness of the feedback systems (Siqueira et al., 2011), (Doyle and Stein, 1993) and (Zhou and Doyle, 1998).

Robustness is a desirable characteristic of control system for at least two reasons. The first concerns the ongoing concern of the projectist that the control systems to work satisfactorily, even if the operating conditions are different from those considered in the design template (nominal) and, secondly, the robustness of conditions must be used in order to intentionally adopt a simplified design model to facilitate analysis, as well as their impact on the complexity of the resultant controller (Bechlioulis et al., 2014) and (Siqueira et al., 2011).

The methodology adopted in this research, robust control LQG/LTR, is a controller design technique used in multivariable dynamic systems. It is justified its use by some of its features, such as the controller robustness is guaranteed in the face of a broad class of modeling errors, the technique is used in $MIMO$ systems, the design procedure is of a systematic nature and that the methodology is based on a frequencial approach in $MIMO$ systems (Da Cruz, 1996).

The tools used for the development of the robust controller design LQG/LTR, are Linear Quadratic Regulator design (LQG) and the Kalman Filter (KF) that justifies the design LQG (Linear Quadratic Gaussian) (Wang et al., 2007) and (Smain and Brahim, 2009). These tools will not be interpreted as usual in stochastic optimal control, but used according to its properties. In this paper, we present a design methodology that allows the determination of a compensator, to be located in the direct branch of the control loop, so that the performance and stability requirements are met. The following describes the robotic manipulator which will be used in the robust control approach.

The recovery procedure of robustness properties, lost with the estimation of the state variables is performed by the LTR (Loop Transfer Recovery) procedure, once the robotic manipulator in question, has transmission zeros in the left half plane (zeros minimum phase) (Doyle and Stein,1993). The dual procedure to be used in this research design concerns the loop transfer function matrix, $K(s)G_N(s)$, which corresponds to the meshgrid opening in the input of the plant in Figure 1. In this methodology, it is assumed a priori conveniently, K compensator matrix, LQR design, while the L matrix of observer states (FK) is variable. This approach is consistent with the analysis of the singular values of $G(s)K(s)$ to translate the specifications of the control system through restrictions on their Bode diagrams (Da Cruz, 1996).

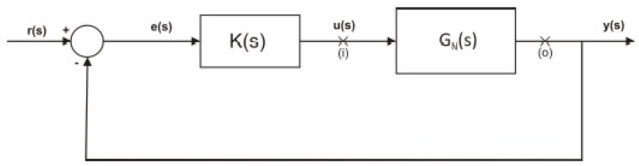

Figure 1. System block diagram for study of the LTR procedure.

2. INTELLIGENT LQR CONTROLLER

LQR problem is formulated by a nonlinear optimization framework, whose focus consists on minimizing a quadratic performance index. This problem has as an obstacle, the selection of the state weighting matrices and control, Q and R, respectively, and solving the algebraic Riccati equation (*ARE*) so it can allocate eigenvalues and eigenvectors specified and ensures, stability and robustness of the control system. Developments are presented in two approaches of computational intelligence in order to solve the LQR problem. These developments are a result of researches using an GA in selection of the weighting matrixes Q and R and solution (*ARE*) by means of a (RNR) (Wang, 1998), (Abreu et al., 2010) and (Neto et al., 2010).

The classic model of LQR design, shown here, can be found at (Athans, 2007) and (Smain and Brahim, 2009). This method provides control law that minimizes the quadratic performance index J and meets the constraints of equations in the state space. The control law is the controller gain. This gain depends on the symmetric solution set positively valued algebraic equation of Riccati (*ARE*):

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{1}$$

2.1 Intelligent modeling using genetic algorithm

A major challenge in control theory, using the linear quadratic regulator design is the selection of weighting matrices Q and R. In this research, it is used a computational intelligence approach to conduct a search of the same, the GA. The matrix parameters of the LQR design are used in order to perform the syntony of the optimal compensator gain. The mathematical model of finding the weighting matrix is observed in (Neto and Abreu, 2007).

Performed computing self-structure "closed loop", the score of each individual strategy is performed as the following: for each eigenvalue, within the range specified in the nonlinear optimization framework by the designer, is designated by a value (01) the individual QR and one hundredth (0.01) for each eigenvector associated having standard sensitivity, $Sz < 1$. The individual who has the highest score will be deemed the great, and the tie criteria will be considered rather it did have smaller sum of standard sensitivities.

2.2 Performance of initial population

The following defines the profile of the initial population values for the cost function in illustration. The profile of the initial population can be defined by the values of the objective function, shown in Figure 2. It can be seen clearly a genetic diversity, and the same low between the twentieth and the thirtieth individual. Others show better diversity. These conclusions about the initial population boot process, imply a satisfactory population in the search process of the optimum individual QR.

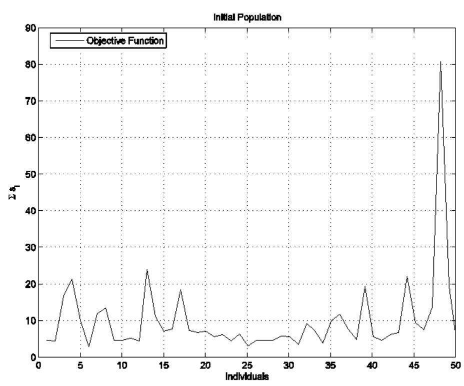

Figure 2. Total sensitivity of each individual

2.3 Performance of the final population

The values of the objective function for the final population compared to the average of the objective function for each individual of the population during the search process is shown in Figure 3. The mean values of each individual final population showed the improvement until the thirty individual; from this individual, it can not be realized many improvements.

The search performed by the GA found five viable individuals who satisfied the auto structure allocation strategies (eigenvalues and eigenvectors) and sensitivities. Among these stands out the individual 41 that generated the optimal weighting matrices:

$$Q_{LQR} = \begin{bmatrix} 41.436 & 5.494 & 4.069 & 5.12 \\ 5.94 & 44.103 & 9.148 & 3.102 \\ 4.069 & 9.148 & 29.682 & 6.951 \\ 5.123 & 3.102 & 9.951 & 22.067 \end{bmatrix}, \quad R_{LQR} = \begin{bmatrix} 15.225 & 2.517 \\ 2.517 & 11.568 \end{bmatrix} \tag{2}$$

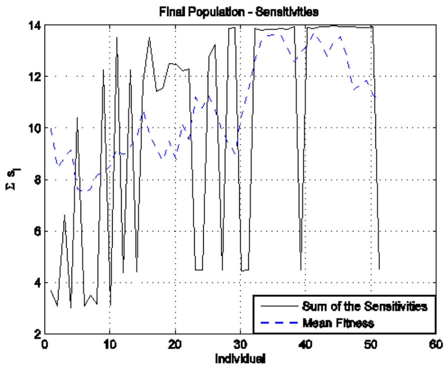

Figure 3. Evolution process-function average goal and better objective function of the generations

As may be seen, the GA search process with the configuration of its parameters, produced solutions very close to each other, i.e., the matrix set QR solution presented by the GA has only minor discrepancies. The sensitivity of the final population and the average fitness function of each generation is shown in Figure 4.

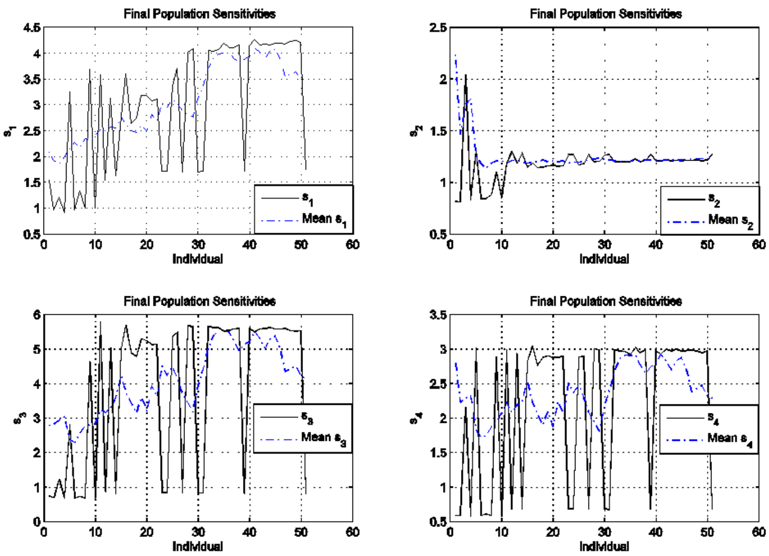

Figure 4. Fitness Standard of every generation and every individual

3. SOLUTION OF RICCATI EQUATION USING NEURAL NETWORK

In order to resolve the RAE, it uses combinatorial optimization problem, with the restrictions matrices of weighting, Eq.2. This structure is formulated to minimize an energy function monotonic, non-decreasing and bounded below (Wang et al.,1998). The matrices A, B, Q and R are presented, and its goal is to find a symmetric positive definite matrix solution, P. Its architecture is formulated as the following:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} \{e_{ij}[g_{ij}(P)] + e_{ij}[H_{i,j}(P, L)]\} \tag{3}$$

which $Z(t) \in R^{n\times n}$ are square matrices of activation states; $F = [.]$ is a vector field function activations not decreasing; $V(t) \in R^{n\times n}$ is the RAE solution, $Z(t) \in R^{n\times n}$ is a lower triangular matrix of the states of $Cholesky factor$. The positive parameters of the neural network tuning are the weights η_v and η_z. The matrix S in the above equation represents the influence of the weighting matrix R and is given by considering that the optimal controller gains are adjusted based on changes in the matrices Q and R.

Figure 5 illustrates the architecture of the neural network to solve the RAE, and made up of four layers connected bidirectionally. The output layer $V(t)$ is the solution RAE, $Z(t)$ represents the Cholesky factor Q. The Runge-Kutta fourth order is used to compute the values V and Z. The function performance to measure the $rRNA$ of the training process is the function Average Error Quadratic, EMQ, which represents the energy function, being the same given the combination of errors associated with RAE and the Cholesky factor. Accordingly, the EMQ_{RNAR} of the learning of $rRNA$, the components being given by: $e_{EAR} = Q - (A^TP + PA - PSP)$ and $e_{FC} = LL^T - P$.

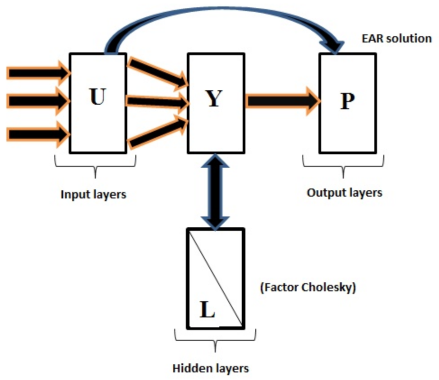

Figure 5. Architecture $rRNA$ for RAE

3.1 RAE solution using recurrent neural network unsupervised

This section comprises the convergence adjustment methods and analysis of $rRNA$ to solve the equation of Riccati. The parameter settings, η_v, η_z, the neuronal model, are supported by the standard infinite surfaces and function of energy Riccati equation solution. The standard of the infinite and the surface energy function, as functions of the parameters (weights) sizing η_v, η_z , are evaluated to define a configuration for improving the stability of convergence and solvency characteristics of $rRNA$. Figure 6 shows the surface of the infinite norm of the input matrix U, whereby the change ranges of the parameters η_v, η_z are respectively $\begin{bmatrix} 10000 & 1000 \end{bmatrix}$ and $\begin{bmatrix} 10 & 1 \end{bmatrix}$. The chosen parameters η_v, η_z and weights are 1000 and 1, respectively. Neuronal solution of Riccati equation, based on neuronal model that addresses the optimization framework that minimizes the energy function of the output layer rRNA, V, is given by 4:

$$P_{RNA} = \begin{bmatrix} 17.884 & 2.228 & 2.533 & 1.838 \\ 2.228 & 19.229 & 2.760 & -2.245 \\ 2.533 & 2.760 & 13.596 & -4.100 \\ 1.838 & -2.245 & -4.100 & 10.859 \end{bmatrix} \tag{4}$$

4. KALMAN FILTER FOR ESTIMATION OF STATES

In this section it is formulated the Kalman filter, FK, in order to allocate the eigenvalues and eigenvectors in stochastic state estimation problem. This new formulation allows the use of evolutionary computation techniques to determine a stochastic observer to estimate the states with a desired dynamic, where you have no access to measure them, from the Kalman filter. By making use of self-structure restrictions, restrictions of eigenvalues and eigenvectors, one can formulate the estimation problem stochastic state, which is found $L = \Sigma(\Xi, \Theta)C^T\Theta^{-1}$ of FK, so that the constraints are satisfied. This problem can be formulated as an optimization problem, allowing the determination of a stochastic estimator directed $L(\Xi, \Theta)$ using random search techniques.

A GA for the search of the covariance matrices Ξ of the state disturbance and Θ measurement noise for the Kalman filter is developed. Therefore, it has been interest in performing a numerical algorithm that can be used in any estimation

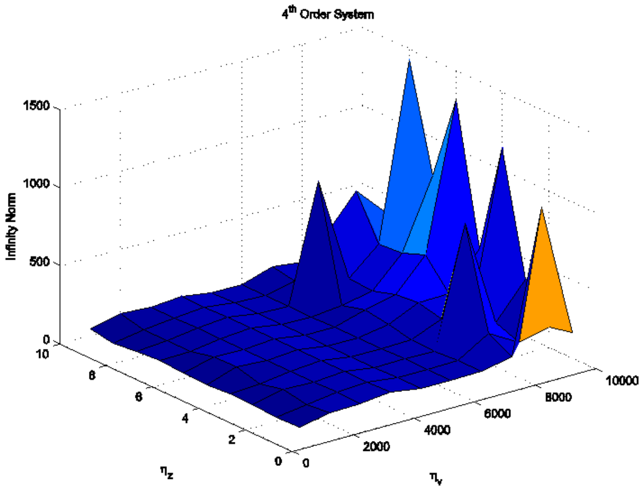

Figure 6. The ifinite norm of the surface to $\eta_v = 1000 - 10000$ and $\eta_z = 1 - 10$.

problem stochastic state, in order to determine a gain $L(\Xi, \Theta)$ in order to allocate eigenvalues and eigenvectors. It uses the same procedure for the LQR design.

The Figure 7 displays the results of Kalman Filtering. In it there is the trajectories of states versus the estimated states. To note that, despite the presence of noise, the regulatory action is detected in all states. For a more precise analysis, Figure 8 shows the estimation error in each state. It was observed that the rule 4 has the lowest estimation error, while the higher estimation error was observed for the state 5 (Abreu and Fonseca, 2010).

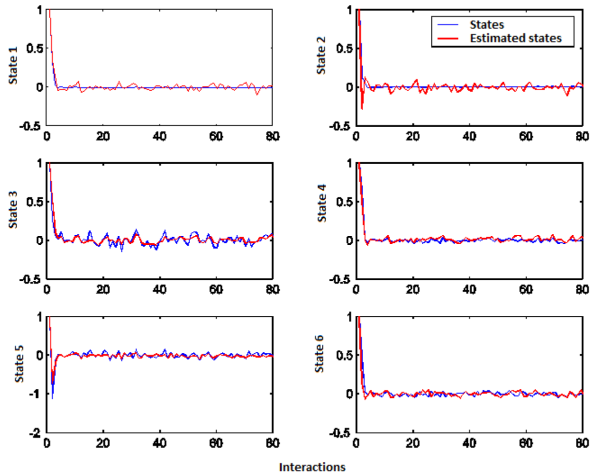

Figure 7. States of the stimated trajectories vesus

5. LQG/LTR CONTROLLER

The LQG/LTR problem with recovery of the feedback loop is formulated as an optimization problem that is applied in the recovery of the input meshgrid. The methods of the LQG/LTR design shown in (Doyle, 1993) provide important theoretical results, but the approach is lacking in models, efficient algorithms and procedures, towards search method, setting the loop gain and its recovery methods except trial and error. In this research, a design procedure to adjust the observers of earnings is presented to recover the mesh at the entrance. The adjustment gain are based on Automatic modification the parameter v, with reference to the covariance matrices determined by the FK, to determine optimal gain state observer, to recover the sturdiness properties optimizing the structure as it will be formulated.

5.1 Gain adjustment model LTR

The problem LQG/LTR to recover the bonding properties of the loop transfer function is formulated as an optimization problem. The performance index is the best choice that minimizes the error of frequency decomposition values of control structure of LQR and LQG designs. Stand out as objectives to determine the controller, Section 3, the selection of weighting matrices Q and $RLQR$ problem, Section 2, and the determination of the observer gain, considering the

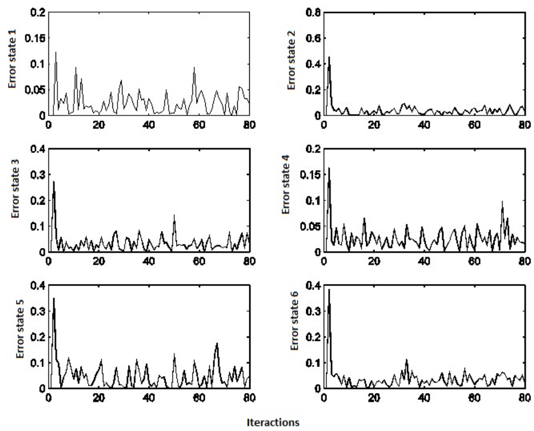

Figure 8. Estimation error for each state

recovery by the entrance, the LQG problem associated with selection of covariance matrices. The use of trial and error procedures for the LTR design warrants the development of an algorithm that provides a good method to determine the loop gain recovery. The gain K controller, set in the procedure was given in the first report.

The procedure for the recovery of the LTR of the loop gain at the entrance, shows that appropriate choices of covariance matrices is a function of a parameter v_i, which can approximate the LTR control loop the loop LQR control. The recovery approach to robustness properties of the research design, using genetic algorithms and neuronal networks was motivated in article (Sergi, 2012).

5.2 LTR model search

The selection of matrices $(Q; R)$ and (Ξ, Θ) re the two alternatives to meshgrid recovery LQR design. The recovery by the entry, the search is directed to the headquarters. Variations of matrices weighting are referenced matrices $(Q_0; R_0)$ and the law for this variation is

$$(Q_i = v_i^2 Q_0 + CC^T, R = v_i^2 R_0) \tag{5}$$

Eq. 6 which is a scalar function v_i. The recovery by the entry, the search is directed to the pair of arrays (Ξ, Θ), and the covariance matrices are referenced to the covariance matrices (Ξ, Θ) and a law establishing its variations $\Xi_i = v_i^2 \Xi_0 + BB^T, \Theta$ as a function of a scalar v_i.

The problem is modeled as a combinatorial optimization framework, enabling the determination of the controller gain $K = R^{-1}B^T P_{LQR}(Q; R)$ or state observer gains $L_{\Xi,\Theta}$ that retrieves the loop gain L_{LQR} through the design $L_{LQR/LTR}$. Recovery of the meshgrid LQR design is performed in the frequency domain. Once the system is opened in Point 1, Figure1,the open loop transfer function is given by $L(s)^{input}_{LTR} = F(s)G(s) = K\Phi_r LC\Phi B$, where $L(s)^{input}_{LTR}$ is the entrance into the transfer function. The problem is formulated to determine that the state observer gains $L_{\Xi,\Theta}$ which recovers the loop transfer function $L(s)^{LQR}$, supported by a combinatorial optimization framework.

$$min \sum \left(\sigma_i^{L_{LQR}} - \sigma_i^{K\Phi_r LC\Phi B}\right) \qquad s.a|L_{\Xi,\Theta}| \leq \varepsilon^{L_{\Xi,\Theta}} \qquad \lambda_{i,left} \leq \lambda_{calc.} \leq \lambda_{righ.} \qquad S_i \leq \varepsilon, i = 1 \cdots n. \tag{6}$$

where $|L_{\Xi,\Theta}| \leq \varepsilon$ are operational constraints of the problem. A recovery solution LTR design at the entrance is given by selecting the pair of covariance matrices, which is held by the search model $LTR_{\Xi,\Theta}$, and these arrays are the parameters of matrix algebraic equation of Riccati, $A\Sigma + \Sigma A^T + G\left(\nu^2 \Xi_0 + BB^T\right) G^T - \Sigma C^T(\nu^2\Theta_0)^{-1}C\Sigma = 0$, which v is the parameter used to adjust climb the covariance matrices. The solution of algebraic Riccati equation is used to compute the observer gains $L_{LTR} = \Sigma C^T(\nu^2\Theta_0)^{-1}$, where L_{LTR} which are the observer gains of the Kalman filter.

5.3 Performance AG-LQR/LTR

Performance $GA - LQG/LTR$ design is verified by comparing the singular values of the LQR design and LTR design. The implementation of the algorithm considers a population composed of fourteen individuals and a search cycle of seven generations. The $crossover$ operation has the parameter values 0.1 and 0.05. The mutation operation is likely to occur $probmut = 5\%$.

Figure 9 shows the singular values of the recovery of strength properties of the LQR design design using LTR design, using the genetic algorithm. It was found that when $\upsilon \to 0$, bonding properties lost with the pet, are completely recovered. According (Lewis and Vassilis, 1995), the limits to secure robustness properties are given in terms of minimum singular values, being large at low frequencies (for performance robustness) and the maximum singular values are small at high frequencies (for robustness of stability).

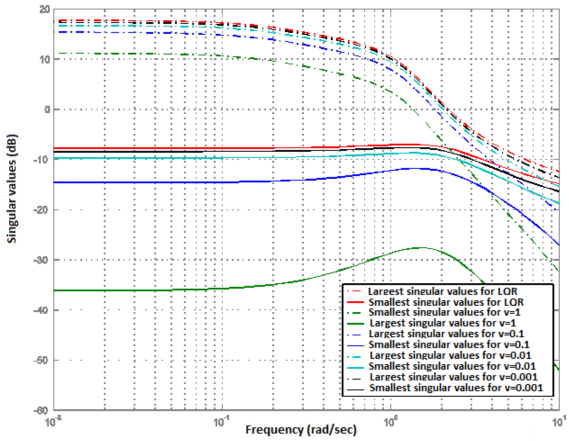

Figure 9. Singular values of recovery of strenght properties

6. PERFORMANCE BARRIERS AND ROBUST STABILITY

Given that this research is a robust controller design, it is important verify commitments performance and robust stability. For this analysis, it was considered as unknown dynamics, the effect of noise caused by two poles as (Skogestad and Postlethwaite, 2007). For this analysis we adopted the following specifications:

•Reference Signal Monitoring and disturbance rejection with error not exceeding a 10% for $\omega \leq 0,5\ rad/s$;

•Sensitivity to variations in the plant does not exceed 15% for $\omega \leq 0,5\ rad/s$.

To get the reference signal monitoring and disturbance rejection with an error not exceeding 10% for $\omega \leq 0,5\ rad/s$, that is, δ_r and $\delta_d \leq 0,1$, the barrier performance for these conditions is given by $20 \log 10(0,1)$ for $\omega \leq 0,5\ rad/s$.

To have sensitivity to plant variation in error not exceeding 15% for $\omega \leq 0,7\ rad/s$, i.e., $\delta_s \leq 0,15$, the barrier performance for these conditions is given $20 \log 10(0,15)$ for $\omega \leq 0,7\ rad/s$.

The stability of the barrier was determined by finding the inverse of the highest singular value of the measurement noise ζ is a constant varying from 0.1 to 1.0 for a given frequency. The variation across frequency provides the barrier stability.

Figure 10 illustrates the unique value of the nominal system increased along with the performance barriers and stability. For this figure, it is noted that the increased nominal system is within limits considered for this design. Therefore, the system meets the system commitments that consider the errors regarding the tracking of the input signal, the disturbance rejection and sensitivity to variation of the plant. Also, it is important to note that the system remains stable even in the presence of noise, G_d.

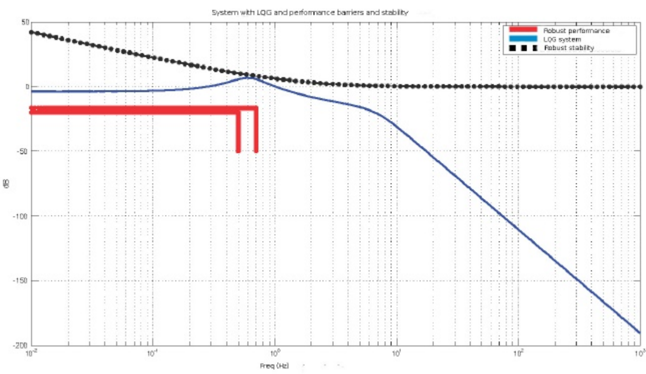

Figure 10. Increased nominal system (in blue) and the performance barriers (in red) and stability (in black).

7. CONCLUSION

A hierarchical model of genetic algorithms for entry into the transfer loop recovery was presented as a solution for controlling gains adjustment and observers in the state space. the development of the LQR controllers, LQG and LQG/LTR and observers of design methods based on a hierarchy of genetic algorithms and artificial neural network models were used to adjust the loop gains. Based on the model implementations, controller and observer, clearly realizes that in the Bode diagram for stability and robust performance, the unique value of the loop gain is allocated according to the design specifications. Thus, the adjustment method proposed can be seen as a good alternative for practical applications in real time.

8. ACKNOWLEDGEMENTS

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