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INVERSE DESIGN OF RADIATIVE ENCLOSURE WITH FILAMENT HEATERS

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Abstract. *In typical designs of ovens, it is desired to determine the positions for the heaters that result in a uniform heating of the surface representing the material undergoing the thermal process, namely the design surface. In this study, the heat flux and temperature are prescribed on the design surface, and the heating elements are positioned in a single filament so as to satisfy prescribed conditions. The oven is modeled as a three-dimensional cavity filled with transparent medium. Due the symmetry of the problem, it is possible solve only one quarter of the cavity. It is assumed that the walls are gray and diffuse and the dominant heat transfer mechanism is thermal radiation. This problem, conventionally, is solved through a trial-and-error procedure, which in general are not able to achieve the prescribed conditions with the expected level of accuracy. In this work, the solution is obtained by inverse analysis. The inverse problem is solved implicitly, as an optimization problem. The solution is obtained by the method of optimization extreme (GEO), a stochastic global optimization method used to find the t shape of the heating filament. The methodology leads to satisfactory results, with maximum error less than 2%, and requiring only two filament heaters.*

Keywords: *inverse analysis, radiative enclosure, filament heater, GEO.*

1. INTRODUCTION (TIMES NEW ROMAN, BOLD, SIZE 10)

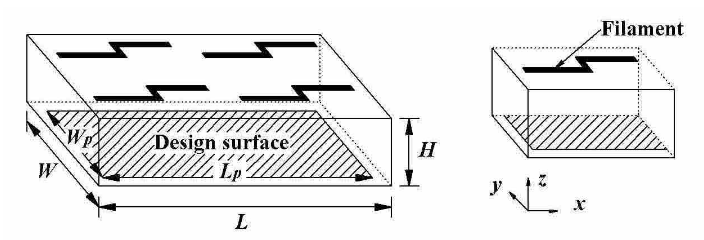
Heat treatment is widely used in the industry to restore or change materials properties under high temperatures, a case where the heat transfer can be dominated by thermal radiation. One example is the annealing treatment, which requires uniform rise of the temperature to avoid internal stresses that may lead to cracking. To obtain the required heating conditions, the heat flux and temperature are necessarily prescribed on the material surface. In this paper, the material surface is termed design surface; the problem consists of determining the spatial positions for the heating elements to achieve the desired conditions on the design surface. The conventional solution techniques are based on the specification of one condition for each boundary of the system, which require a trial-and-error approach to find the needed information for the heating elements. Alternatively, the analysis can be formulated as an inverse problem, in which two boundary conditions are specified on some boundaries, while other boundary is left unconstrained.

The inverse analysis of radiative systems is formulated by a set of Fredholm equations of the first kind, which are known to be ill-posed. Special methodologies, such as optimization methods, are required to estimate answers for such problems. Optimization techniques have been applied to solve radiative enclosures. The conjugate gradient method was applied for the estimation of temperatures over the heater surface in an enclosure with diffuse-spectral design surface (Bayat et al., 2010). Cassol et al., (2011) used the generalized extremal optimization (GEO) algorithm to find the location and the luminosity of the light sources that satisfied a prescribed luminous on the working area. Brittes and França (2013) applied a hybrid method that combined GEO optimization to solve for the position of the heaters, and TSVD regularization to obtain the required heat input in the heaters. These two studies considered punctual heaters, so to achieve a solution with good accuracy a large number of independent heaters were required, which is not the best practical alternative.

This work considers the inverse design of a three-dimensional rectangular enclosure with diffuse-gray surfaces. Inside the enclosure there is a non-participating medium. The spatial distribution of the heaters is left unconstrained while two conditions are imposed on the design surface – the radiative heat flux and the temperature. This study proposes the solution through optimization by the GEO algorithm (Souza et al., 2003). The objective function to be minimized is a measure of the deviation between the prescribed heat flux on the design surface and the heat flux that is obtained from the conditions found for the heaters. The major contribution of the present study is to propose a methodology to design filament shaped heaters which is more practical than punctual heaters. In spite of constraining the heaters to be filament shaped, the inverse analysis led to designs with quite acceptable accuracy.

2. PHYSICAL AND MATHEMATICAL FORMULATION

Figure 1(a) shows a rectangular radiative enclosure with length, height and width equal to L, H and W, respectively. The medium in the interior of the enclosure is non-participating so that the heat transfer involves solely the radiation exchanges between the surfaces, which are assumed to be diffuse-gray. The enclosure is formed by three types of surface elements: design surface elements, for which the temperature and heat fluxes are prescribed; heating elements, for which locations are to be determined so that the two conditions on the design surface are attained; and adiabatic wall elements, which are insulated from outside. The design surface ought not to cover the entire extension of the enclosure base; otherwise the portions close to the corners would be mainly affected by the reflections from the side walls, not by the radiation from the heating elements on the top surface. The three dimensional enclosure is divided into finite-sized square elements with $\Delta x = \Delta y = \Delta z$. In the analysis to be presented, the design surface, heating and wall elements are designated by *jd*, *jh* and *jw*, respectively, with $1 \le jd \le JD$, $1 \le jh \le JH$ and $1 \le jw \le JW$. Index *j* will be used when referring to a general surface element. Uniform heat flux and temperature, designated by q_{DS} (W/m²) and T_{DS} (K), respectively, are prescribed on the design surface. Due to symmetry, only one quarter of the system will be simulated. One filament heater will be assembled for each quarter, so that a total of four similar heaters will be required.



(a) (b)

Figure 1. (a) Radiative enclosure; (b) Solution domain corresponding to one-quarter of the enclosure

The radiosity method (Siegel and Howell, 2002) is used to solve radiative transfer in this paper. In the method, the surfaces are subdivided into N surface elements, for which one equation is set according to the known boundary condition, that is, temperature, Eq. (1), or the net radiative heat flux, Eq. (2).

$$q_{o,j} = \varepsilon_j e_{b,j} + (1-\varepsilon_j)\sum_{k=1}^{N} F_{j-k} q_{o,k} \tag{1}$$

$$q_{o,j} = q_{r,j} + \sum_{k=1}^{N} F_{j-k} q_{o,k} \tag{2}$$

where $q_{o,j}$ and $q_{r,j}$ are, respectively, the radiosity and net radiative heat flux leaving surface element *j*. The $e_{b,j} = \sigma T_j^4$ is the blackbody emissive power, σ is the Stefan-Boltzmann constant, σ = 5.67×10^{-8} W/(m^2·K^4). The view factor between two elements, F_{j-k}, is the fraction of the radiative energy that leaves surface element *j* and reaches element *k*, and ε_j is the hemispherical total emissivity of surface element *j*. Equations (1) and (2) are valid for diffuse-gray surfaces. Equations (1) and (2) can be written in dimensionless form according to:

$$Q_{o,j} = \varepsilon_j t_j^4 + (1-\varepsilon_j)\sum_{k=1}^{N} F_{j-k} Q_{o,k} \tag{3}$$

$$Q_{o,j} = Q_{r,j} + \sum_{k=1}^{N} F_{j-k} Q_{o,k} \tag{4}$$

In the above equations, the dimensionless heat flux and the radiosity are given by $Q_{r,j} = q_{r,j} / \sigma T_{DS}^4$ and $Q_{o,j} = q_{o,j} / \sigma T_{DS}^4$, respectively. The dimensionless temperature is defined as $t_j = T_j / T_{DS}$.

2.1 The generalized extremal optimization method (GEO)

The generelized extremal optimization (GEO) is a stochastic method where the N variables of the optimization problem are encoded by only one binary string and the bits are randomly aligned to form an initial bit configuration. The number of bits m to attain a desired precision q for each design variable is given by the following equation:

$$2^m \geq \frac{x_i^u - x_i^l}{q} + 1 \tag{5}$$

where x_i^u and x_i^l are the upper and lower bounds, respectively, of variable x_i, with $i = 1, N$. All the aligned bits provide a single binary string whose length is $L_b = mN$. Conversely, the physical value of each design variable is given by:

$$x_i = x_i^l + \left(x_i^u - x_i^l\right)\frac{I_i}{2^m - 1} \tag{6}$$

where I_i is an integer number obtained in the transformation of the variable x_i from its binary form to a decimal representation.

The objective function applied in this paper measures the deviation between the specified heat flux on the design surface and the one obtained from algorithm on the heater, can be written as:

$$F\left(x_{jh}, y_{jh}\right) = \sqrt{\sum_{jd=1}^{JD}\left(Q_{DS} - Q_{r,jd}\right)^2} \tag{7}$$

In the GEO method, one bit is flipped (0 to 1 or vice-versa) and the objective function value, Eq. (7), is calculated and compared to the previous one, then the bit is returned to its original value. This process is repeated until all bits in the string are flipped. Next, the bits are ranked according to the change in the objective function. The lowest rank (k = 1) corresponds to the worst modification in the objective function while the highest one ($k = L_b$) is attached to the best modification. Then, a bit is picked to mutate (from 0 to 1 or vice versa) and a random number RAN in the range [0, 1] is generated. The probability of a bit to mutate is determined from:

$$P(k) = k^{-\tau}, \quad 1 \leq k \leq L_b \tag{8}$$

where τ is a positive adjustable parameter. However, a bit is confirmed to mutate only if $k^{-\tau}$ is equal or greater than RAN. Otherwise, a new candidate bit is chosen and the process is repeated until a bit is confirmed to mutate, Eq. (8) reveals that the bits with bad fitness (low k) are more likely to mutate than the bits with good fitness (high k). This process is repeated until a given stopping criterion is reached. Then, the best value for the objective function is found and the corresponding configuration is returned.

The build of the filament heater requires restrictions for position. The position x_i of the first heater in the filament is obtained according to Eq. (6) with the restrictions, x_i^l and x_i^u, defined by the grid. The restrictions to the next heater are updated according to the position of the previous heater, so $x_{i+1}^l = x_i - 1$ and $x_{i+1}^u = x_i + 1$. The same is imposed to y, the initial position y_i is defined by the grid with y_i^l and y_i^u, and the next position the equations are addicted: $y_{i+1}^u = y_i + 1$ and $y_{i+1}^l = y_i - 1$.

3. RESULTS AND DISCUSSION

In this study, the emissivities of the design surface and of heat sources are $\varepsilon_{jd} = 0.9$ and $\varepsilon_{jh} = 0.9$, respectively. The other elements of the enclosure are adiabatic, so their emissivities do not affect the solution. The aspect ratio of the enclosure base is $W/L = 0.8$; the dimensionless height is $H/L = 0.2$. Due to the geometry symmetry, one quarter of the enclosure is solved. The adopted number of elements in the x, y and z directions are 15, 12, and 6, respectively. Grid independency tests revealed that the chosen number of elements provides sufficient numerical accuracy in face of the errors that are expected from the inverse analysis. The design surface dimensions are taken as $W_p/L = 0.6$ and $L_p/L = 0.8$. In this configuration, the number of elements on the design surface is JD = 108.

The specified conditions on the design surface elements are the net radiative heat flux, equal to $q_{DS} = -3.22\times10^3$ W/m², and temperature, equal to $T_{DS} = 673$ K, which are possible values found in industrial heating processes. The negative value for the radiative flux follows from the convention that heat out of a surface is positive. In dimensionless form, the specified conditions are therefore $Q_{r,jd} = Q_{DS} = -0.277$ and $t_{jd} = t_{DS} = 1.0$, respectively. The remaining elements are all assumed perfectly insulated, that is, $Q_{r,jw} = 0$.

The accuracy of the solution can be accessed by simply running the forward problem using the found information for the heating elements and then verifying the deviation between the specified and the resulting radiative heat fluxes on each design surface element. The error is defined as:

$$\gamma_{jd} = \left| \frac{Q_{DS} - Q_{r,jd}}{Q_{DS}} \right| \tag{9}$$

To illustrate the methodology, this study compares one filament heater with forty elements and fifty elements. The restrictions for the positions of the heaters are defined by the grid, that is $1 \le i_x \le 15$ (x direction) and $1 \le i_y \le 12$ (y direction). The dimensionless heat input $Q_{r,jh}$ is defined using the quantity of energy necessary on the design surface for obtained the prescribed heat flux so, for forty heat elements, $Q_{r,jh} = 0.7479$. The number of evaluation of objective function was 500,000 for determine the best value of parameter τ, the interval of search was [0,50; 2,50] with steps for 0.25, and ten repeats of the solution, that is, ten different seeds were used to define the shape and position of the first filament. Since each seed leads to different results, for the result to become independent of the seed, repetitions are carried out, and the best value of parameter τ is obtained with the medium value of the result obtained for each seed. Once defined the best τ, the problem is solved again with 1,000,000 evaluations of the objective function to obtain the final result.

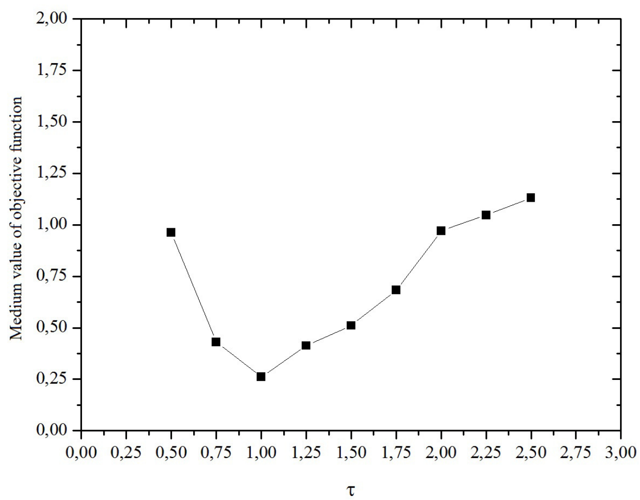


Figure 2. Evaluation of parameter τ for the case with 40 elements.

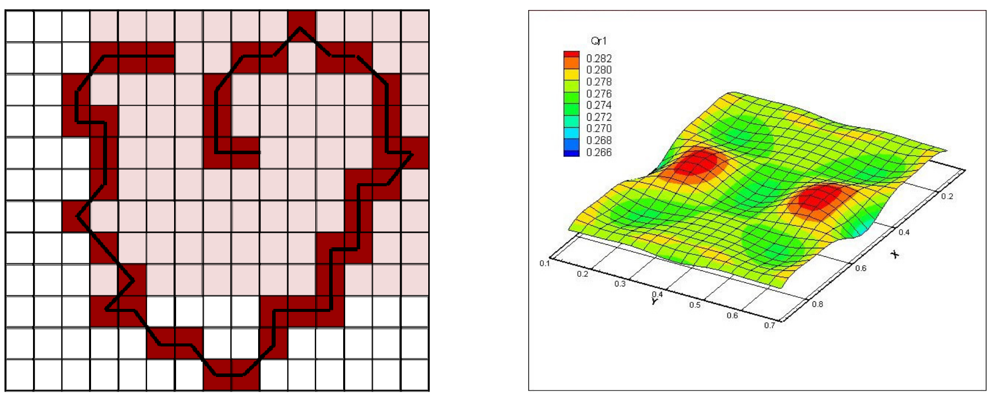
The best value of parameter τ is defined by the minimum value of objective function. Analyzing Eq. (7), it is possible to verify that the objective function is related to the error of the solution. Therefore, for this reason, the minimum value of objective function leads to the best solution. Observing Figure 2, the best τ clearly is equal to 1. Table1 presents results for ten different seeds, for the best τ.

The minimum value of the objective function was obtained with seed 2. The position of the filament heater for this case is presented in Fig. 3 (a). The corresponding heat flux on the design surface is shown in Fig. 2(b). As can be seen, the dimensionless heat flux shows a satisfactory uniformity around the target value of 0.277. The solution leads to the errors shown in Table 2. This solution leads to four filament heaters for the entire cavity, that is, one heater for each quarter of the domain. Errors lower than 5% are considered acceptable for this type of problem, so this solution can be considered satisfactory.

The fourth seed, even if does not produces the lower value of objective function, produces an interesting solution for the position of the element heaters. These results are presented in Fig. 4.

Table 1. Value of objective function for each seed. Filament heater formed with 50 elements.

Seed	Value of objective function
1	0.2347
2	0.1904
3	0.2162
4	0.2161
5	0.2246
6	0.3361
7	0.2147
8	0.2461
9	0.2105
10	0.2203

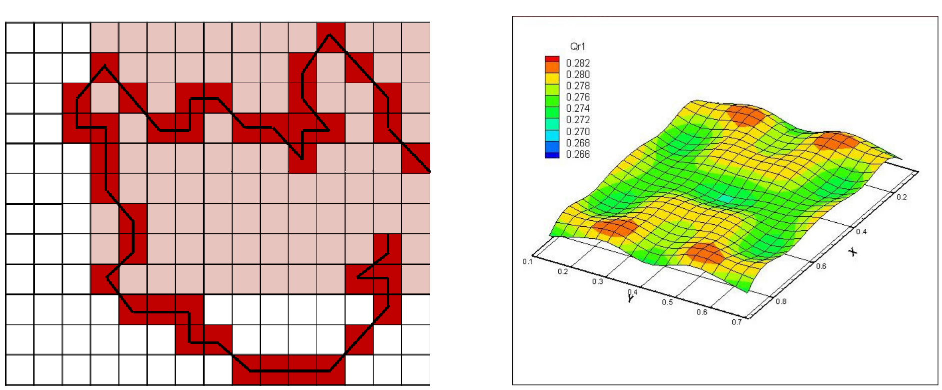


(a) (b)

Figure 3. (a) Representation of the filament heater in black, positions of the element heaters in red. The shaded area corresponds to the design surface. (b) Radiative heat flux on design surface..

Table 2. Errors related for the solution with 40 elements in filament heater.

Average error (%)	0.63
Maximum error (%)	2.86



(a) (b)

Figure 4. (a) Representation of the filament heater in black, positions of the element heaters in red. The shaded area corresponds to the design surface. (b) Radiative heat flux on design surface.

Table 3. Errors related for the solution with 50 elements in filament heater.

Average error (%)	0.72
Maximum error (%)	1.87

One feature of this solution is that one end of the filament lies on the symmetry line, depicted in Fig. 1. So, the entire cavity would be built with just two filament heaters rather than four filament heaters. The errors related with this case are presents on Table 3, indicating another satisfactory solution. As seen, the maximum error was lower than the previous case, but the average was higher. This apparent contradiction is that the objective function defined by Eq. (7) is related but not exactly the average or the maximum errors. When the objective functions is minimized, it can be expected that the both errors will also be minimized, but they will not be necessarily the minimum possible values.

As a final example for this study, it is considered the case with fifty elements forming the filament heater. The evaluation of parameter τ is presented in Fig. 5. Observing this figure, the best τ is clearly equal to 1. Table 4 presents ten different seeds for τ equal to 1, and the respective values of the objective functions. Observing Table 4, it can be seen that the lower value of objective function was for the first seed.

The position of the filament heater and the respective radiative heat flux, obtained for the first seed, are presented in Fig. 6. It can be again observed that the radiative heat flux on the design surface for this case shows a satisfactory uniformity around the target value of 0.277. The difference of this case in comparison to the solution with forty elements is that the two ends of the filament lies in the symmetry lines. This means that the entire cavity would be built with just one filament heater, as is shown Fig. 7. The error of the solution is shown in Fig. 5, which can be considered fully satisfactory.

The two examples show that the inverse methodology can be a useful tool in the design of heaters to achieve prescribed conditions on the design surface. In this analysis, it was prescribed uniform temperature and heat flux on the design surface, but other conditions could be proposed as well. The filaments presented in Figs. 2, 3 6 and 7 illustrate that is possible to achieve uniformity of heating on the design surface with satisfactory level of accuracy. Moreover, the relatively complex, non-intuitive shapes of the heaters indicates that it would be very difficult, if not impractical, to attempt to find the solutions with a trial-and-error solution.

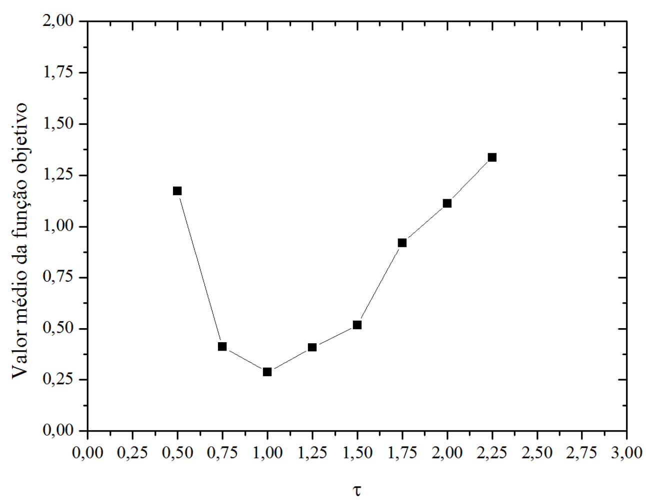
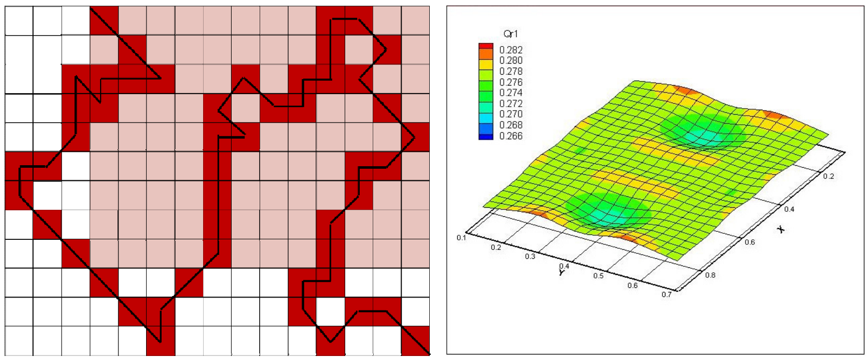


Figure 5. Evaluation of parameter τ for the case with 50 elements.

Table 4. Value of objective function for each seed. Filament heater formed with 50 elements.

Seed	Value of objective function
1	0.1186
2	0.2752
3	0.1795
4	0.1408
5	0.1861

6	0.1504
7	0.1811
8	0.2497
9	0.3433
10	0.1820



(a) (b)

Figure 6. (a) Representation of the filament heater in black, positions of the element heaters in red. The shaded area corresponds to the design surface. (b) Radiative heat flux on design surface.

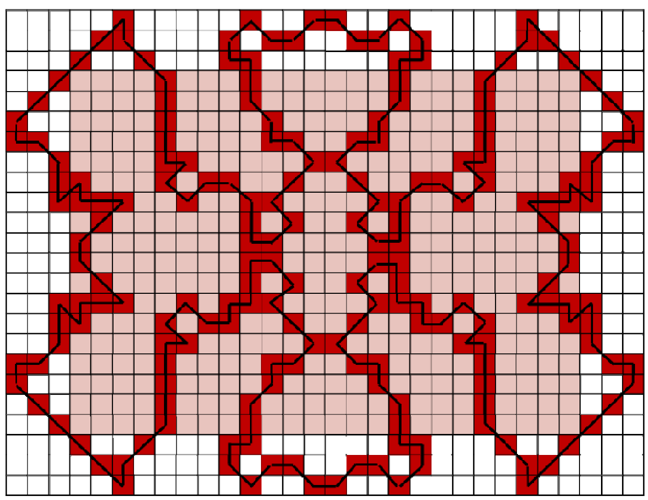


Figure 7. (a) Representation of the filament heater in black, for entire cavity in case with 50 elements. The positions of the element heaters in red. The shaded area corresponds to the design surface.

Table 5. Errors related for the solution with 50 elements in filament heater.

Average error (%)	0.39
Maximum error (%)	2.43

4. CONCLUSION

This study showed application of the an inverse analysis for a solution of a three-dimensional radiative enclosure bounded by diffuse-gray surfaces. It was determined the location and shape of the filament heaters to attain prescribed uniform temperature and radiative heat flux on the design surface. The heaters elements were placed on the top surface, obeying some restrictions to form filaments using the generalized extremal optimization (GEO) method.

Since the initial objective was to obtain errors less than 5%, this was fully achieved with the solutions shown in the paper, with errors less than 2%. The test cases used in this paper to illustrate the methodology involved filaments heaters formed with 40 and 50 elements, in both cases leading to uniform heat flux distribution around the target value. Some of the solutions led to only two or one single filament heating for the entire system. Finally, these values of errors corroborate the idea of designing filament heaters instead of a set of punctual independent heaters, since it is of more practical interest.

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