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SENSITIVITY ANALYSIS OF A ROTORDYNAMIC SYSTEM WITH MAGNETIC BEARINGS AND INTERNAL SEAL

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Abstract. *Gas machine seals with bearing support systems are one of the most important components of common rotating turbomachines. Despite many works are centered on understanding and predicting their rotordynamic characteristics, uncertainties remain in prediction of machinery behavior meaning risks of instability. Industrial turbomachines rotors can reach speeds beyond 15.000 RPM, whereby active magnetic bearings (AMB) become an interesting and achievable solution for supporting radial and thrust rotor forces in these machines. This paper presents a sensitivity analysis and discussion about the rotordynamics of a rotor with two radial AMB and internal seal, using a Jeffcott model with 4 DOF, and a simplified equivalent model for AMB and annular gas seal. For the sensibility analysis has been implemented an algorithm based on the Monte Carlo Methods.*

Keywords: *Rotordynamics, Annular seal, Sensitivity analysis.*

1. INTRODUCTION

Most of annular seals can have a significant effect on the dynamic behavior of high-pressure systems like steam turbines and centrifugal compressors (Childs & Vance, 1997). Internal seals might cause instability in the rotordynamical system, which can be operating beyond the first critical speeds. Instability occurs when a real part of any state-space system eigenvalues is positive. The model used with two flexible bearings supporting a rigid shaft, an axis-symmetrical cylinder located in the center of the shaft that represents the internal annular seal.

The stability behavior of a rotordynamical system with a fixed value of pressure deferential and different values of rotordynamical model spinning speed of the rotor is discussed in the present work. Experimental direct and cross-stiffness and direct damping coefficients obtained on previous work (Childs & Ramsey, 1990) are used for simulations. Several seal models and parameters estimations can also be observed in detail in (Galera, 2013) and (Brol, 2011).

The internal seal parameters are considered uncertain, and are modeled by a random vector, with the independent components. The Monte Carlo method is used to approximate the statistics of the response. The stochastic stability analysis performed in the present work is inspired in the work of Ritto et. al. (Ritto et al., 2014) and Cavalini Jr et. al. (Cavalini Jr et al., 2015). A stochastic model using fuzzy and fuzzy-random parameters for a flexible rotor model was made by (Lara-Molina et al., 2015).

This paper is organized as follows. Section 2 presents the 4 DOF rotordynamical model considered for the analysis, and, Section 3 presents the stochastic rotordynamical system. The results are discussed in Section 4, and finally, the concluding remarks are made in the last section of the paper.

2. ROTOR MODELING

In order to establish the rotor model, a rigid shaft with a rigid disk is taken into account, rotating with a spinning speed ω, in the center and its center of gravity G located just at the center of the shaft, as shown in Figure 1. It is assumed that the mass of the shaft is negligible (i.e. the mass of seal disk is quite higher than the mass of the shaft). Active Magnetic Bearings that support the shaft and disk can be represented as a flexible bearing with non-rotational stiffness and damping K_1, K_2, C_1, C_2 (Chiba et al., 2005). The disk represents the rotational annular seal, and has radius R, length L and mass m. The distance between bearings are defined as L_b, a and b are the distances between the center of gravity and the left and right bearing respectively.

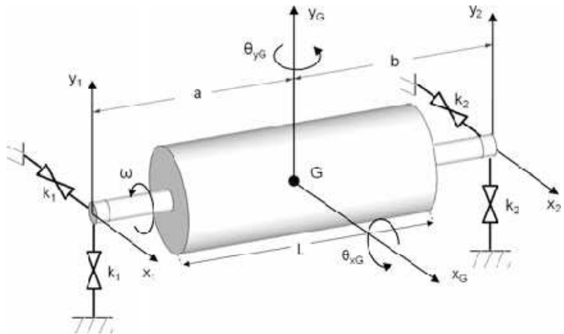


Figure 1. Rotor model with isotropic flexible bearings (Vance, 1988)

The rotordynamic of the model behavior in four DOF, can be described with displacements in the directions (X_1, Y_1, X_2, Y_2)**,** according to Fig. 1**.** Another way to represent it is considering the translations and rotations about the gravity center of mass $(X_G, Y_G, \theta_{XG}, \theta_{YG})$.

2.1 Rotordynamic equations

The polar and transversal moment of inertia of the disk can be expressed respectively as:

$$J_P = \frac{mR^2}{2} \quad (1)$$

$$J_T = \frac{m}{4}\left(R^2 + \frac{1}{3}\mathrm{L}\right) \quad (2)$$

For the tilting angle approach, magnitude of rotations are assumed as being small, i.e. $sin(\theta) \approx \theta$ and $L_b = a + b$. The displacements in $X$and Y axis, and rotations about the center of mass can be expressed respectively as $(\text{Young et al.}, 2013)$:

$$X_G = \frac{1}{L_b}(bx_1 + \mathrm{a}x_2) \quad (3)$$

$$Y_G = \frac{1}{L_b}(by_1 + \mathrm{a}y_2) \quad (4)$$

$$\theta_{XG} \approx \frac{1}{L_b}(y_2 - y_1) \quad (5)$$

$$\theta_{YG} \approx \frac{1}{L_b}(x_2 - x_1) \quad (6)$$

The equations of motion for the rotation and translation are established using Newton's law of motion, resulting in:

$$m\ddot{X}_G + \varepsilon\dot{X}_G + \alpha X_G - \gamma\theta_{YG} = F_X \quad (7)$$

$$m\ddot{Y}_G + \varepsilon\dot{Y}_G + \alpha Y_G - \gamma\theta_{XG} = F_Y \quad (8)$$

$$J_T\ddot{\theta}_{XG} + \omega J_P\dot{\theta}_{YG} + \gamma X_G + \delta\theta_{XG} = 0 \quad (9)$$

$$J_T\ddot{\theta}_{YG} - \omega J_P\dot{\theta}_{XG} + \gamma Y_G + \delta\theta_{YG} = 0 \quad (10)$$

where $\alpha = K_1 + K_2$, $\gamma = -K_1a + K_2b$, $\delta = K_1a^2 + K_2b^2$ and $\varepsilon = C_1 + C_2$. F_X and F_Y are the annular seal forces acting in the disk frame.

For small motions about an equilibrium position, the reaction force model for an annular gas seal can be denoted by (Childs & Ramsey, 1990):

$$F_X = -K_{sd}X_G - K_{sc}Y_G - C_{sd}\dot{X}_G \quad (11)$$

$$F_Y = K_{sc}X_G - K_{sd}Y_G - C_{sd}\dot{Y}_G \quad (12)$$

Direct and cross-stiffness coefficient (K_{sd} and K_{sc} respectively**)**, and the direct damping coefficient C_{sd} depend on the spinning rotation ω, the inlet pressure P and the diference between inlet and outlet pressure ΔP. In this case, a linear function is used to model the seal behavior with these coefficients (using the canonical form $v = px + s$ and keeping P and ΔP constant), resulting in:

$$K_{sd} = \omega K_{sd1} + K_{sd2} \tag{13}$$

$$K_{sc} = \omega K_{sc1} + K_{sc2} \tag{14}$$

$$C_{sd} = \omega C_{sd1} + C_{sd2} \tag{15}$$

This means that two points (often experimental) can be used to give a simplified approximation for the actuating forces in the system.

Again remembering that seal coefficients depend of the spinning speed ω, let $q = [X_G \quad Y_G \quad \theta_{XG} \quad \theta_{YG}]^T$, thus Eqs. 7, 8, 9 and 10 can be expressed in matrix form as:

$$M\ddot{q} + \omega C\dot{q} + Kq = -K_s q - C_s \dot{q} \tag{16}$$

where:

$$M = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & J_T & 0 \\ 0 & 0 & 0 & J_T \end{bmatrix} \tag{17}$$

$$C = \begin{bmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

$$K = \begin{bmatrix} \alpha & 0 & 0 & -\gamma \\ 0 & \alpha & -\gamma & 0 \\ \gamma & 0 & \delta & 0 \\ 0 & \gamma & 0 & \delta \end{bmatrix} \tag{19}$$

$$C_s = \begin{bmatrix} \omega C_{sd1} + C_{sd2} & 0 & 0 & 0 \\ 0 & \omega C_{sd1} + C_{sd2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{20}$$

$$K_s = \begin{bmatrix} \omega K_{sd1} + K_{sd2} & \omega K_{sc1} + K_{sc2} & 0 & 0 \\ -\omega K_{sc1} - K_{sc2} & \omega K_{sd1} + K_{sd2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{21}$$

Equation 16 can be rewritten as:

$$\ddot{q} = -M^{-1}\{(C + C_s + \omega G)\dot{q} + (K + K_s)q\} \tag{22}$$

In order to obtain the deterministic stability of the rodordynamic model proposed, Eq. 22 is expressed as a linear state-space system with the form $\dot{U} = AU$, where:

$$A = \begin{bmatrix} 0_{4x4} & I_{4x4} \\ -M^{-1}(K + K_s) & -M^{-1}(C + C_s + \omega G) \end{bmatrix} \tag{23}$$

$$U = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \tag{24}$$

Finally, the real part of matrix A eigenvalues determines the stability of the system. Positive values indicate that system is unstable, and they vary with the spinning speed ω.

3. STOCHASTIC MODEL

In the present analysis only uncertainties in the couple stiffness of the seal K_{sc} are considered. To obtain the linear relation between the stiffness and damping coefficients, two experimental points of reference (Childs & Ramsey, 1990) are taken at two constant pressure and pressure differential. Then a linear interpolation is performed. The cross and direct-stiffness, and direct damping coefficients, of the seal model are show in Tab. 1.

Table 1. Deterministic parameters used in the linearization of direct and cross stiffness and direct damping of seal model for $P = 1830\ kPa$ and $\Delta P = 915\ kPa$

Parameter	Unit	Value
K_{sd1}	N/(m RPM)	260.4354
K_{sd2}	N/m	-8.6364e4
K_{sc1}	N/(m RPM)	56.4277
K_{sc2}	N/m	3.0455e4
C_{sd1}	N s/(m RPM)	0.0174
C_{sd2}	N s/m	270.9091

These parameters are modeled as random variables at different variables at two different frequencies of shaft rotation, and then a linear interpolation between experimental data (Childs & Ramsey, 1990) is used. Therefore, a random vector with four independent variables is used since we do not have any information about their dependency.

Given the following information: (1) all random variables are positive, (2) all random variables have finite mean, and (3) when all random variables approach zero, the probability measure tends to zero; then the maximum entropy principle (Jaynes, 1957) yields the Gamma probability density function.

For the stochastic analysis, the matrix given by Eq. 23 is random, and so are its eigenvalues. The statistics of the response are obtained by means of the Monte Carlo method.

4. NUMERICAL RESULTS

For the simulation of the implemented model the parameters presented in Tab.2 are used.

Table 2. Parameters used in the simulation of rotordynamic system

Parameter	Unit	Value
Spinning speed (ω)	RPM	0 to 25000
Disk mass (m)	kg	100
Distance between left bearing and disk (a)	m	0.5
Distance between right bearing and disk (b)	m	0.5
Disk radius (R)	m	0.1
Disk length (L)	m	0.2
Left bearing equivalent stiffness coefficient (K_1)	N/m	3e6
Right bearing equivalent stiffness coefficient (K_2)	N/m	3e6
Left bearing equivalent damping coefficient (C_1)	N s /m	150
Right bearing equivalent damping coefficient (C_2)	N s/m	150

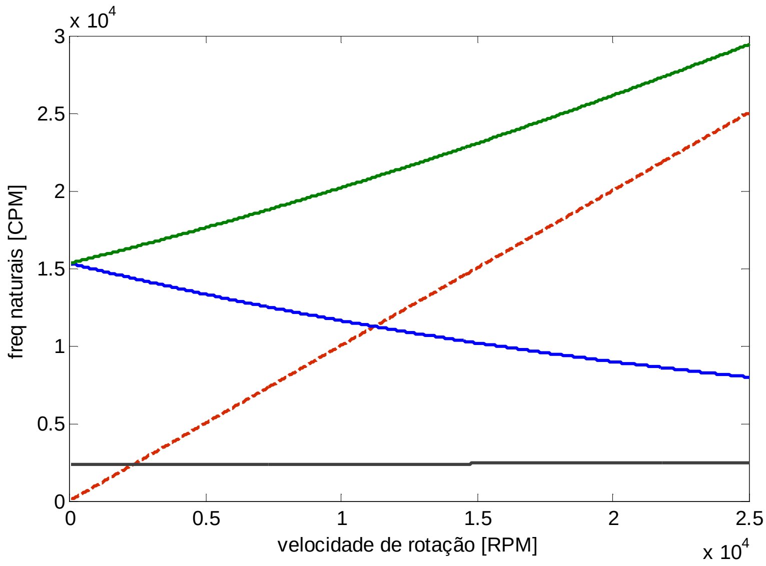


Figure 2. Campbell Diagram of analyzed system

Obtained Campbell diagram is shown in Fig. 2. It can be observed that in the range from 0 to 25000 RPM the system has two critical speeds; the first critical speed is about 3000RPM. Analysis of the Campbell diagram for flexible rotor-shaft model is detailed by Ritto et. al. (Ritto et al., 2010).

Figure 3 shows the real part of eingevalues as a function of spinning speed. It can be noted that deterministic system model is stable in speeds from 0 to about 21500 RPM. Beyond this value, the system becomes unstable.

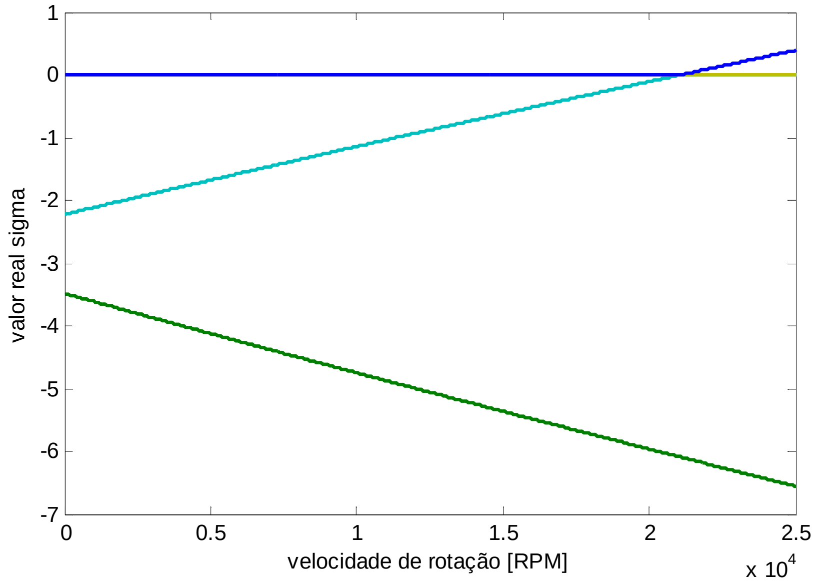


Figure 3. Real part of matrix A eingevalues

The random eigenvalues of the system matrix A are computed for three levels of uncertainties, measured by the standard deviation over the mean of the random variables, which are related to the annular seal cross stiffness. For flexible rotors stochastic model can be observed in (Koroishi et al., 2012).

Figure 4 shows the probability of instability (or probability of failure p_{fail}) of the system as function of the rotation speed, for uncertainties of 5%, 10% and 15%. If the probability of instability should not exceed 20%, one should operate below 20000 RPM. Furthermore, the higher uncertainty, the lower this limit is. All the three lines cross 21500 RPM at about 50% of probability of instability. This value rotation speed corresponds to the operation limit, if no uncertainties are taken into account.

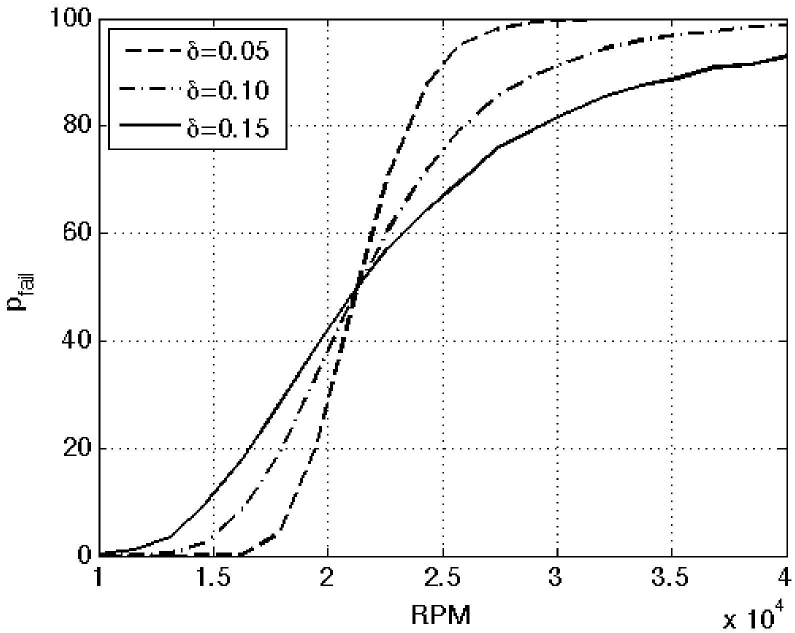
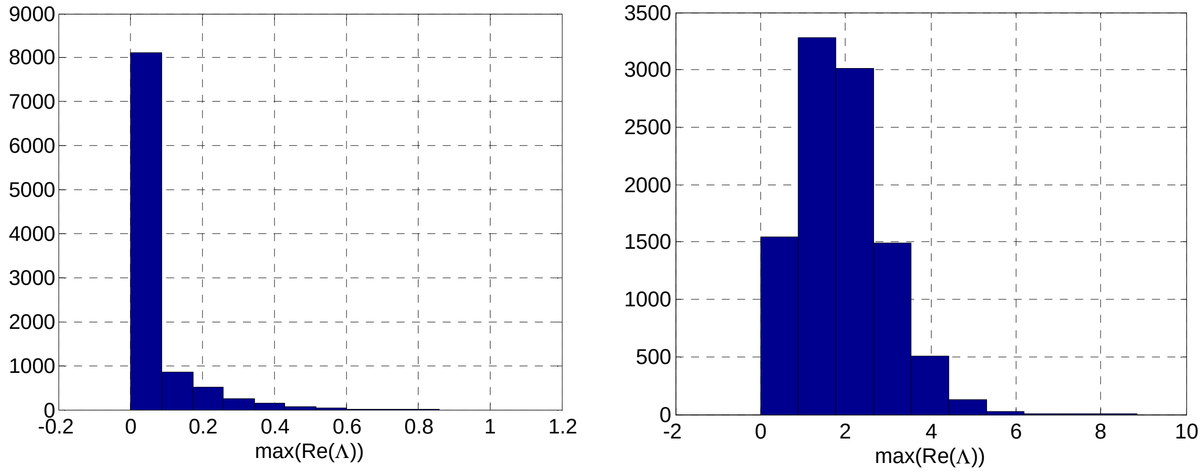


Figure 4. Probability of instability (fail) for different spinning speeds and different levels of uncertainty.

Finally. Fig. 5 details two histograms for a level of 5% of uncertainty. Figure 5a shows the histogram of the maximum real part of the eingevalues of the system matrix at 2000 RPM and Fig.5b shows the same histogram at 40000 RPM. When the system is stable, the max real part of eigenvalues is zero. This is why there is a high probability of zero, when the probability of instability is small. In Fig 5a, the probability of instability is about 40%, therefore, the probability of a maximal real part of eigenvalues be greater than zero is about 40%, and the probability of a maximal real part of eigenvalues be zero is about 60%. In the Fig.5b, the probability of instability is 100%, therefore, all the maximal real part of the eingevalues are greater than zero.



(a) (b)

Figure 5. Histogram of max real part of eingevalues for (a) 20000 RPM and (b) 40000 RPM, considering 5% of uncertainty.

5. CONCLUSIONS

A 4-DOF rotordynamical system is analyzed in this paper. The system includes two equivalent active magnetic bearings and an annular seal. The stability of the system is assessed by computing the maximal real part of the eingevalues of the system matrix. A positive real part yields instability.

Uncertainties in the cross-stiffness coefficients, an important parameter for the system stability, are taken into account. Four independent random variables with gamma distribution are considered, and the uncertainties in the output are computed by means of the Monte Carlo method.

The probability of instability is computed as a function of the rotation speed of the machine. Three levels of uncertainties are analyzed. Also, the histogram of the maximal real part of the eingevalues of system is investigated.

A way to improve the safety factor is increasing the direct stiffness coefficient. That means that AMB may require more current to control the auto induced excitation of critical and unstable spinning speeds.

For further works, the previous model with more uncertainty values will be simulated, and a prototype will be developed in order to validate the model proposed here.

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