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CONTROL OF A BALL AND BEAM USING TRACKER WITH ENTIRE EIGENSTRUCTURE ASSIGNMENT IN REAL TIME

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Abstract. *The control of ball and beam system is a theoretical classical control problem. In the present contribution, an acquisition system with wireless transmission is coupled to an experimental ball and beam system in order to build a test bed for new advanced control strategies in real time. For this purpose, the ball is substituted by a car with a embedded microcontroller enabling the use of a linear encoder to measure the position and this measurement is transmitted wirelessly. The model of the system is obtained by using Newton-Euler equations with two degrees of freedom of a known ball and beam. The movement equations are simplified and linearized. The controllers used in the tracking control are designed using the entire eigenstructure assignment technique, considering a state-space model to motion equations. Simulations are performed in the software Matlab® / Simulink to test the tracking control with auto-assignment structure. The experimental results have demonstrated the effectiveness of the developed control system.*

Keywords: *Ball and Beam, Tracking System, Real-time, Wireless Communication, Entire Eigenstructure Assignment.*

1. INTRODUCTION

Most control problems in practical world are straightforward to control. For a fixed input signal the output stays more or less constant. An important set of systems however are, either by design or nature, unstable and feedback control is essential to make them operate safely. Many moderns industrial processes and technological systems are intrinsically unstable could be used without stabilizing feedback control (Wellstead,2014).

They pose difficulties from the perspective of control engineering and the design of controllers, in particular. In order to overcome these difficulties in the design of a controller, various architectures of the controllers have been developed whose construction is supported by pertinent algorithms (Oh et al. 2011).

Some plants are complex and have a large number of degrees of freedom, thus became hard to obtaining a mathematical model and a complete instrumentation (Amjad, 2010). Because of the difficulties mentioned above, an alternative to study control technics is the development of teaching platforms (Campos, 2007), which provide challenges similar to those found in real systems (Chang, 2011), and the possibility of monitoring all problem variables

Ball and beam systems are produced by several companies such as Quanser®, AMIRA® among others in view of the popularity of ball and beam control problem. Furthermore, several authors proposed modifications in the system in order to highlight some specific control strategies. (Wieneke, 2011) studied assemble methods and different kinds of sensors. (Lin, 2010) uses magnetic actuator. (Hasanzade, 2008) uses a camera to determine the position of the ball and (Ruth, 2015) uses shape memory alloy as a sensor and actuator to modify the angular position of the beam. The ball and system is also an introduction for the study of many techniques of control, like in (Zhang, 2015) and (Andreeva, 2002).

The ball and beam is a system frequently used in the analysis of control problems (Rana, 2011). The system is a ball on a long beam, where the ball is free to move on the beam. The purpose of this system is to control the position of the ball varying the angular position of the bar (Sathiyavathi, 2013). Some changes on ball and beam system are introduced in this contribution. The ball is substituted by a cart, which makes possible the use of an encoder sensor along the beam and an embedded microcontroller in the cart. This microcontroller decodes the encoder pulses and it transmits the actual position of the cart by Radio Frequency (RF) to the computer.

For a better understanding, this work has been divided in six sections and in the second one it is discussed the obtaining of the mathematical model of the cart and the beam. The control method is shown on the third section. A method of wireless communication, real time processing and discretization of the plant control system are described in the fourth section. In the fifth results are displayed for a given set of eigenvalues, which are discussed in the final section.

2. MATHEMATICAL MODEL

The dynamic model of the ball and beam system is obtained from the Newton-Euler approach considering the generalized coordinates x and alpha. x is the ball position and α is the angular position of the beam as shown at Fig. 1. In this case, eq. (1) represents the dynamics of the ball and eq. (2) models the dynamics of the beam, as it can be seen in Wei(2010)

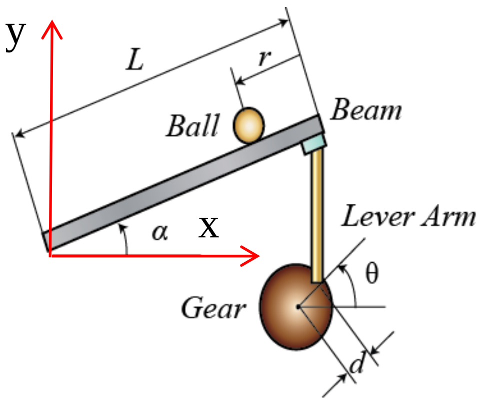


Figure 1 - Ball and Beam

$$\left(\frac{J_b}{R^2} + m\right)\ddot{x} - mg\sin(\alpha) - mx\dot{\alpha}^2 = 0 \tag{1}$$
$$(mx^2 + J)\ddot{\alpha} + 2mx\dot{x}\dot{\alpha} - mg\cos\alpha = \tau \tag{2}$$

Because of the ball was replaced by a cart in our modified system, the term J_b, which represents the moment of inertia of the ball, is neglected. Considering small angles for alpha, some terms can be canceled, so the simplified model of the system is presented in eq. (3) and eq. (4).

$$m\ddot{x} - mg\alpha = 0 \tag{3}$$
$$J\ddot{\alpha} - mg = \tau \tag{4}$$

Additionally, it is considered that the torques produced by the cart weight and moment of inertia of the beam are small compared with the torque of the motor. Therefore, the eq. (4) can be ignored. Consequently, the dynamic of the system is summarized to eq. (5).

$$\ddot{x} = g\alpha \tag{5}$$

The system equation can be represented through states space eq. (6) and eq. (7). Where A is the state matrix, B is the input array, C is output array and D is the direct transmission array, x is the input vector, y is the system output and u is the control action vector.

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu} \tag{6}$$

$$\boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{Du} \tag{7}$$

Where g is the gravity acceleration. Applying the space states formulation to eq.(5), considering alpha as an input control, the position of the cart x is the variable to be controlled. With $\boldsymbol{x_1} = x$ and $\boldsymbol{x_2} = \dot{x}$, thus:

$$\dot{\boldsymbol{x}}_1 = x_2 \tag{8}$$

$$\dot{\boldsymbol{x}}_{\boldsymbol{2}} = g\alpha \tag{9}$$

The system in space state can be seen in eq. (10) and (11), the units it is mm/s^2.

$$\begin{bmatrix}\dot{\boldsymbol{x}}_{\boldsymbol{1}}\\ \dot{\boldsymbol{x}}_{\boldsymbol{2}}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 9810\end{bmatrix}\alpha \tag{10}$$

$$\boldsymbol{y} = [1 \quad 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix} \tag{11}$$

3. CONTROL SYSTEM

A controllable open-loop system is represented by the *n* th-order state and *p* th-order output equations Eq. (6) e (12):

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}$$

$$\boldsymbol{y} = \boldsymbol{Cx} = \begin{bmatrix}\boldsymbol{E}\\ \boldsymbol{F}\end{bmatrix}\boldsymbol{x} \tag{12}$$

where $\boldsymbol{y}$ is an $p\times 1$ vector and $\boldsymbol{w} = \boldsymbol{Ex}$ is a $m\times 1$ vector representing the outputs which are required to follow a $m \times 1$ input vector $\boldsymbol{r}$.

According to D'Azzo and Houpis, (1995, p.541) the design method consists on the addition of a vector comparator and an integrator, which satisfies the equation:

$$\dot{\boldsymbol{z}} = \boldsymbol{r} - \boldsymbol{w} = \boldsymbol{r} - \boldsymbol{Ex} \tag{13}$$

D'Azzo and Houpis, (1995) present the state feedback control law to be used here, which is:

$$\boldsymbol{u} = \boldsymbol{K_1x} + \boldsymbol{K_2z} = [\boldsymbol{K_1} \quad \boldsymbol{K_2}]\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{z}\end{bmatrix} \tag{14}$$

This control law assigns the desired closed loop eigenvalues spectrum if and only if the matrices pair $(\overline{\boldsymbol{A}}, \overline{\boldsymbol{B}})$ is controllable (D'Azzo and Houpis, 1995). It has been shown that this condition is satisfied if$(\boldsymbol{A}, \boldsymbol{B})$ is a controllable pair and

$$rank\begin{bmatrix}\boldsymbol{B} & \boldsymbol{A}\\ \boldsymbol{0} & -\boldsymbol{E}\end{bmatrix} = n + m \tag{15}$$

To $(\boldsymbol{A}, \boldsymbol{B})$ be controllable it is necessary to satisfy the controllability condition (Eq. 21).

$$rank\ \boldsymbol{M_c} = rank[\boldsymbol{B}\ \boldsymbol{AB}\ \ \boldsymbol{A^2B} \ldots \boldsymbol{A^{n-m}B}] = n \tag{16}$$

Satisfaction of the condition of Eq. (15) and (16) guarantees that a control law at Eq. (17) can be synthesized such that the closed-loop output tracks the command input. In that case the closed-loop state equation is:

$$\dot{\boldsymbol{x}}' = \begin{bmatrix}\dot{\boldsymbol{x}}\\ \dot{\boldsymbol{z}}\end{bmatrix} = \begin{bmatrix}\boldsymbol{A}+\boldsymbol{BK_1} & \boldsymbol{BK_2}\\ -\boldsymbol{E} & \boldsymbol{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{x}\\ \boldsymbol{z}\end{bmatrix} + \begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{I}\end{bmatrix}\boldsymbol{r} = \boldsymbol{A}'_{cl}\boldsymbol{x}' + \boldsymbol{B}'\boldsymbol{r} \tag{17}$$

The feedback matrix must be selected so that the eigenvalues are in the left-half plane for the closed-loop plant matrix of Eq. (17). Thus, the outputs $\boldsymbol{W}(t)$ track the piecewise constant command vector $\boldsymbol{r}(t)$ in the steady state. The control system is illustrated in Fig. 2.

The $\ker S(\lambda_i)$ imposes constraints on the eigenvector $\boldsymbol{v_i}$ that may be associated with the assigned eigenvalue λ_i. The $\ker S(\lambda_i)$ identifies a specific subspace, and the selected eigenvectors $\boldsymbol{v_i}$ must be located within this subspace. In addition, the selected eigenvectors must be linearly independent so that the inverse matrix $\boldsymbol{V^{-1}}$ exists (Montezuma, 2010).

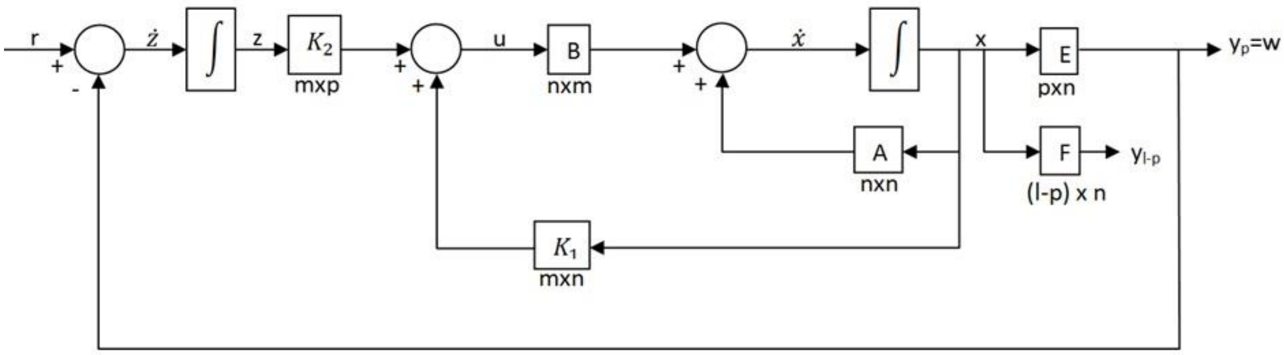


Figure 2 – Tracking System

The synthesis of state-feedback control, assumes that all the states **x** are measurable or that they can be generated from output. The ability to reconstruct the plant states from output requires that all the states be observable. The necessary condition for complete observability is given by Eq. (18).

$$Rank(\boldsymbol{M_o}) = Rank[\boldsymbol{C}^T \ \ \boldsymbol{A}^T\boldsymbol{C}^T \ \ (\boldsymbol{A}^T)^2\boldsymbol{C}^T \ \cdots (\boldsymbol{A}^T)^{n-p}\boldsymbol{C}^T] = n \tag{18}$$

The reconstructed state vector $\hat{\boldsymbol{x}}$ may then be used to implement a state-feedback control law $\boldsymbol{u} = \boldsymbol{K}\hat{\boldsymbol{x}}$. Since, the physical plant may be subjected to unmeasurable disturbances, which cannot be applied to the mathematical model, therefore, the difference between the actual plant output, $\boldsymbol{y}$ and the simulation output $\hat{\boldsymbol{y}}$ is used as another input in the simulation equation. Thus, the observer state and output equations became:

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{L}(\boldsymbol{y} - \hat{\boldsymbol{y}}) \tag{19}$$

$$\hat{\boldsymbol{y}} = \boldsymbol{C}\hat{\boldsymbol{x}} \tag{20}$$

where $\boldsymbol{L}$ is the *nxp* observer matrix the synthesis of $\boldsymbol{L}$ can be seen in D'Azzo and Houpis, (1995).

The eigenvalues of $(\boldsymbol{A} - \boldsymbol{LC})$are usually selected so that they are to the left of the eigenvalues of $\boldsymbol{A}$**.** Thus, the observer states rapidly approach the plant states.

The representation of the physical plant represented by eqs. (6) and (12) and the observer represented by eqs. (19).and (20) is shown in Fig. 3.

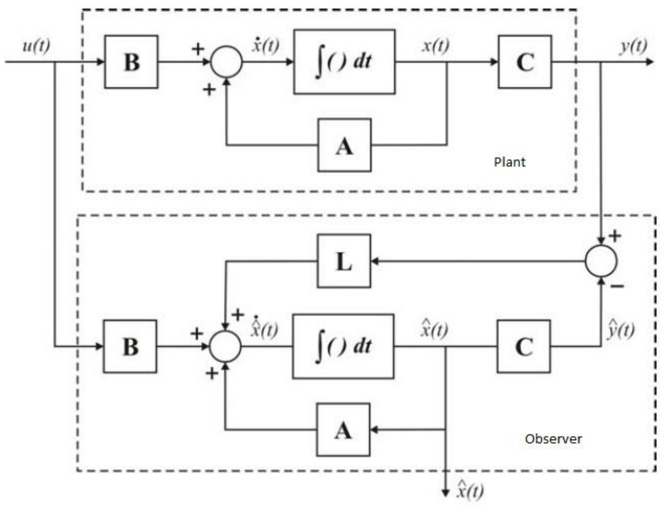


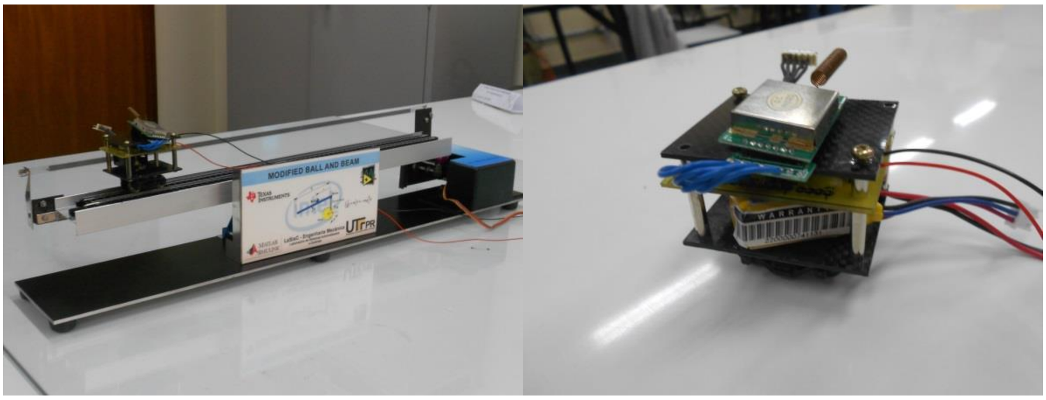
Figure 3 – Plant with Observer

4. METHODOLOGY

With the replacement of the ball by a cart, it becomes possible to use a linear encoder embedded to the car, instead of analog sensors, like resistive, infrared or ultrasound, to determine the cart position. This change ensures accuracy in the position measurement, and eliminates some problems common to the sensors mentioned above. In the literature, some sensor problems are related , for example, in Lin (2010) it is necessary to use a filter to reduce noise from resistive sensor, increasing processing time, and in Wiekene (2010) on the position of an ultrasound sensor is difficult.

The real time plant was built in Simulink® using the Real-Time mode at 50 Hz. The control system input (cart position) used is a serial interface. For output is used a PWM (Pulse-width modulation) generated by the PCI-6602 board from National Instruments. The control action is the angle α of the mathematical model. The constructed system can be seen in detail in Fig. 4.

The car allowed to embedded a microcontroller (dsPIC), which has the function of decoding the quadrature encoder signal and thus, determine the position of the cart and transmit it through a transmitter / receiver with wireless communication via RS-232 interface.



(a) (b)

Figure 4 – (a) Ball and beam system; (b) Cart built

The transmission of the position through wireless communication works as follows: the control system sends an interrupt request to the microcontroller, on the interrupt routine is the cart's position. Therefore, discretization is determined only by Simulink® and not by the microcontroller. Avoids reprogram the microcontroller to change the discretization rate of the control system. The car position is an integer of 16 bits, when using a transmitter / receiver with RS-232 interface, it is only possible to transmit 8 bits at a time, so it is necessary send the cart position in 2 bytes. The transformation of the integer in the car's position with floating point is made in Simulink®. The baud rate used for serial communication is 9600 bps. Figure 5 illustrate the process.

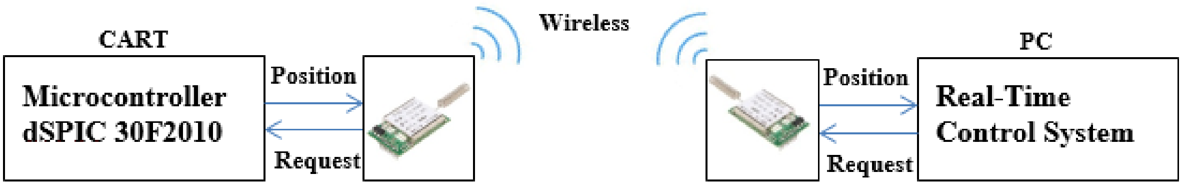


Figure 5 - Wireless Communication

The control action is restricted by physical characteristics of the plant, so the eigenvalues set should result in a control action within the limits of performance. The bar has a range of activity from -6 ° to 4.87 °, the presented difference happens because of the motor arm touches the system base, as seen in Fig.6. Simulations were performed to find a set of eigenvalues.

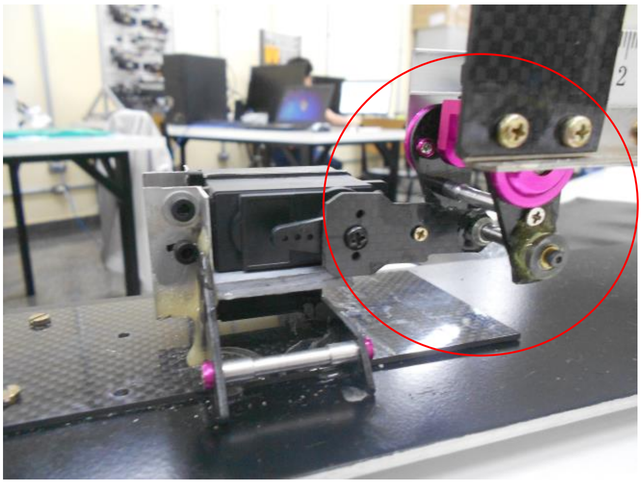


Figure 6 – Action System

The control system used in the experiment was discretized with a time of 0.02 seconds. The mathematical model was discretized by zero-order hold method (ZOH).

During the first tests was observed a difference between the simulation and the experimental results, this difference is due to the friction and inertia. These were modeled as a dead time in the simulations, this value was empirically established by comparing the simulated and the real to a step input. The dead time was set at approximately 0.08 seconds. Figure 7 shows the control plant, in detail input, output, control action and control system

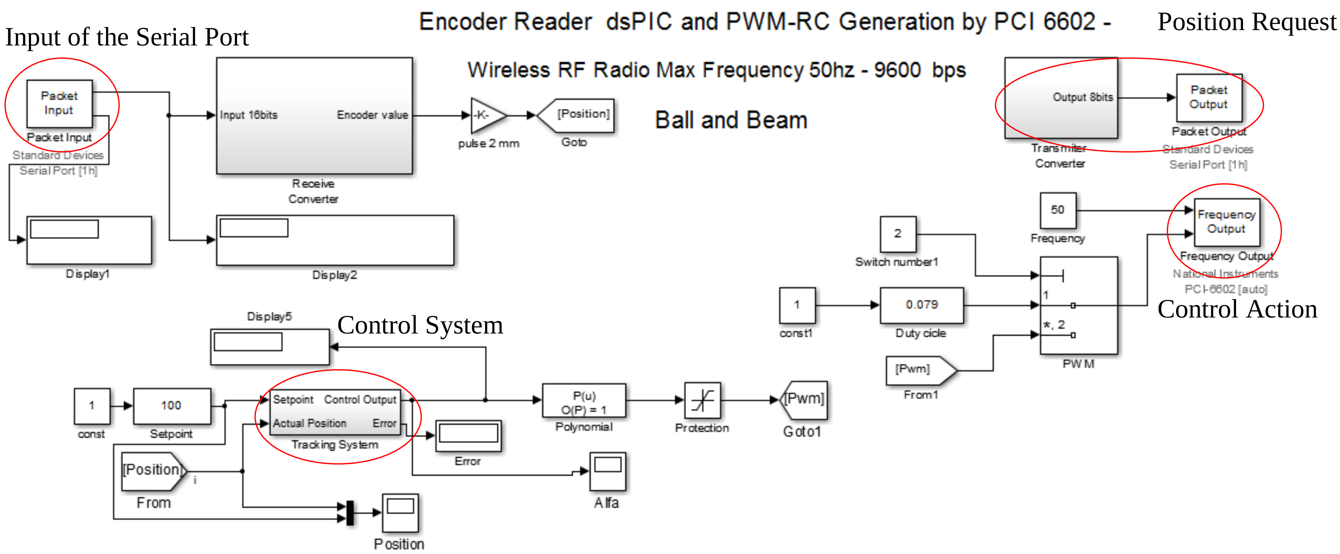


Figure 7 - Experimental Plant

5. RESULTS

The eigenvalues assigned to the system were [1,8 2 2,2]rad/s, the controller gains are obtained through the methodology presented in section 3. The purpose of the system is that the cart tracks the set point. The mathematical model was validated by comparing the simulations with the experimental results. Different inputs were applied as the following: step input, sine and square wave.

The cart has friction that influence in some system response characteristics. At the input step the control action rises to overcome the friction, because the angle α is increasing, but the car does not move. This is due to the gravity component is less than maximum frictional force. When the cart starts to move it overcomes the static friction and the coefficient of friction changes to the dynamic coefficient of friction, which is lower than the previous one. This generates an oscillation in the control action for the step input, as can be seen in Fig. 8. The tracking system as well as many other control systems may not be efficient for high dead time. It is possible to model the friction, but it would be difficult because the dry friction is not linear, the other option is to build a car with lower friction, which seems to be more feasible, because the idea is to use linear control systems. Fig. 8 shows the response to a step input simulated and experimentally obtained.

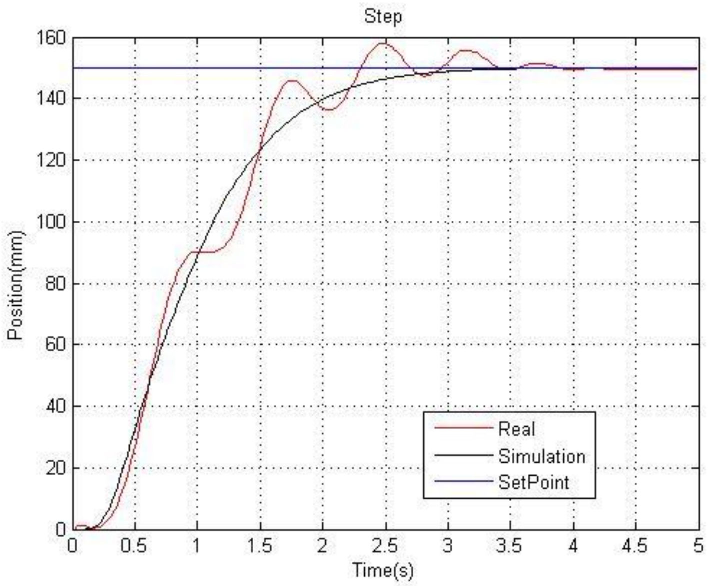


Figure 8 – Step Input

In Fig. 9 the system input is a sine function, the error relative to the set point is characteristic of the tracking system, as the ideal for this control is a piecewise continuous input, this error can be minimized by increasing the eigenvalues, provided it does not saturate the system performance. The friction causes the cart has a dead time, so it remains in a certain position until it overcomes the friction force. When the cart stops, it is necessary a minimum value of α to move it. It can be seen the effect of friction in the changes of direction in the sine function. The steady state error is also caused by the friction into the square wave input Fig. 10.

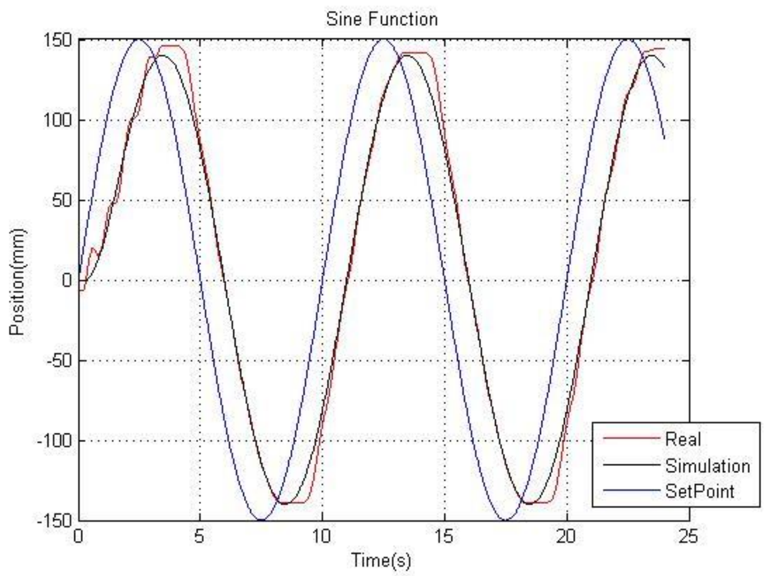


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Figure 9 - Sine Function

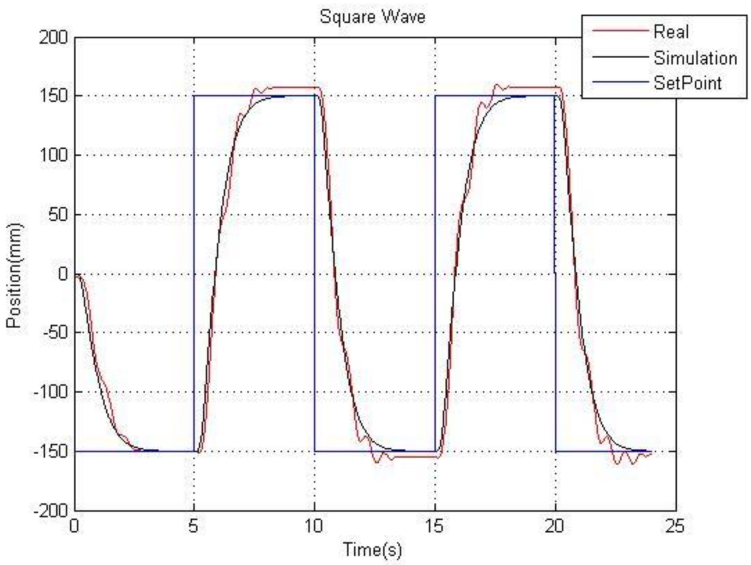

Figure 10 - Square Wave

6. CONCLUSION

The proposed model by mathematical modeling proved effectiveness. The eigenvalues found were sufficient to control the plant without saturate the system performance. The differences between the simulations and experimental tests are due to static friction in the car. The control remained stable, accurate and provided good repeatability on the tests, but it can notice a small steady state error, a delay to change the direction and oscillation in the step input due to the static friction of the plant that was not modeled. There is also the possibility of developing other types of linear and non-linear controls to analyze the best results and characteristics of desired responses. The shaft of the cart will be revised to decrease the friction of the actual plant.

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