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APPLICATION OF THE FORCE CONE METHOD IN TOPOLOGY OPTIMIZATION: A CASE STUDY ON TRUSS DESIGN

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Abstract. *The present paper discusses the application of the Force Cone Method (*Kraftkegelmethode*, in German) in the generation of optimized truss conceptual designs. One difficulty of truss design is the initial positioning of nodes and linkages. Once the conceptual design is generated, optimized layouts can be more readily generated from the initial concept using optimization techniques, and thus better results are obtained. The Force Cone Method (FCM) uses graphic heuristics to rapidly generate simple yet robust initial designs given the load and support positions. A case study, in which the method is applied, was performed, generating satisfactory results with respect to the load index (ratio of maximum load supported and structure mass). The paper concludes that the FCM is an effective tool in generating initial conceptual designs of trusses.*

Keywords: *Force Cone Method, topology optimization, truss design, Kraftkegelmethode*

1. INTRODUCTION

Conceptual design is one of the fundamental steps of integrated product design. Although there are several well-known tools which an engineer can employ in the creative process (Back et al., 2008), those are often general and can lead to dead-ends down the road of product design. This is no different in structural engineering, where a good initial starting point may spare time and resources by reducing the computational efforts and construction of prototypes. This paper will explore the usage of a fast graphical method that can be applied primarily during the conceptual phase of product design.

The Force Cone Method (FCM) was developed by Mattheck (Mattheck and Haller, 2013) for the generation of topologically efficient structures. Its main advantage lies in the easy generation of conceptions that may serve as starting point in the design of structures and uses a process that is purely geometrical, thus reducing the usage of computationally demanding techniques, and, at the same time, obtaining resulting structures that are very similar (Haller, 2013). Therefore, the FCM is a useful tool for generating, in a simple and efficient way, initial designs that can then be further optimized using more complex methods.

As Haller (2013) points out, for the cases analyzed by Michell (1904), the structures obtained with the Force Cone Method are similar to the ones developed by the latter, who was one of the pioneers of structural optimization, and whose theory for optimized structures allows finding analytically optimized topologies for simple cases. Although Haller (2013) has studied several cases to which the FCM can be applied, the analysis focused mainly on theoretical and numerical calculations, and until this moment few experimental testing was done, especially for multidimensional trusses, where a three-dimensional state of tensions is more likely to appear. The current paper focuses on enhancing the understanding of the FCM by applying the method to the design and construction of 3D trusses, as well as comparing and experimentally testing several other unbiased designs with the one generated by the Force Cone Method.

2. METHODOLOGY OF THE FORCE CONE METHOD

The Force Cone Method is based on the idea that a force, applied at a point, produces a compressive stress field in the direction that it is applied, and a tractive field along the opposite direction. The stress fields can be graphically represented by a double circular right cone with an aperture of 90° with an axis that is parallel to the direction of the applied force and a vertex collocated with the position where the force is applied. Haller (2013) points out that if a force is applied to an infinite two-dimensional elastic plate, the region covered by both cones, whose two-dimensional projection in the truss plane are triangles, encompasses 80% of all radial stresses caused by the force. Furthermore, in the two-dimensional case, the edges of the cones coincide with the maximal shear stress directions for the one-dimensional stress state. If multiple forces are present, the two-dimensional projections of the intersections of the edges (i.e., the generatrices) of the several cones are called primary points (*Primärpunkt*, in German) and are good candidates for joints of the resulting structure (Haller, 2013). The linkages between primary points furnish a suggestion of structure.

Therefore, according to Haller (2013), the application of the Force Cone Method for the two-dimensional case is accomplished by the following steps:

1) loads and reactions are drawn with their respective directions (including bearing reactions);
2) the 90° force cones, represented by triangles, are drawn for each load;
3) the intersection between force cone delimiting lines are marked as primary points;
4) the dots are connected so that the initial design of the structure is complete.

Figure 1 illustrates each step. Note that Frame 2 illustrates the construction of the force cones with respect to the force directions. The load force, in the case exemplified, points downwards and thus the compression cone is positioned under the force and the traction cone is positioned above. The opposite is true for support reactions.

It is worth mentioning, however, that the Force Cone Method optimizes a structure for elastic failure (Haller, 2013) and other failure modes should be kept in mind when designing a trussed structure. Those failure modes will be briefly discussed afterwards.

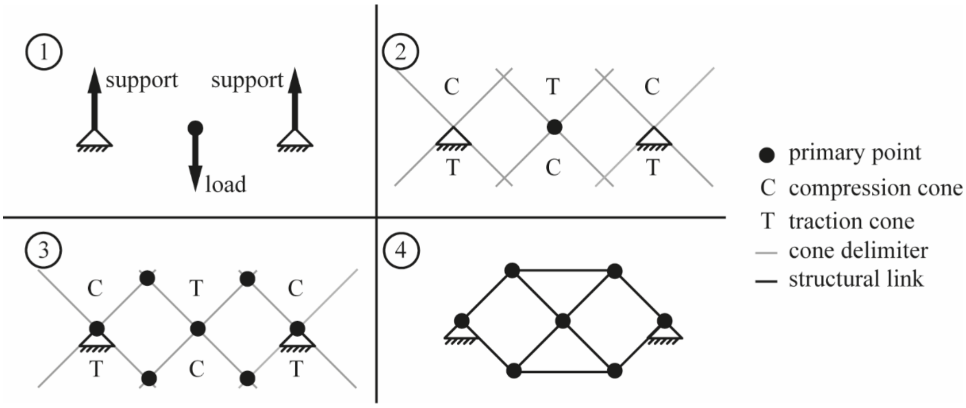


Figure 1. Steps of the Force Cone Method. Adapted from Haller (2013).

3. CASE STUDY: TRUSS DESIGN

The FCM was implemented and experimentally tested in the Laboratory of Mechanical Properties at Universidade Federal de Santa Catarina (UFSC). The goal is to optimize a structure with respect to its load index, the ratio of the maximum load supported by the structure's mass, i.e.

$$\text{load index} = \frac{\text{maximum load before failure}}{\text{mass of the structure}} \tag{1}$$

The optimization problem was constrained in order to ensure the topological variables were dominant. Therefore, certain constructive characteristics were limited. The bar elements are constructed with an aluminum alloy and are cylindrical tubes with an outer diameter of 6.5mm and a wall thickness of 1mm. The most relevant mechanical properties of the aluminum alloy were determined experimentally and will be further described later in this paper.

Other geometric constraints were also imposed in order to minimize the search-space of the optimization problem. Those constraints include the boundary conditions (i.e. where the load and supports are located) and material limit so that the total length of bar elements is bounded and no very large structure can be a solution to the problem. Figure 2 illustrates the boundary conditions. Note that the supports are of roller-type and the load is applied in a position for which the internal stress distribution is not symmetric. Furthermore, the total length of all bar elements should be no larger than 4000 mm and the width of the structure at least 60mm and no larger than 150 mm.

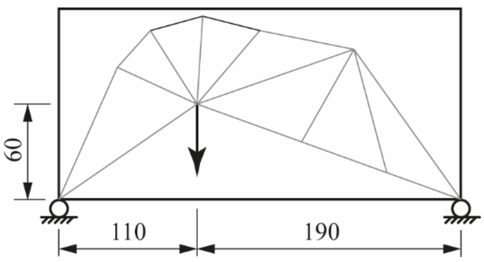


Figure 2. Geometric constraints of the optimization problem and a valid structure drawn within the search space (not to scale).

The structure is assumed to operate only in its linear-elastic region so that the optimal topology is independent of load, i.e. the internal stresses are linear with the load F. Furthermore, the topology is also independent of the material as long as the material has a linear-elastic region and the structure operates in it. However, even if the topological solution is the same, the maximum load and, consequently, the load index depends on the material and the bar cross-section. That being said, the failure criteria considered in the optimization process were: yielding, joint failure and/or buckling.

In order to compare the structure resulting from the FCM with other designs, the optimization problem was also passed to teams of mechanical engineering undergraduate students at Universidade Federal de Santa Catarina (UFSC). Those teams designed and constructed structures using any method they felt adequate, as long as the constraints discussed in this section were not violated.

3.1 Mechanical properties of the aluminum alloy

In order to determine the mechanical properties of the aluminum alloy used in the construction and optimization process of the trussed structure's bar elements, two different tensile test machines were employed.

The first one, with a lower load capacity (DL3000, EMIC), was used to test the elastic regime of the material. The test specimens followed the standard NBR 6152 (ABNT, 2002). Two extensometers (PA=13-125BA-350L, Excel Sensors) were glued perpendicular to each other. From the stress-strain curve, the Young modulus and the Poisson coefficient of the alloy were obtained by linear regression.

The second tensile testing machine, with a larger load capacity (DL2000, EMIC), was employed to determine a larger portion of the stress-strain curve and thus capture the yielding point. The yield stress was obtained using the criteria indicated by Callister (2007), in which the material is assumed to have yielded when the plastic deformation is 0.2%. The data from both tensile tests were obtained using an eight channel acquisition system (Spider 8, HBM).

The tensile tests showed that the Young modulus of the aluminum alloy is 59.42 MPa, the Poisson coefficient is 0.33 and the yield strength is 125.72 MPa.

3.2 Failure criteria

As previously discussed, the failure criteria considered were: yielding, buckling and joint failure. For experimental purposes, any of those criteria is met whenever the structure exits the elastic regime, i.e., whenever the force-displacement curve provided by the testing machine shows a non-linear behavior.

For analysis purposes, the structure is said to have yielded whenever any point reaches the material's yield strength. However, as any computational solution to the internal stresses of the structure provides, more generally, a stress tensor rather than a scalar, the von Mises criterion is used. Therefore, yielding is said to have been reached whenever any point on the structure as a von Mises stress larger than the yield strength. However, the stress tensor of any point in the midst of a bar element has only one non-zero component in its preferential direction (i.e. in the direction of the bar element) which can be directly compared to the yield strength. Therefore, the von Mises criterion is only applied when a triaxial stress state is present, i.e. during the analysis of the joints.

Buckling is the expected failure mechanism expected for the bar elements under compression, especially in the slender ones. A bar element is said to have buckled whenever the maximum buckling stress is reached, i.e. (Hibbeler, 2010)

$$\sigma_{\max} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \tag{2}$$

where r is the radius of gyration, L the length of the bar element and K is a geometric factor which depends on the boundary conditions employed.

3.3 Analysis methods

The main analysis method employed is the finite element method, which is implemented in all software used in this paper: SolidWorks 2013 (by Dassault Systemes) and MATruss 1.5 (by MA Software). The former was used to obtain the triaxial stress state in the joints and the latter to solve for the stress in each bar element. The buckling failure criterion is also present in MATruss, which outputs the safety factor

$$S_b = \frac{\sigma_{\mathrm{crit}}}{\sigma} \tag{3}$$

and each internal stress σ is obtained through the finite element method.

3.4 Joints

A trussed structure composed of several bar elements requires joints to link all bars into a rigid body. There are several ways to do that, each with its own advantages and disadvantages, which are usually correlated with the internal stresses to which the joint is subject. In the present work, two types of joint construction methods were employed.

The first joint type was used to link bar elements in the same truss plane. In order to do that, the extremities of the cylindrical bars were flattened using a manual vise and holes were drilled using a portable vertical drill. Two bar elements were then linked using an ISO M5 bolt and nut, as is shown on Fig. 3.


(a)


(b)

Figure 3. (a) Photo of the manufactured flat joint; (b) Bolt used in the linking of bar elements in perpendicular planes.

Note that the type of joint shown on Fig. 3a induces a triaxial stress state and stress concentration due to the presence of the hole and the transition between flattened and cylindrical surfaces, which may lead to catastrophic failure of the structure. SolidWorks Simulation 2013 was employed in order to determine the maximum von Mises stress due to the stress state and the stress concentration factor, defined as the ratio between the maximum von Mises stress and the bar element stress far from the hole, was found to be around 3.3.

The stress concentration factor found is only valid if the material is isotropic and linear-elastic. This is not the case. The joint manufacturing process relied on the plastic deformation of the extremities, which increased the material's yield strength due to work hardening. Furthermore, in order to guarantee that the structure would not fail in the joints, the bolt and nut were strongly tightened so that the stress concentration is reduced due to a better distribution of the incoming load from the adjacent bar element.

In order to link the bar elements of perpendicular truss planes, a ISO M5 bolt was tightened inside the aluminum bar as shown in Fig. 3b. Figure 4 shows how the integration of all bar elements occurred.

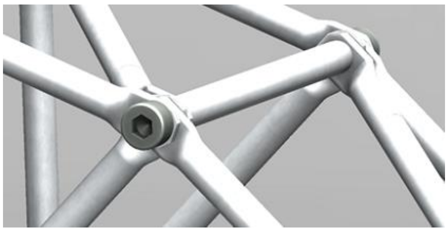


Figure 4. Joint showing the linking of bar elements in parallel and perpendicular planes.

4. CONCEPTUAL DESIGN AND OPTIMIZATION

As previously discussed, Fig. 2 presents the search space of the optimization problem along with the geometric constraints and the boundary conditions. An important remark with respect to the positioning of loads and supports is that the BÂF angle between the right support B, the left support A and the force F is smaller than 45°. Whenever this happens, Haller (2013) discusses that the standard direct solution (as shown in Section 2) of the Force Cone Method may not be optimal. However, because the method serves more as a tool for generating suggestions of optimized structures, thus not being a straight method, the FCM can still be employed with adaptations. One way this can be done, for example, is by transferring the external loads, by means of bar elements, in such a way that the angle BÂF' becomes larger than 45°, where F' is the point to which the force F is transferred. If this is done, the direct standard Force Cone method can be applied.

Figure 5 shows some initial designs (full lines) made using the FCM's precepts to the optimization problem posed in this paper. Note that the Design A is the straightforwardly obtained by applying the FCM to the optimization problem. Note that, however, not only the resulting structure violates the allowed search space, but this solution also differs from suggestions provided by other optimization methods (Haller, 2013). Hence, other structures, all of which still use the FCM framework were elaborated.

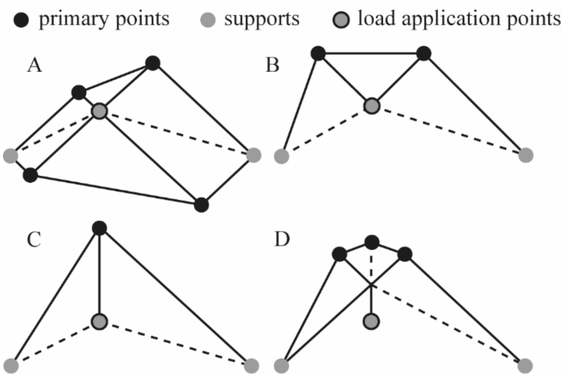


Figure 5. Initial designs using the Force Cone Method (relative dimensions are to scale).

Design B is an adaptation from the solution provided by the FCM to the symmetric optimization problem where the force is positioned midway between supports. The bars connected to the load are placed along the generatrix of the force cone, as proposed by the Force Cone Method. Analogously, the rightmost element bar is placed along the generatrix of the right reaction cone. The other two bars of the external arc are positioned so that the arc is symmetric. Design C is constructed by positioning a vertical bar above the external load application point. Because it is the only element attached to the load, the bar functions by transporting the load from its original application point to a new one, and the latter can be thought as if it were the new load application point. Thus, Design C transports the applied force upwards simply by using a vertical bar in such a way that the 45° criterion is met for both reactions. Design D is a combination of B and C. However, it differs from Design C because it employs more bars in order to smoothen the external arc, better distributing the force to the reactions. Mattheck and Haller (2012) discuss this practice as an enhancement to the Force Cone Method.

The initial designs were altered in order to make them statically stable, i.e., truss triangles were "closed". The bars added to the conceptions are shown as dashed lines. Regarding Design A, the bars and primary points which violated the search space (i.e., the lower ones) were simply removed. All four resulting two-dimensional structures were tested using MATruss, a finite element solver for trussed structures. The material properties used as an input to the software were the same as the ones presented in Section 3.1. A force of 1000N was applied in the force application point. Note, however, that because the structure should operate in its elastic region, the maximum stress found by the finite element solver is linear with the force. Therefore, the choice of 1000N is arbitrary and has no influence whatsoever on the location of the largest internal stress.

Table 1. Results of the first set of simulations using MATruss.

Structure	Maximum tractive stress [MPa]	Maximum compressive stress [MPa]	Buckling safety factor [-]
A	114	122	1.66
B	64.5	62	1.66
C	84.5	62	0.72
D	59.0	78	0.71

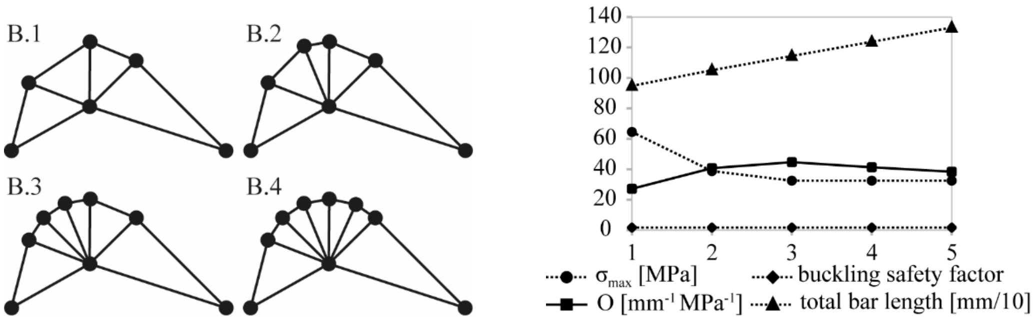
The main results of the simulations are shown on Tab. 1. It is seen that Structure A has generated the largest maximum stress. Structure D exhibits a smaller value of maximal traction, but performs badly at buckling. Therefore, Structure B is chosen for further optimization. Note that all results are in agreement with the underlying theory of the Force Cone Method. As previously mentioned, Structure A, which is obtained directly from the application of the FCM, performs badly because it does not meet the 45° criterion. However, when the structure is modified in such a way that the criterion is obeyed, the maximum tractive and compressive stresses are considerably reduced.

The results shown on Tab. 1 highlight another important feature of the FCM: inclusion of radial bars – in order to create an arc – along the tractive region of the force cone of the external load tends to further reduce internal stresses (Mattheck and Haller, 2012; Michell, 1904). Therefore, in order to continue the optimization of Structure B through a graphical method – the FCM – instead of more complicated topology optimization techniques, radial bars were inserted one by one and each structure was simulated using MATruss. However, even though the addition of more bars along the tractive cone is guaranteed to reduce internal stress, it also increases the structure's total mass. Thus, an optimization parameter was developed to balance and quantify both factors, as follows:

$$O = \frac{S}{L\sigma_{t,\max}} \times 10^6 \tag{4}$$

where O is the optimization parameter, S the buckling safety factor, L the bars' total length, $\sigma_{t,\max}$ the maximal tractive stress, and 10^6 is simply a coefficient to increase the order of magnitude of the optimization variable. Note that O is analogous to the load index defined in Section 3.

The process of adding radial bars is illustrated in Fig. 6. As it can be seen, the insertion of bars improves the optimization parameter up until Structure B.2, after which the maximal tractive stress remains nearly constant. The buckling safety factor is constant along the entire optimization process because the critical bar for buckling is not within the tractive stress cone, in which the radial bars were added. Thus, after Structure B.1, the structure fails by buckling and on a bar outside the region improved by the arch. The decision to not optimize for buckling is two-fold: the main objective of the paper is to analyze the viability of the FCM, which is concerned with elastic failure rather than buckling, and because further increasing the number of bars in order to avoid buckling would not reduce the index load. The optimal two-dimensional structure found is Structure B.1.

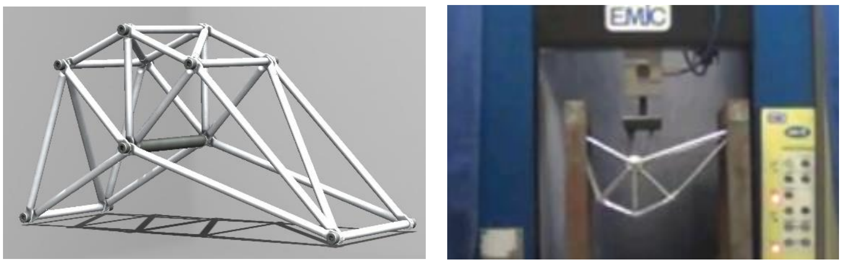


(a) (b)

Figure 6. (a) Further optimization of Structure B from Fig. 5 showing the addition of radial bars (relative dimensions are to scale); (b) Performance of the optimized structures.

It remains necessary to convert Structure B.1, which is two-dimensional, to a 3D structure. Although the FCM uses cones for the design of three-dimensional structures (and not their projections), we chose to convert the two-dimensional structure to a 3D, by simply replicating the 2D structure. Therefore, the resulting 3D structure consisted of two equal truss planes separated by 100mm and connected by horizontal and diagonal bars. The three-dimensionalized Structure B.1 is shown in Fig. 7.

The total length was calculated to be 3315.23 mm, which is less than the constraint of 4000 mm. The total mass, after assembly, is 275.9g. The resulting tridimensional structure was again simulated using finite element methods implemented in both MATruss and SolidWorks Simulation and the maximum admissible load before failure was found to be 3320 N, after which buckling should occur.



(a) (b)

Figure 7. (a) Rendered photo of the optimized 3D structure and (b) assembly of the structure on the test machine

5. RESULTS

As previously mentioned in Section 3, the optimization problem, along with its constraints, was passed to several teams of mechanical engineering students at Universidade Federal de Santa Catarina. Each team designed a trussed structure based on their best knowledge. This comparison is important in order to reduce bias in the design methodology. Fig. 8 illustrates a two-dimensional version of each structure (note that all of them were extended to 3D upon manufacturing) along with identification codes. The variation in proposed designs in clearly seen in Fig. 8, which suggests that even students who had recently taken design and solid mechanics courses found it difficult to come up with efficient designs and ended up suggesting structures without much technical justification.

All structures were expanded to 3D by replicating truss planes, except for Structure E, which consisted of three parallel planes instead of two. All structures were tested on a universal testing machine (DL 2000, EMIC) with a constant displacement rate of 5 mm/min and the tests run until plastic regime was identified or catastrophic failure occurred. The force in which either condition happens is registered as the maximum load before failure (MLBF). The mass of each structure was also registered before testing began. The mass and maximum load before failure of each structure are shown on Fig. 9b and the load indexes on Fig. 9a.

The FCM structure achieved the best load index, slightly better than Structure D. Although the joints may have influenced the MLBF, so that the rank of the structures could have been other, it is clear from the results that the FCM is a suitable method for the conception of nearly optimized structures with little effort spent on the optimization process. The FCM structure failed by buckling on the predicted bar with a load of 3640 N, hence the deviation from the predicted maximum load was 8.8%.

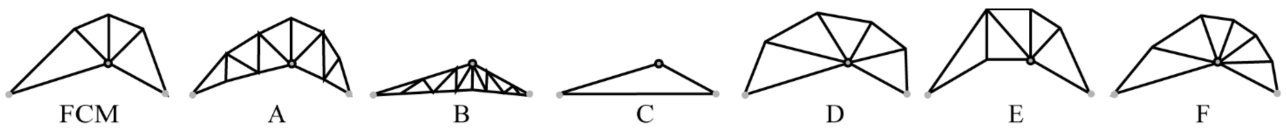


Figure 8. Two-dimensionalized structures designed by several teams along with the one resultant of the FCM.

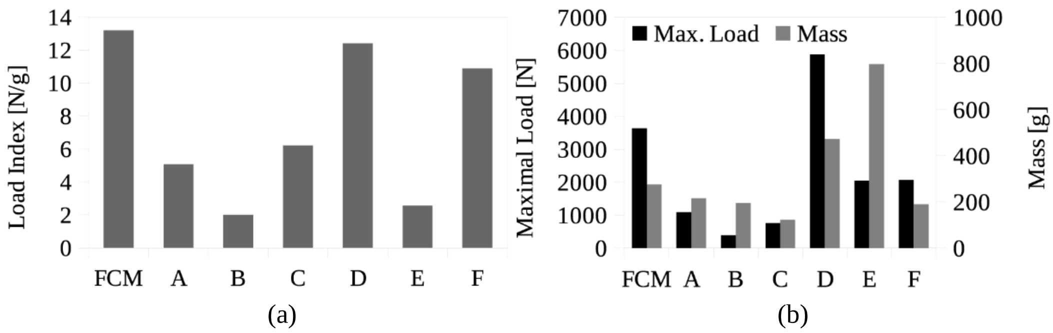


Figure 9. Performance parameters of each structure.

Structures B and C, both with a triangular shape, supported the least load. Although Structure B performed poorly because of its joints, we can safely assume that its maximum load would be similar to Structure C because of their analogous topology. Both structures are outlined by the triangle whose vertices are the load application point and the supports. This is, however, a region the FCM mainly avoids, since most of it lays outside of the force cones' region of coverage. Recalling the meaning of the force cones, this implies that Structures B and C contain its main elements outside of regions where main fluxes of forces (i.e., inside the force cones) are present, and the external load is transferred to the supports with bars that mostly violate the 45° criterion. This means that the transmission of load to the supports is only possible with large internal forces in order to produce a reaction in a direction opposite to the load that satisfies equilibrium conditions.

The structures that achieved the best load indexes were the ones with a similar shape to the one generated by the FCM. They were mainly composed of an arc connected to the load application point with radial bars. Structure E had these features but its mass was excessive, due to its three planes framework (i.e., upon expanding to 3D, the team decided to create three truss planes instead of two). Both Structures G and D, which had their load indexes closest to the FCM structure, were also the ones that most resemble the latter. However, both had a pair of internal radial bars too horizontal lying outside of the force cone region. Structure D resembled FCM structure and achieved the largest load, but also had a larger mass due to ill-positioned bars.

Despite the fact that the structure generated by the FCM had the largest load index, it should be noted that it failed by buckling. Further topology optimization by adding more radial bars as suggested by the FCM would not compensate for this failure mode. Therefore, the recommendation of not positioning bars outside of the force cone region (which is the core of the FCM method) should be made with caution. If buckling is to be avoided, a more detailed optimization procedure is required.

6. CONCLUSION

The Force Cone Method (Haller, 2013), a graphical method for the generation of optimized topologies of two-dimensional structures, was shown to be a good first step into the initial design of 3D trussed structures. Even though the FCM has its limitations, such the 45° criterion, the current paper showed that its mindset and general rules, because of its simplicity and flexibility, can be applied even in such cases. Solutions can be further enhanced depending on the experience of the designer, but the method allows unexperienced designers to get a deeper insight on the behavior of forces in a structure. In fact, the present study has shown that when comparing different types of trussed structures satisfying the same basic geometric constraints and boundary conditions, the best performing structures were the ones which roughly resembled the result of the FCM. However, it is important to highlight that the structures resultant of the FCM should not be though as final because the method does not account for failure modes other than yielding, such as buckling and crack propagation. Even so, FCM is a fast method for generating designs which can serve as inputs to more complex optimization techniques, thus saving computational time and money spent on the construction of badly performing prototypes.

Naturally, the viability of this method in generating initial designs can be further analyzed by extending its application to three dimensions. Although the resulting truss shown in Fig. 7 is 3D, the expansion from 2D to 3D as done in this paper is naïve, and it is expected that the FCM should provide insights in this process as well. The analysis of the viability of the FCM for the construction of 3D structures would also benefit from controlled experimental analysis of geometric constraints and boundary conditions, specifically regarding the 45° criterion.

Finally, the variety of topologies shown in Fig. 8, and the differences among the resultant load indexes, some far from being optimized, shows that even mechanical engineering students, for which solid mechanics and statics are mandatory coursework, may lack the notion of optimized design. Therefore, it is safe to say that the FCM is useful to assist not only experienced designers in generating initial conceptions, but it can also be employed as a learning tool for students. The simple and flexible process of the Force Cone Method makes it easy to grasp the basic physical driving forces in optimized design in contrast with other optimization techniques which involve advanced mathematics that an undergraduate student is often yet unprepared to undertake.

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