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DESIG OF AN EIGENSTRUCTURE ASSIGNMENT CONTROLLER USING GENETIC ALGORITHM APPLIED TO A BALL AND BEAM SYSTEM

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Abstract. *The present contribution aims at describing an optimal design procedure of a ball and beam control system based on the so-called eigenstructure assignment control. The complete dynamic model of the ball and beam system was obtained by the Lagrange-Euler equations. This model was simplified to obtain a one degree of freedom model. The states variables and output of the system are feedbacked in the eigenstructure assignment controller. The gains of the controller depend on the imposed eigenvalues of the system. The optimal design procedure consists in solving a constrained nonlinear optimization problem to find the imposed eigenvalues. Therefore the optimal eigenvalues of the controlled system should minimize the settling time of the response with a constrained control input (maximum alignment angle of the beam). The optimization problem was solved by using a Genetic Algorithm. A prototype of the ball and beam system was implemented by substituting the ball by a cart, additionally it was used a linear encoder and a wireless system to measure the position of the cart along the beam. The control system was implemented with hardware-in-loop framework using Matlab / Simulink in real-time. The experimental results show the improvement of the dynamic performance of the system in terms of position accuracy and the transient response.*
Keywords: *Ball and Beam, Eigenstructure Assignment Control, Genetic Algorithm, Optimization.*

1. INTRODUCTION

The ball and beam system is a classical framework used to study several control techniques as studied by Zhang *et al.* (2015) and Andreeva *et al.* (2002). The typical configuration of the ball and beam system consists in a ball that rolls over a long beam. Additionally, the beam is joined to a motor by means of a revolute joint, the motor modifies the inclination of the beam. The current position of the motor is measured by a sensor, this allows to feedback the inclination of the beam into a control system to regulate the position of the ball around a set point position of the beam (Sathiyavathi and Krishnamurthy, 2013).

The ball and beam system is produced by several companies such as $Quanser^{®}$, $AMIRA^{®}$ among others in view of the popularity of ball and beam system. Furthermore, several authors proposed modifications in the system. Wieneke and White (2011) studied assemble methods and different kinds of sensors. Lin *et al.* (2010) use magnetic actuator. Hasanzade *et al.* (2008) use a camera to determine the position of the ball and Ruth *et al.* (2015) uses shape memory alloy as a sensor and actuator to modify the angular position of the beam.

Some changes on ball and beam system are introduced in this contribution. The ball is substituted by a cart, this make possible the use of an encoder sensor along the beam and an embed microcontroller in the cart. This microcontroller decodes the encoder pulses and it transmits the actual position of the cart by Radio Frequency (RF) to the computer. The prototype of the modified ball and beam system used in this contribution is presented in the Fig. 1.

On the other hand, ball and beam systems present a high degree of uncertainty, nonlinearity and instability increasing the difficulties to design the controller. Some methods addressing control and optimization techniques have been developed to deal with the controller design difficulties (Oh *et al.*, 2011). Among these works, Castillo *et al.* (2015) uses a modified optimization technique based on an optimization method with colonies of ants to find the optimal membership functions and rules of fuzzy controller; Mahmoodabadi *et al.* (2014) use particle swarm optimization to tune a decoupled

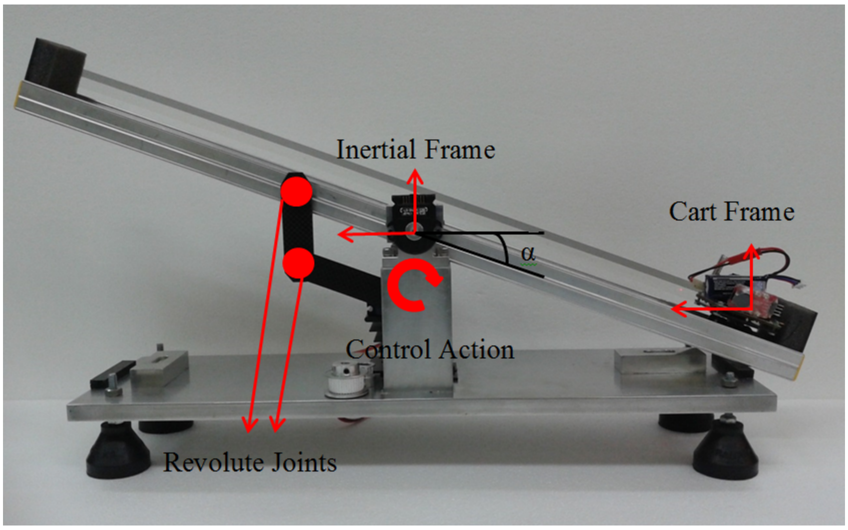

Figure 1. Cart and Cart system

sliding mode control.

This contribution proposes an optimal design method to tune an eigenstructure assignment controller using Genetic Algorithms (GA). Consequently, an optimization problem is proposed to select the eigenvalues of the controller aiming at minimize the settling time of the cart for a desired set-point position. This optimization problem was solved by numerical simulations considering the complete model of the ball and cart system. The performance of the controlled ball and cart system is evaluated experimentally with the gains obtained with the optimal eigenvalues. The experimental results indicated an improvement of the dynamical performance of the ball and cart system.

This paper has five sections. In section 2 it is presented the classical equations of ball and beam system, the complete model is simplified and it is adapted to the ball and cart proposes. The section 3 presents the eigenstructure assignment controller. In section 4, the optimal design method to tune the control system based on an optimization problem is introduced. The numerical and experimental results are shown in section 5. Finally, the conclusion are presented in the section 6.

2. BALL AND BEAM MODELLING

The dynamic model of the ball and beam system is obtained from the Langrange-Euler approach considering the generalized coordinates x and α. x is the ball position and α is the angular position of the beam as shown at Fig. 2. In this case, equation (1) represents the dynamics of the ball and equation (2) models the dynamics of the beam.

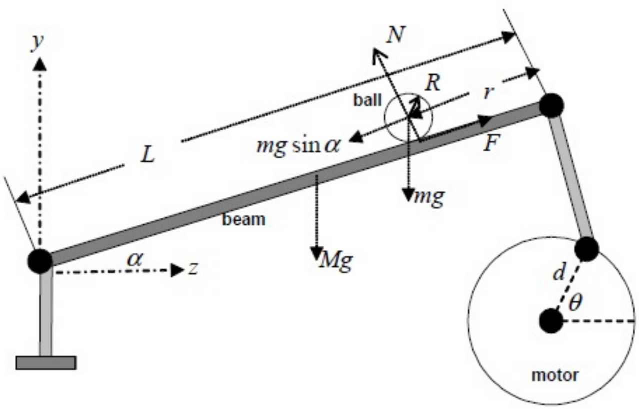

Figure 2. Typical Ball and Beam System

$$[(J_b/R^2) + m]\ddot{x} - mg\sin\alpha - mx\dot{\alpha}^2 = 0 \tag{1}$$
$$(mx^2 + J)\ddot{\alpha} + 2mx\dot{x}\dot{\alpha} - mg\cos\alpha = \tau \tag{2}$$

Because of the ball was repalced by a cart in our modified system, the term J_b, which represents the moment of inertia of the ball, is neglected. Considering small angles for α, some terms can be canceled, so the simplified model of the system is presented in eq. (3) and eq. (4).

$$m\ddot{x} - mg\alpha = 0 \tag{3}$$
$$(mx^2 + J)\ddot{\alpha} + 2mx\dot{x}\dot{\alpha} - mg = \tau \tag{4}$$

Additionally, it is considered that the torques produced by the cart weight and moment of inertia of the beam are small compared with the torque of the motor. Therefore, the eq. (4) can be ignored. Consequently, the dynamic of the system is summarized to eq. (5).

$$\ddot{x} = g\alpha \tag{5}$$

Where g is the acceleration of the gravity. Applying the space states formulation to eq. (5), considering α is an input control, the position of the cart x as the variable to be controlled. With $x_1 = x$ and $x_2 = \dot{x}_1$, thus:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 9810\end{bmatrix}\alpha \tag{6}$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \tag{7}$$

3. EIGENSTRUCTURE ASSIGMENT CONTROLLER

The open-loop system is represented by the eqs. (8) and (9). The state vector $\mathbf{x}$ has nth-order.

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \tag{8}$$
$$\mathbf{y} = \mathbf{Cx} = \begin{bmatrix}\mathbf{E}\\ \mathbf{F}\end{bmatrix}\mathbf{x} \tag{9}$$

where $\mathbf{y}$ is the $p \times 1$ output vector, $\mathbf{w} = \mathbf{Ex}$ is a $m \times 1$ vector. It is required that $\mathbf{y}$ tracks an input $p \times 1$ reference vector $\mathbf{r}$. According to D'Azzo and Houpis (1995) the design method consists in the addition of a comparator vector and an integrator which satisfies the eq. (10).

$$\dot{\mathbf{z}} = \mathbf{r} - \mathbf{w} = \mathbf{r} - \mathbf{Ex} \tag{10}$$

D'Azzo and Houpis (1995) present the state feedback control law to be used here which is:

$$\mathbf{u} = \mathbf{K_1x} + \mathbf{K_2z} = \begin{bmatrix}\mathbf{K_1} & \mathbf{K_2}\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \mathbf{z}\end{bmatrix} \tag{11}$$

A block diagram representing the feedback control system is shown in Fig. 3. The control system consisting of the plant dynamics and output equations given by eqs. (8) and (9) and the control law given by eq (10) and eq (11)

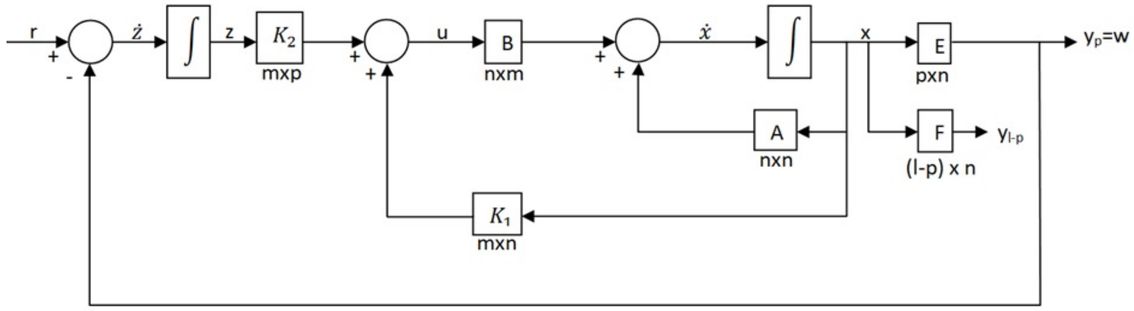

Figure 3. Eigenstructure assignment control system

This control law assigns the desired closed loop eigenvalues spectrum if and only if the system modeled by matrices $(\bar{\mathbf{A}}, \bar{\mathbf{B}})$ is controllable. It has been shown that this condition is satisfied if $(\mathbf{A}, \mathbf{B})$ is a controllable pair and

$$Rank \begin{bmatrix} \mathbf{B} & \mathbf{A} \\ \mathbf{0} & -\mathbf{E} \end{bmatrix} = n + p \tag{12}$$

To $(\mathbf{A}, \mathbf{B})$ be controllable it is necessary to satisfy the controllability condition of eq. (13).

$$Rank(\mathbf{M_c}) = Rank\begin{bmatrix} B & AB & A^2B & ... & A^{n-m}B \end{bmatrix} = n \tag{13}$$

Satisfying the conditions of eq. (12) and eq. (13), it is guaranteed that the control law of eq. (14) can be synthesized such that the closed-loop output tracks the input set-point. In that case the closed-loop state equation is:

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{z}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} + \mathbf{BK} + \mathbf{1} & \mathbf{BK_2} \\ -\mathbf{E} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{z} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \mathbf{r} \tag{14}$$

The feedback matrix must be selected so that the eigenvalues are in the left-half plane for the closed-loop plant matrix of eq. (14). The state feedback is applied to assign the closed loop eigenvalue spectrum.

The $Ker(\mathbf{S}(\lambda_i))$ imposes constraints on the eigenvector υ_i that may be associated with the assigned eigenvalue λ_i. $Ker(\mathbf{S}(\lambda_i))$ identifies a specific subspace, and the selected eigenvectors υ_i must be located within this subspace. In addition, the selected eigenvectors must be linearly independent so that the inverse matrix $\mathbf{V^{-1}}$ exists (Montezuma, 2010).

The synthesis of state-feedback control assumes that all the states x are measurable or that they can be generated from output. In many practical control systems it is physically or economically impractical to install all the transducers which would be necessary to measure the states (D'Azzo and Houpis (1995)). The ability to reconstruct the plant states from output requires that all the states be observable. The necessary condition for complete observability is given by Eq. (15).

$$Rank(\mathbf{M_o}) = Rank\begin{bmatrix} \mathbf{C}^T & \mathbf{A}^T\mathbf{C}^T & (\mathbf{A}^T)^2\mathbf{C}^T & ... & (\mathbf{A}^T)^{n-m}\mathbf{C}^T \end{bmatrix} = n \tag{15}$$

$$\hat{\mathbf{y}} = \mathbf{C}\hat{\mathbf{x}} \tag{16}$$

The purpose of this section is to present methods of reconstructing the states from the measured output by a dynamical system which is called an observer. The reconstructed state vector $\hat{x}$ may then be used to implement a state-feedback control law $\mathbf{u} = \mathbf{K}\hat{\mathbf{x}}$. The basic method of reconstructing the states is to simulate the state and output of the plant. These equations are simulated with the same input u applied to the physical system. The states of the simulated system and of the physical system will then be identical only if the initial condition of both be the same. However, the physical plant may be subjected to unmeasurable disturbances which cannot be applied to the simulation, therefore, the difference between the actual plant output y and the simulation output $\hat{y}$ is used as another input in the simulation equation. Thus, the observer state and output equations became:

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{Bu} + \mathbf{L}(\mathbf{y} - \hat{\mathbf{y}}) \tag{17}$$

where $\mathbf{L}$ is the $n \times p$ observer matrix, the synthesis of L can be seen in D'Azzo and Houpis (1995).

The eigenvalues of **(A-LC)** are usually selected so that they are to the left of the eigenvalues of $\mathbf{A}$. Thus, the state observer rapidly approaches the plant states. The representation of the physical plant represented by Eqs. (8) and (9) and the observer represented by Eqs. (16) and (17) are shown in Fig. 4.

4. Optimization problem

The optimization problem of eq. (18) aims at finding the eigenvalues λ_i of the control system that minimize the settling time of the controlled ball and beam system response. The objective function is the settling time t_e which is function of the eigenvalues $\boldsymbol{\lambda} = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix}$ of the controlled system. A constraint is imposed in the control to assure that the revolute joint α not exceed the maximum reachable angle. Moreover the three eigenvalues $\boldsymbol{\lambda}$ should be different to ensure the stability of the eigenstructure assignment controller.

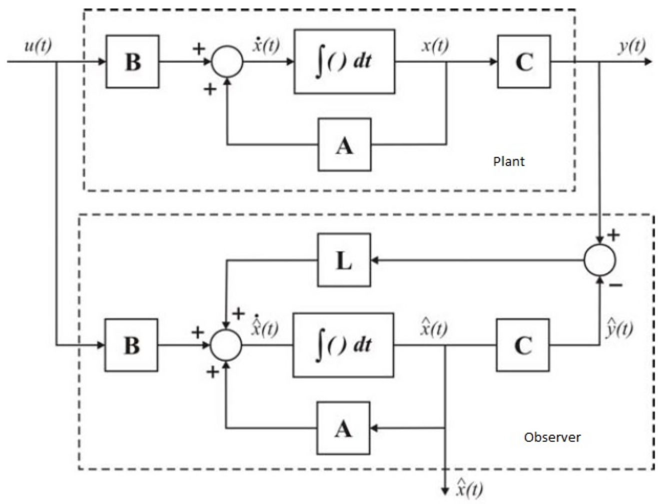

Figure 4. Control system diagram with state observer.

$$\min_{\lambda} t_e(\boldsymbol{\lambda})$$
subjected to
$$|\alpha| \leq \alpha_{max}$$
$$\lambda_1 \neq \lambda_2 \neq \lambda_3 \qquad (18)$$

The optimization problem of eq. (18) is nonlinear and constrained since the response of the controlled system depends of the controller gains which in turn depends on the eigenvalues $\boldsymbol{\lambda}$ of the as stated in eq. (14). The control action which is the inclination of beam is constrained to avoid the saturation of the motor, thus $|\alpha| \leq \alpha_{max}$. Heuristic techniques have been used in order to solve this sort of optimization problems(Lara-Molina *et al.*, 2014; Koroishi *et al.*, 2014), specifically Genetic Algorithms have been used in several optimization problems of engineering (Lara-Molina *et al.*, 2011).

4.1 Genetic Algorithm

Genetic Algorithms (GAs) are heuristic search algorithms based on the mechanism of natural selection and natural genetics initially proposed by Holland (Holland, 1992). GAs are high performance and robust optimization methods to solve engineering problems.

In general, a genetic algorithm has four basic characteristics: $i)$ A genetic representation of solutions to the problem; $ii)$ A way to create an initial population of solutions; $iii)$ Selection of the population for next generation, an evaluation function rating solutions in terms of their fitness; $iv)$ Genetic operators that alter the genetic ascendants during reproduction. A fluxogram of a standard genetic algorithm is presented in Fig. 5.

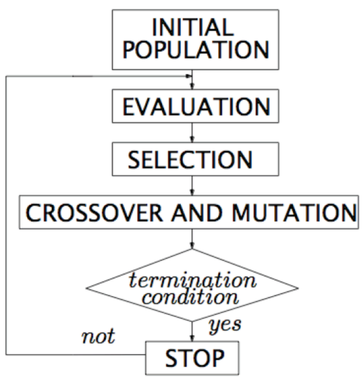

Figure 5. Fluxogram of a standard Genetic Algorithm.

GAs start with an initial set of random solutions, this set of solutions are called the population. Each individual of the population, which is a chromosome, represents a potential solution to the problem. The encoding is a genetic representation of the chromosome. In the evaluation, a measure of fitness is assigned to each individual. Individuals called parents are selected, the parents contribute to the population at the next generation. Some individuals of the population suffer genetic operations to create new individuals through stochastic transformations. There are two types of genetic operations: crossover and mutation. Crossover creates new individuals by combination of the parts of two parents; and mutation creates new individuals by randomly altering chromosome characteristics to guarantee genetic diversity in the population. New individuals of the population are called offspring. A new population is formed by selecting the more fit individuals from the present population and the offspring population. After successive iterations called generations, the algorithm converges to the best individual, which hopefully represents an optimal solution to the problem.

5. Results

The simulations of the model were implemented using MATLAB. After some preliminary simulations, the tuning parameters used in the GA optimization algorithm are presented in Table 1.

Table 1. Parameters used in the GA.

Parameter	GA
Max. Generation number	100
Encoding Type	Real
Population size	30
Crossover probability	0.5
Mutation rate	0.08

In order to find the optimal set of the eigenvalues $\boldsymbol{\lambda}$ to determine the gains of the controller, the following methodology was used. First, the optimization problem of eq. (18) was solved with different constraints in the control action α_{max} minimize the setting time in the regulation of the position of the cart at $100mm$. The solutions for different constraint in the control action α_{max} are presented Tab. 2. It was verified that increasing the constraint α_{max} the magnitude of the optimal eigenvalues $\boldsymbol{\lambda}$ increases too.

Table 2. Optimal $\boldsymbol{\lambda}$ obtained with several values of α_{mas}.

α_{max} (rad)	λ_1(rad/s)	λ_2(rad/s)	λ_3(rad/s)
0.005	1.356942668	1.477015601	1.552328977
0.01	1,913269581	2.060862589	2.231463561
0.015	2.443027943	2.538950259	2.601279831
0.02	2.811474014	2.973049064	2.973876922
0.025	3.203312615	3.231752491	3.351833777
0.027	3.186871169	3.331554309	3.442817997

Figures 6 and 7 show the convergence of the optimization after 9 generations. The Fig. 6 shows the settling time versus generations, the settling time is minimized after 9 generation.

Figure 7 presents the behavior of the eigenvalues along the generations. The results indicates that the magnitude of the eigenvalues increases to minimize the setting time. After 9 generations, the eigenvalues reach their optimal values that minimize the setting time. As seen in Fig. 7 the GA was executed by 100 generations to avoid an early stoping of the optimization.

In order to validate the method, the optimal eigenvalues $\boldsymbol{\lambda}$ were used in the real plant. All eigenvalues of Tab. 2 were tested in the real plant. It was verified that the system diverges for a control action α equal or superiors to 0.028rad (1.6Âř). The system exhibits an acceptable behavior for $\alpha_{max} > 0,02$rad. Nevertheless, for $\alpha_{max} < 0,02$rad the system exhibits a considerable steady error because the control system can not compensate the friction between the cart and the beam. However, the last eigenvalues $\boldsymbol{\lambda}$ in Tab. 2 exhibits goods results specially the last one that executes the regulates the system with the minimum setting time.

The optimal solution for $\alpha_{max} = 0.027$ rad of Tab. 2 were used in the prototype. The Fig. 8 shows experimental results to regulate set-point position of the cart at 100mm. The experiment was performed 4 times to illustrate the repeatability of the system and the small steady state error are shown in Fig. 8. Additionally, it can be seen that the simulation response is very closed to the experimental results.

The constraints imposed in the optimization problem limit the control action, therefore the selected eigenvalues do not saturate the motor. Additionally they minimize the time to stabilize the system in a desired set-point. The experiments in

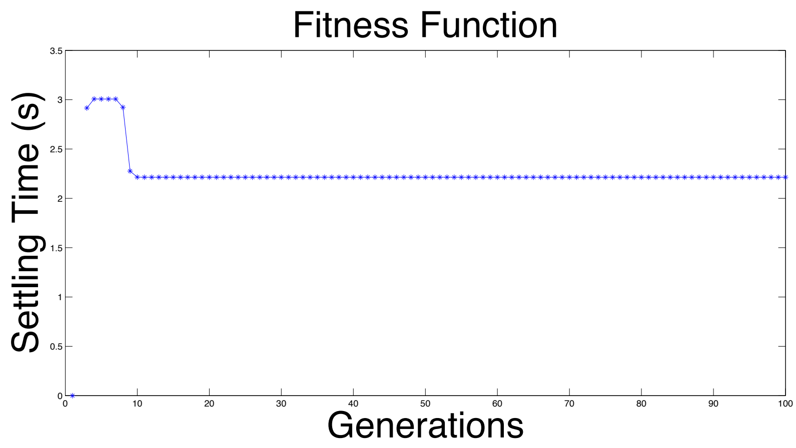

Figure 6. Fitness Function

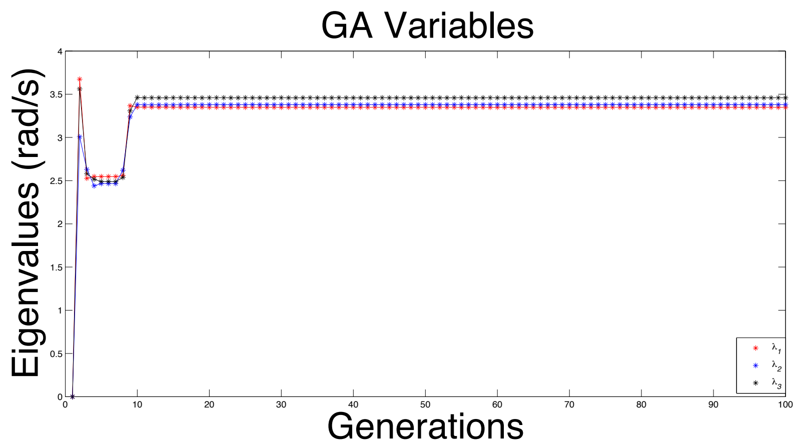

Figure 7. Design variables: eigenvalues $\boldsymbol{\lambda}$

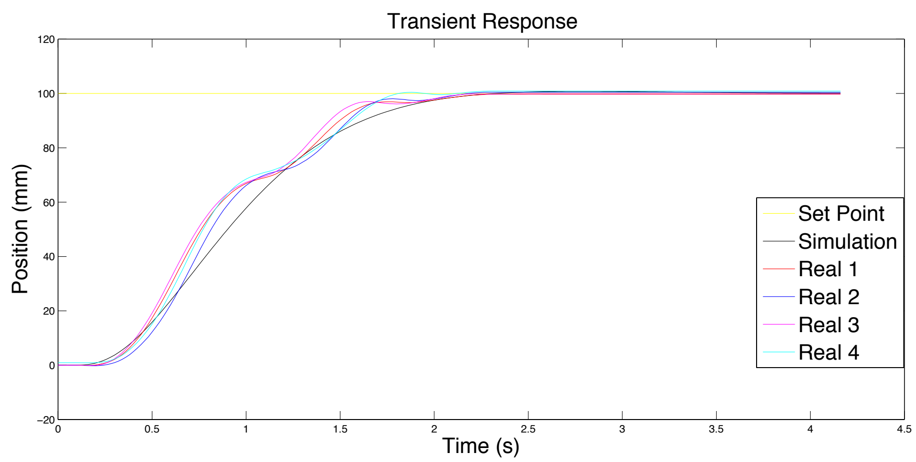

Figure 8. Position control: simulation and experimental results

the prototype show the efficiency of the design method. Additional benefits were obtained by using this design method, the steady state error was reduced considerably. Using imposed eigenvalues was obtained a steady error of 5 mm. Using the optimal design method to find the optimal eigenvalues the steady error was reduced to 0.5 mm.

The necessary time to solve the optimization process is smaller than 2 minutes using Desktop Computers Core I7. This shows that the design method to tune the control system is not computationally intensive.

6. Conclusion

An optimal design method to tune an eigenstructure assignment controller using Genetic Algorithms (GA) was described in this contribution. This optimal design method was evaluated by means of numerical simulation and experimental test in the prototype to verified that the the purposed methodology enhance the dynamic performance of the ball and beam system. Additional benefits were obtained by using this design method, e.g. the steady state error was reduced considerably.

The next step of the work will enhance the dynamic model of the system to approximate simulation to the real dynamics of the prototype. Additionally, a nonlinear model of the friction between the cart and the beam will be also considered. Nonlinear control techniques using fuzzy approaches will be considered to enhance the efficiency of the control system.

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