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Estimate of a Two-Dimensional Thermal Contact Conductance Between Two Different Materials Using the Reciprocity Functional Approach

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Abstract. *This paper aims to estimate two-dimensional thermal contact conductance profiles between two bodies made from different materials. For this purpose, an approach based on the reciprocity functional method was used, where the problem was divided in two parts. In the first part, two auxiliary problems that do not depend on the thermal contact conductance were solved. With the solution of these problems, it was possible to obtain the temperature jump and the heat flux at the inaccessible interface between the two bodies and therefore estimate the thermal contact conductance. The authors previously conducted a similar study, but considering that both bodies were made from the same material. Non-intrusive simulated temperature measurements were used, with very good estimates. The method is non-iterative and therefore requires very low computational time when compared with traditional techniques.*

Keywords: *Reciprocity Functional, Thermal Contact Conductance, Inverse Problems*

1. INTRODUCTION

Thermal contact resistance (Özisik, 1985; Çengel, 2003) is defined as the resistance to the heat flux between two or more materials at their contact interfaces. In general, the surfaces of the materials may have microscopic imperfections and their contact will never be perfect. These imperfections form valleys that work as a thermal insulation, leading to a temperature drop at the interface. This resistance depends on different factors: the type of materials that are in contact, the surface roughness of the materials, the temperature and pressure at the interface, and the type of fluid that fulfills the voids at the interfaces (Çengel, 2003). The thermal contact conductance, defined as the reciprocal of the thermal contact resistance is another quantity of interest.

The study of the thermal contact resistance is present in several areas of research. In Yovanovich (2005), the author presented a timeline showing different areas of research where thermal contact resistance is important, including cryogenics, nuclear, and microelectronics industry, among others.

Thermal contact resistance is difficult to determine and, for this reason, several works are found in the literature with experimental models (Rosochowska *et al.*, 2004) and prediction/estimation models (Cooper *et al.*, 1969; McWaid and Marschall, 1992; Gill *et al.*, 2009; Milošević *et al.*, 2002). The main problem with estimation studies is the fact that most of them include intrusive measurements.

The problem proposed in this work involves the estimate of the thermal contact conductance at an inaccessible interface between two different materials using non-intrusive measurements. This type of problem is reported as an inverse problem, which consists in estimating one or more parameters, functions, boundary conditions or properties that are unknown in a given problem. This class of problems is increasingly being used in different areas of knowledge, such as engineering, and medicine, among others.

There are different techniques for the solution of inverse problems, such as the deterministic methods (Özisik and Orlande, 2000) and methods based on the Bayesian Inference (Kaipio and Somersalo, 2004), among others. In this work we will use a method based on the *Reciprocity Functional* (Andrieux and Abda, 1993).

Two works of the authors Stéphane Andrieux and Amel Ben Abda (Andrieux and Abda, 1993, 1996) are extremely important to understand the main concepts of the Reciprocity Functional. In Andrieux and Abda (1993), some identification problems were considered and based on the concept of reciprocity *theorem of Maxwell-Betti*, the authors defined the reciprocity functional, showing that its calculation involves boundary integrals of known quantities.

From the method proposed by Stéphane Andrieux and Amel Ben Abda for crack identification, several other studies using the reciprocity functional began to emerge in different areas (Delbary *et al.*, 2008; Shifrin and Shushpannikov, 2014; Colaço and Alves, 2013, 2014; Abreu *et al.*, 2014).

The reciprocity functional approach for thermal contact conductance identification was previously investigated by the authors. In Colaço and Alves (2013), the authors estimated a one-dimensional thermal contact conductance in a two-dimensional stationary problem using the reciprocity functional approach. Abreu *et al.* (2014) made an extension of the methodology developed by Colaço and Alves (2013), simplifying the technique and making it more robust.

This paper aims to estimate a two-dimensional thermal contact conductance between two different materials using the reciprocity functional approach. For this purpose, two auxiliary problems are defined and, subsequently, through the methodology presented in (Colaço and Alves, 2013, 2014; Abreu *et al.*, 2014), the thermal contact conductance profile is obtained. This technique is considered fast, since the auxiliary problems are not dependent on the thermal contact conductance and need to be solved one time only.

2. MATHEMATICAL FORMULATION

This work considers two different materials in non-perfect contact, i.e., it is considered one domain Ω, divided in three parts $\Omega = \Omega_1 \cup \Gamma \cup \Omega_2$, where the first domain, Ω_1, has thermal conductivity κ_1 and the second domain, Ω_2, has thermal conductivity κ_2 with a contact surface Γ between them. For this problem a two-dimensional thermal contact conductance is considered.

The boundary conditions are given as follows: Γ_1 and Γ_2 (lateral surfaces) are assumed to be thermally insulated, Γ_0 (upper surface) has a prescribed heat flux q imposed on it, and Γ_∞ (lower surface) is subjected to a prescribed temperature. Figure 1 shows the geometry of the problem.

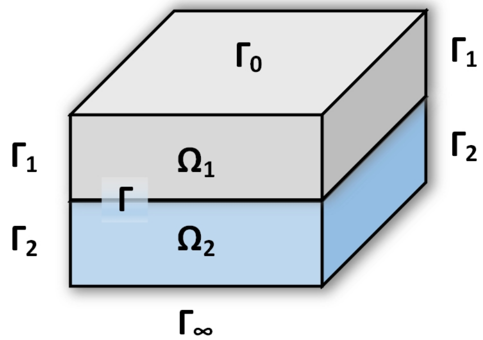


Figure 1: Geometry of the problem.

The mathematical formulation of this three-dimensional steady-state heat transfer problem with different thermal conductivities (κ_1 and κ_2), can be written as follows:

$$\nabla^2 T_1 = 0 \qquad \text{in} \quad \Omega_1 \tag{1a}$$

$$\frac{\partial T_1}{\partial \mathbf{n_1}} = 0 \qquad \text{on} \quad \Gamma_1 \tag{1b}$$

$$-\kappa_1 \frac{\partial T_1}{\partial \mathbf{n_1}} = h(T_1 - T_2) \qquad \text{on} \quad \Gamma \tag{1c}$$

$$-\kappa_1 \frac{\partial T_1}{\partial \mathbf{n_1}} = q \qquad \text{on} \quad \Gamma_0 \tag{1d}$$

$$\nabla^2 T_2 = 0 \qquad \text{in} \quad \Omega_2 \tag{1e}$$

$$\frac{\partial T_2}{\partial \mathbf{n_2}} = 0 \qquad \text{on} \quad \Gamma_2 \tag{1f}$$

$$T_2 = 0 \qquad \text{on} \quad \Gamma_\infty \tag{1g}$$

$$\kappa_2 \frac{\partial T_2}{\partial \mathbf{n_2}} = -\kappa_1 \frac{\partial T_1}{\partial \mathbf{n_1}} \qquad \text{on} \quad \Gamma \tag{1h}$$

The objective of this paper is to estimate the thermal contact conductance h at the inaccessible contact interface Γ. As stated above, two auxiliary problems must be defined to obtain the temperature jump and the heat flux on the inaccessible interface, and subsequently, through the methodology developed in (Colaço and Alves, 2013, 2014; Abreu *et al.*, 2014), which uses functional reciprocity (Andrieux and Abda, 1993), it is possible to obtain different profiles of the thermal contact conductance.

The thermal contact resistance can be obtained dividing the estimated temperature jump by the estimated heat flux:

$$R = \frac{\Delta T}{q} \tag{2}$$

The thermal contact conductance can be defined as

$$h = \frac{1}{R} \tag{3}$$

The first auxiliary problem will be used to determine the unknown temperature jump and the second auxiliary problem to determine the unknown heat flux, both located on the contact surface Γ.

Auxiliary Problem 1

The first auxiliary problem is defined by equations (4a-4h) (Colaço and Alves, 2013; Abreu *et al.*, 2014), where $F_1 \in C^2(\Omega_1)$ and $F_2 \in C^2(\Omega_2)$ are some harmonic test functions.

$$\nabla^2 F_{1,j} = 0 \qquad \text{in} \quad \Omega_1 \tag{4a}$$
$$\frac{\partial F_{1,j}}{\partial \mathbf{n_1}} = 0 \qquad \text{on} \quad \Gamma_1 \tag{4b}$$
$$F_{1,j} = F_{2,j} \qquad \text{on} \quad \Gamma \tag{4c}$$
$$F_{1,j} = \psi_j \qquad \text{on} \quad \Gamma_0 \tag{4d}$$
$$\nabla^2 F_{2,j} = 0 \qquad \text{in} \quad \Omega_2 \tag{4e}$$
$$\frac{\partial F_{2,j}}{\partial \mathbf{n_2}} = 0 \qquad \text{on} \quad \Gamma_2 \tag{4f}$$
$$F_{2,j} = 0 \qquad \text{on} \quad \Gamma_\infty \tag{4g}$$
$$\kappa_2 \frac{\partial F_{2,j}}{\partial \mathbf{n_2}} = -\kappa_1 \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} \qquad \text{on} \quad \Gamma \tag{4h}$$

The ψ_j is a $L^2(\Gamma)$ orthonormal basis function (Eq. (4d)). In this work, we used a combination of sine and cosine functions. From this auxiliary problem, described by Eqs. (4a-4h), we may develop the reciprocity functional methodology (Colaço and Alves, 2013).

For the domain Ω_1, let us consider the identity (5),

$$0 = \int_{\Omega_1} \left[F_{1,j}(\nabla^2 T_1) - T_1(\nabla^2 F_{1,j}) \right] d\Omega_1 \tag{5}$$

The following expression is obtained, using the Green's second identity,

$$\int_{\Omega_1} \left[F_{1,j}(\nabla^2 T_1) - T_1(\nabla^2 F_{1,j}) \right] d\Omega_1 = \int_{\partial\Omega_1} \left[F_{1,j} \frac{\partial T_1}{\partial \mathbf{n_1}} - T_1 \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} \right] d(\partial\Omega_1) \tag{6}$$

Then,

$$0 = \int_{\Gamma_0 \cup \Gamma 1 \cup \Gamma} \left[F_{1,j} \frac{\partial T_1}{\partial \mathbf{n_1}} - T_1 \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} \right] d(\partial\Omega_1) \tag{7}$$

Using the boundary conditions (1b) and (4b), we have

$$0 = \int_{\Gamma_0 \cup \Gamma} \left[F_{1,j} \frac{\partial T_1}{\partial \mathbf{n_1}} - T_1 \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} \right] d(\partial\Omega_1) \tag{8}$$

The temperature measurements Y are available on the boundary Γ_0, using this fact and the boundary condition (1d), we have

$$0 = \int_{\Gamma_0} \left[F_{1,j} \left(-\frac{q}{\kappa_1} \right) - Y \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} \right] d\Gamma_0 + \int_{\Gamma} \left[F_{1,j} \frac{\partial T_1}{\partial \mathbf{n_1}} - T_1 \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} \right] d\Gamma \tag{9}$$

Using another identity, now for the domain Ω_2, we can write

$$0 = \int_{\Omega_2} [F_{2,j}(\nabla^2 T_2) - T_2(\nabla^2 F_{2,j})]\, d\Omega_2 \tag{10}$$

Using again the Green's second identity, and Eqs. (1f),(1g),(4f) and (4g), the following expression is obtained

$$0 = \int_{\Gamma} \left[F_{2,j} \frac{\partial T_2}{\partial \mathbf{n_2}} - T_2 \frac{\partial F_{2,j}}{\partial \mathbf{n_2}} \right] d\Gamma \tag{11}$$

As κ_1 and κ_2 are constants, summing Eqs. (9) and (11) we obtain

$$0 = \int_{\Gamma_0} \kappa_1 \left[F_{1,j} \left(-\frac{q}{\kappa_1} \right) - Y \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} \right] d\Gamma_0 + \int_{\Gamma} \kappa_1 \left[F_{1,j} \frac{\partial T_1}{\partial \mathbf{n_1}} - T_1 \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} \right] d\Gamma +$$

$$+ \int_{\Gamma} \kappa_2 \left[F_{2,j} \frac{\partial T_2}{\partial \mathbf{n_2}} - T_2 \frac{\partial F_{2,j}}{\partial \mathbf{n_2}} \right] d\Gamma \tag{12}$$

or

$$\int_{\Gamma_0} \kappa_1 \left[F_{1,j} \left(-\frac{q}{\kappa_1} \right) - Y \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} \right] d\Gamma_0 = \int_{\Gamma} \left[-\kappa_2 F_{2,j} \frac{\partial T_2}{\partial \mathbf{n_2}} - \kappa_1 F_{1,j} \frac{\partial T_1}{\partial \mathbf{n_1}} \right] d\Gamma +$$

$$+ \int_{\Gamma} \left[\kappa_2 T_2 \frac{\partial F_{2,j}}{\partial \mathbf{n_2}} + \kappa_1 T_1 \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} \right] d\Gamma \tag{13}$$

Using Eqs. (1h), (4c) and (4h), we can obtain

$$\int_{\Gamma_0} \kappa_1 \left[F_{1,j} \left(-\frac{q}{\kappa_1} \right) - Y \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} \right] d\Gamma_0 = \int_{\Gamma} \kappa_1 \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} [T_1 - T_2]\, d\Gamma \tag{14}$$

Then, we can define the Reciprocity Functional in terms of the function F_1, through of the methodology developed by Andrieux and Abda (Andrieux and Abda, 1993) as

$$\Re(F_{1,j}) = \int_{\Gamma_0} \left[F_{1,j} \left(-\frac{q}{\kappa_1} \right) - Y \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} \right] d\Gamma_0 \tag{15}$$

Using Eqs. (14) and (15), we obtain

$$\Re(F_{1,j})\kappa_1 = \left\langle T_1 - T_2, \kappa_1 \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} \right\rangle_{L^2(\Gamma)} \tag{16}$$

We can now define

$$\kappa_1 \frac{\partial F_{1,j}}{\partial \mathbf{n_1}} = \beta_j \quad \text{on} \quad \Gamma \tag{17}$$

and write Eq. (16) as follows

$$\Re(F_{1,j})\kappa_1 = \langle T_1 - T_2, \beta_j \rangle_{L^2(\Gamma)} \tag{18}$$

Considering that the temperature jump at the inaccessible interface Γ can be written as (Abreu *et al.*, 2014)

$$[T_1 - T_2]_\Gamma = \sum_i \alpha_i \beta_i \tag{19}$$

and using Eqs. (18) and (19) we can write (Abreu *et al.*, 2014)

$$\Re(F_{1,j})\kappa_1 = \langle \beta_i, \beta_j \rangle_{L^2(\Gamma)}\, \alpha_j \tag{20}$$

Through solving the system given by Eq. (20), the unknown coefficients (α_j) are determined and therefore, using the Eq. (19), we can obtain the temperature jump at the inaccessible interface Γ.

Auxiliary Problem 2

The second auxiliary problem is defined by Eqs. (21a-21d) (Colaço and Alves, 2013; Abreu *et al.*, 2014). Where $G_1 \in C^2(\Omega_1)$ are some harmonic test functions.

$$\nabla^2 G_{1,j} = 0 \qquad \text{in} \quad \Omega_1 \tag{21a}$$

$$G_{1,j} = \psi_j \qquad \text{on} \quad \Gamma_0 \tag{21b}$$

$$\frac{\partial G_{1,j}}{\partial \mathbf{n_1}} = 0 \qquad \text{on} \quad \Gamma_1 \tag{21c}$$

$$\frac{\partial G_{1,j}}{\partial \mathbf{n_1}} = 0 \qquad \text{on} \quad \Gamma \tag{21d}$$

The function ψ_j again, appearing in Eq. (21b), is a $L^2(\Gamma)$ orthonormal basis. Using the same process described in the auxiliary problem 1, we obtain

$$\int_{\Gamma_0} \kappa_1 \left[G_{1,j} \left(-\frac{q}{\kappa_1} \right) - Y \frac{\partial G_{1,j}}{\partial \mathbf{n_1}} \right] d\Gamma_0 = \int_{\Gamma} -\kappa_1 G_{1,j} \frac{\partial T_1}{\partial \mathbf{n_1}} \, d\Gamma \tag{22}$$

Defining the Reciprocity Functional in terms of the function G_1 as

$$\Re(G_{1,j}) = \int_{\Gamma_0} \left[G_{1,j} \left(-\frac{q}{\kappa_1} \right) - Y \frac{\partial G_{1,j}}{\partial \mathbf{n_1}} \right] d\Gamma_0 \tag{23}$$

and using Eqs. (22) and (23), the following expression is obtained

$$\Re(G_{1,j})\kappa_1 = -\left\langle G_{1,j}, \kappa_1 \frac{\partial T_1}{\partial \mathbf{n_1}} \right\rangle_{L^2(\Gamma)} \tag{24}$$

Now, defining

$$G_{1,j} = \phi_j \quad \text{on} \quad \Gamma \tag{25}$$

it is possible to write Eq. (24) as follows,

$$\Re(G_{1,j})\kappa_1 = -\left\langle \phi_j, \kappa_1 \frac{\partial T_1}{\partial \mathbf{n_1}} \right\rangle_{L^2(\Gamma)} \tag{26}$$

Considering that the heat flux at the inaccessible interface Γ can be represented as (Abreu *et al.*, 2014)

$$-\left[\kappa_1 \frac{\partial T_1}{\partial \mathbf{n_1}} \right]_{\Gamma} = \sum_i \gamma_i \phi_i \tag{27}$$

and using the Eqs. (26) and (27), we can write (Abreu *et al.*, 2014)

$$\Re(G_{1,j})\kappa_1 = \langle \phi_i, \phi_j \rangle_{L^2(\Gamma)} \gamma_j \tag{28}$$

Through solving the system given by Eq. (28), the unknown coefficients (γ_j) are determined and therefore, using the Eq. (27), we can obtain the heat flux at the interface $\left[\kappa_1 \frac{\partial T_1}{\partial \mathbf{n_1}} \right]_{\Gamma}$.

We must emphasize that the thermal contact conductance is two-dimensional and, for this reason, the function $\psi_j(x, y)$ which appears in the two auxiliary problems is defined as the product of an orthonormal basis function in the *x* direction and another orthonormal basis function in the *y* direction. The total number of functions used is the product of the number of functions in x direction by the number of functions in y direction.

Thermal contact conductance

According to Eq. (3), knowing the temperature jump and the heat flux, we can obtain the thermal contact conductance at the inaccessible interface Γ. Therefore, using Eqs. (3), (19) and (27), we have

$$h = \frac{\sum_i \gamma_i \phi_i}{\sum_i \alpha_i \beta_i} \tag{29}$$

3. RESULTS

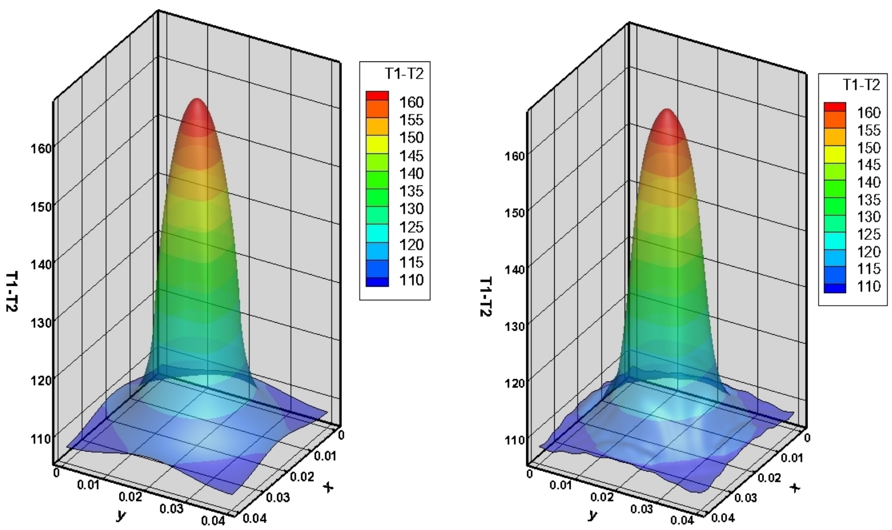
In this paper we considered two materials with $a = 0.04$ m of length, $b = 0.04$ m of width and $c = 0.01$ m of height for each domain. The thermal conductivities were $\kappa_1 = 14$ W$/(m°\mathrm{C})$ for Ω_1 and $\kappa_2 = 54$W$/(m°\mathrm{C})$ for Ω_2, corresponding to Inconel and AISI 1050 steel, respectively. A prescribed heat flux of -10^5 W/m^2 was imposed in upper surface Γ_0, which resulted in a maximum temperature of $217.58°$C. In this work we used simulated measurements, where the function used for the exact thermal contact conductance is presented in Tab. 1. A grid convergence analysis was performed and a grid with $120 \times 120 \times 30$ points was used in each domain.

Table 1: Function used for the thermal contact conductance

Profile - h W$/(m^2°\mathrm{C})$
1000 for $(x < \frac{a}{3})$ and $(x > \frac{2a}{3})$, $(y < \frac{b}{3})$ and $(y > \frac{2b}{3})$
0 for $(\frac{a}{3} \leq x \leq \frac{2a}{3})$ and $(\frac{b}{3} \leq y \leq \frac{2b}{3})$

After solving the two auxiliary problems previously defined, the reciprocity functionals can be calculated by Eqs. (15) and (23). After calculating the functionals, the linear systems given by Eqs. (20) and (28) are solved, and by using Eqs. (19) and (27), the temperature jump and heat flux are estimated at the inaccessible boundary Γ. Finally, using the Eq. (29) the thermal contact conductance is obtained.

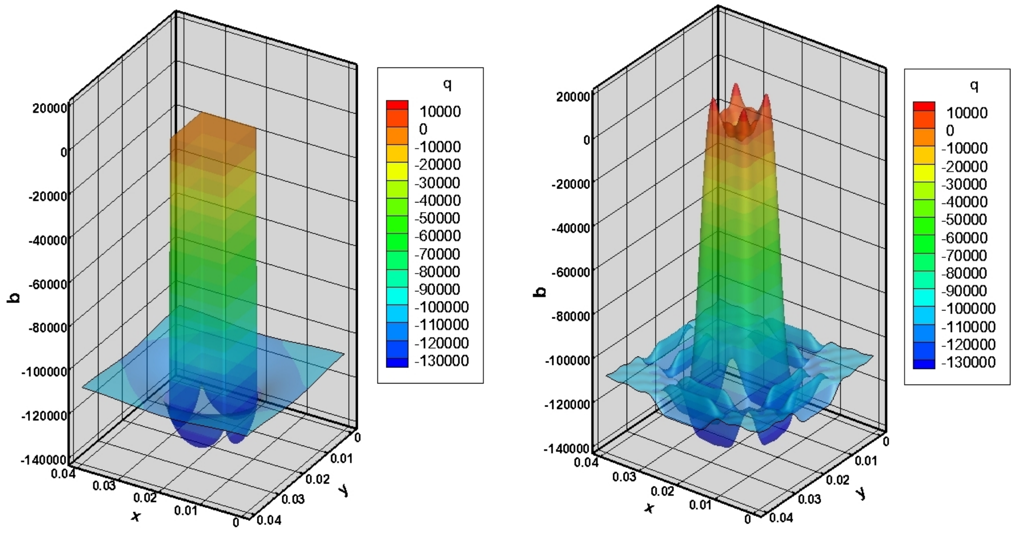
Figures 2a and 2b show the exact and estimated temperature jump on Γ, respectively. We can see that the estimate has small oscillations at the edges, but is in good agreement with exact solution.



(a) Exact. (b) Estimated.

Figure 2: Exact (left) and estimated (right) temperature jump (°C).

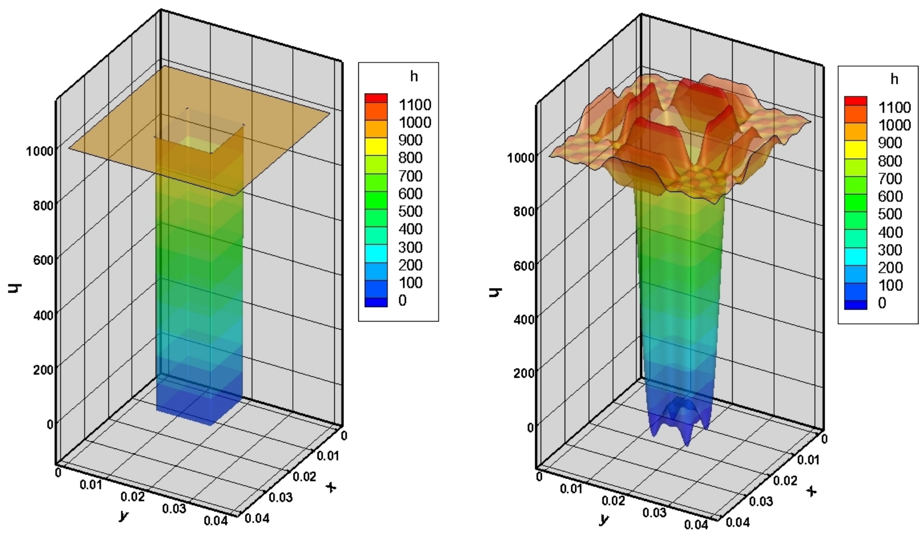
As can be seen in Fig. 3, which shows the exact and estimated heat flux at the inaccessible boundary Γ, such estimate is not as good as the temperature jump, since the function is discontinuous with large variations. However, the estimated function, presented in Fig. 3b, is still very close to the exact heat flux, presented in Fig. 3a.



(a) Exact. (b) Estimated.

Figure 3: Exact (left) and estimated (right) heat flux (W/m^2).

Figure 4 shows the exact and estimated thermal contact conductance. Although some oscillations are present, the estimate is in relative good agreement with the exact solution, as it can be seen comparing Figs. 4a and 4b.



(a) Exact. (b) Estimated.

Figure 4: Exact (left) and estimated (right) thermal contact conductance (W/m$^{2\circ}$C).

All estimates were obtained using 15 orthonormal functions in each x and y direction, resulting in a system with 225 by 225 linear equations. As general rule, the estimates improved with increasing the number of functions until reaching a limit, which in this case was 15 functions. After this limit, estimates begin to deteriorate, due to ill-posed character of the problem. A further analysis is necessary to investigate the ill-posed character of this problem.

4. CONCLUSIONS

In this paper a method based on the reciprocity functional approach was used to estimate a two-dimensional thermal contact conductance between two different materials. Non-intrusive temperature measurements were obtained by a direct problem solution considering a known variation of the thermal contact conductance. Results are presented for errorless measurements, and are in good agreement with the exact functions. The technique is non-intrusive and non-iterative, resulting in a small CPU time. A further analysis considering measurement with errors is needed, as well as an analysis of the regularization terms needed for the solution of the problem.

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