23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

A NEW FRICTION FACTOR EQUATION FOR TURBULENT FLOW OF PURELY VISCOUS NON-NEWTONIAN FLUIDS

Mateus G. Ramirez
Cecilia M.M. Santos
Hamidreza Anbarlooei
Daniel O.A. Cruz
Mechanical Engineering Program (PEM/COPPE), Universidade Federal do Rio de Janeiro, C.P. 68503, 21941-972, Rio de Janeiro, Brazil
doac@mecanica.coppe.ufrj.br

Paulo Couto
Civil Engineering Program (PEC/COPPE), Universidade Federal do Rio de Janeiro, C.P. 68503, 21941-972, Rio de Janeiro, Brazil
pcouto@petroleo.ufrj.br

Abstract. *The friction factor of the turbulent flow of purely viscous non Newtonian fluids is a parameter of great importance in many engineering applications in chemical and oil industry. The accurate and fast determination of such parameter is therefor of great interest. Some formulations can be found in the literature, which can be used to calculate the fiction factor of the turbulent flow of non-Newtonian fluids most of them however, uses implicit equations and involves some interactive and sometimes expensive process. In the present work a new explicit friction factor equation for turbulent flow of purely viscous fluid is developed, based on physical principles making it valid for a more larger application range compared with existing equations. The proposed explicit equation reproduces adequately the experimental data, avoids any interactive process and can be used to generate very precise and fast just in time numerical codes.*
Keywords: *Turbulent flow, Friction equation, Fanning factor, non-Newtonian fluids*

1. INTRODUCTION

Turbulent flow of purely viscous non-Newtonian (PVNN) fluids is present in many industrial applications. Typical examples are the transport of some polymeric substances, sewage sludges, coal, china clay and many other mineral suspensions. The friction factor of the turbulent flow of PVNN fluids is a parameter of great importance in many engineering applications in chemical and oil industry. The power law (P.-L.) fluid is one of the most typical PVNN fluids. This model (in scalar form) is expressed as $\tau = k\,\dot{\gamma}^{\,n}$ where k stands for the consistency factor, n is flow index.

The common method in calculating pressure drop of turbulent flow of a P.-L. fluid in pipes is to adopt the Darcy-Weisbach equation, where the key is to determine the value of the Fanning friction factors (f). There are abundant number of correlations available for calculating friction factors for turbulent pipe flow of the P.-L. fluids. To name some of most important correlations, Dodge and Metzner (1959) friction equation (D&M)

$$\frac{1}{\sqrt{f}} = \frac{4.0}{n^{0.75}} \log\left(Re_{MR}\, f^{1-\frac{n}{2}}\right) - \frac{0.4}{n^{1.2}}, \tag{1}$$

friction equation of Clapp (1961)

$$\frac{1}{\sqrt{f}} = \frac{2.69}{n} - 2.95 + \frac{4.53}{n} \log\left(Re_{MR}\, f^{1-\frac{n}{2}}\right) + 0.68(5n - \frac{8}{n}), \tag{2}$$

friction equation purposed by Stein, Kessler and Greendar (1980)

$$\frac{1}{\sqrt{f}} = 1.7373 \ln\left(Re_{MR}\, f^{\frac{1}{2}} - 0.398\right), \tag{3}$$

and friction equation of Hanks and Ricks (1975)

$$f = 0.0682\, n^{\frac{-1}{2}}\, Re_{MR}^{\frac{-1}{1.87+2.39n}} \tag{4}$$

should be mentioned, where Metzner-Reed Reynolds number is defined as

$$Re_{MR} = \frac{8\,\rho\, U^{2-n}\, D^{n}}{k\,\left(\frac{2}{n} + 6\right)^{n}}. \tag{5}$$

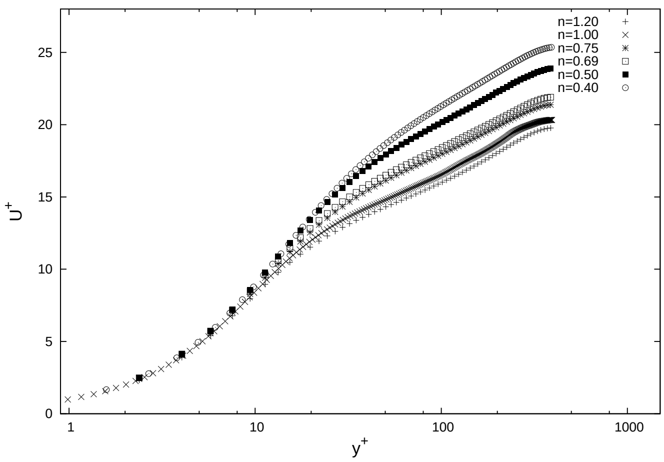

Figure 1. Mean axial velocitie for different flow indices.

Gao and Zhang (2007) have listed 14 different friction equations for P.-L. fluids. These equations can be divided into two categories, implicit equations (equations 1-3) and explicit equations (equ. 4). The main advantage of implicit equations is their wider range of application (larger Reynolds), while the explicit equations are simpler to use.

Almost all of these equations are empirical. Recently, the authors have developed two new analytical friction equations. One is implicit type obtained by integration of the velocity profile (Anbarlooei *et al.*, 2015a). The other is explicit type and is developed by relating micro-scales of turbulence to macro-scale properties of the flow (Anbarlooei *et al.*, 2015b). There are short-comes in both formulations. The explicit equation is Blasius-type equation restricted to low and mid range Reynolds numbers. In the other hand in developing the implicit equation, the velocity profile of the turbulent flow is over-simplified. The idea of current research is to combine both formulation to obtain a new explicit equation (corrected equation) with wider range of applicability.

The structure of the paper is as follows: after briefly reviewing the process resulted in development of implicit and explicit equations, in section 2 the new explicit equation is presented. Section 3 contains the examination of this equation and comparisons with other equations and experimental results. Section 4 presents our concluding remarks.

2. FRICTION EQUATION

In this section the procedure followed to obtain the corrected friction equation is described. As mentioned before, the corrected equation is obtained by combing two other equations, named here as implicit and explicit equations. Two first sub-section review these equations and the way they obtained briefly. For more detailed description, interested readers are refereed to Anbarlooei *et al.* (2015a) and Anbarlooei *et al.* (2015b). Then the procedure to find the corrected equation is discussed.

2.1 Implicit friction equation

Mean velocity profiles for turbulent flow of P.-L. fluids have been investigated before. To name few, Dodge and Metzner (1959), Clapp (1961), Rudman *et al.* (2004) and Anbarlooei *et al.* (2014) are among the most important works. Figure 1 shows the mean axial velocity for different flow indices from DNS results of Anbarlooei *et al.* (2014). As can be seen the viscous, buffer and fully turbulent regions are clearly identified. Also, the existence of a log-layer can be readily seen in this figure, with the same slope and different levels for different flow indices.

Several relations are purposed to express the dependency of this level to the rheological parameters of the fluid. The more recent DNS analysis of Anbarlooei *et al.* (2015a) suggests the level of the log-law in P.-L. fluids to varies with the n as below

$$u^+ = 2.5\ln(y^+) + 5.0 - 5.44\ \ln(n). \tag{6}$$

An expression for the friction coefficient can be obtained through an integration of the velocity profile over the cross section of a pipe. However, a general equation or equations expressing the turbulent velocity profile over all distances from the wall is still unknown. A simple estimation can be obtained using the law of the wall over the cross section of a pipe. Of course, results obtained with this procedure deviate slightly from the experimental data since the integration process does not consider the existence of the viscous and buffer sub-layers and also of the outer layer. It should be noticed that the viscous and buffer layers expand just very narrow portion of the velocity profile. Thus the main source of the difference is due to outer region of the velocity profile. Usually, to incorporate these effects, the resulting additive integration constant is determined from experimental data.

Considering that the log-profile (Eq. 6) spans across the pipe, the average velocity can be obtained from:

$$\frac{u_{av}}{u_\tau} = \frac{1}{\pi R^2}\int_0^R 2\pi r u \,\mathrm{d}r = 2.5\ln\left(\frac{\rho^{\frac{1}{n}}\, u_\tau^{\frac{2}{n}-1}}{k^{\frac{1}{n}}} R\right) - \frac{15}{4} + 5.0 - 5.44\,\ln(n) + C \tag{7}$$

where u_τ is friction velocity and C stands for the correction mentined above.

With $\tau_w = R/2\,(dp/dx)$ and the friction factor definition ($f = 2\tau_w/\rho u_{av}^2$), it is easy to show that

$$\left(\frac{u_\tau}{u_{av}}\right)^2 = \frac{f}{2}. \tag{8}$$

With these definitions, simple mathematical operations result in the following equation for f

$$\sqrt{\frac{2}{f}} = 2.5\ln\left(Re_{MR}^{\frac{1}{n}}\, f^{\frac{1}{n}-\frac{1}{2}}\left[\frac{3+\frac{1}{n}}{2^{\frac{4}{n}-\frac{1}{2}}}\right]\right) - \frac{15}{4} + 5.0 - 5.44\,\ln(n) + C. \tag{9}$$

2.2 Explicit friction equation

The explicit friction equation is based on Gioia and Chakraborty (2006) theory which related the micro-scales to friction (marco-scale) properties. To start from micro-scales, Kolmogorov's K41 theory should be considered. Kolmogorov's theory Kolmogorov (1941) describes how energy is transferred from larger to smaller eddies; how much energy is contained by eddies of a given size; and how much energy is dissipated by eddies of each size. K41 theory is based on three important hypotheses combined with dimensional arguments and experimental observations.

It is discussed in Anbarlooei *et al.* (2015b) that to extend this theory to PVNN fluids, only the first similarity hypotheses should be changes as *in every turbulent flow at sufficiently high Reynolds number, the statistics of the small scale motions (micro-scales) have a universal form that is uniquely determined by* ε *(dissipation rate) and* k. Given these two parameters (ε and k), one can redefine Kolmogorov micro-scalces as below:

$$\eta \sim k^{\frac{3}{2(n+1)}}\,\varepsilon^{\frac{n-2}{2(n+1)}} \;\;;\;\; \tau_\eta \sim k^{\frac{1}{n+1}}\,\varepsilon^{\frac{-1}{n+1}} \;\;;\;\; u_\eta \sim k^{\frac{1}{2(n+1)}}\,\varepsilon^{\frac{n}{2(n+1)}} \tag{10}$$

Thanks to phenomenological theory developed by Gioia and Chakraborty (2006), the first similarity hypotheses in extended version of K41 theory can be related to friction. The first step is to drive an expression for τ , the shear stress at the wall. We consider that the wall shear stress balances the momentum transfer through the near the wall viscous layer. On top of this layer, fluid has a high velocity which scales with U. However, in the full turbulent layer, the axial momentum transfer is negligible so that the net rate of momentum transfer is due to the normal velocity, which is induced by the small eddies. Thus, the wall shear stress scales as $\tau \sim \rho U\, u_n$, where u_n is the velocity of the dissipating eddies. Considering dissipation occurs at micro-scale, u_n can be approximated by u_η. Combining (10) with dissipation calculated from integral scale, results in the following equation for velocity at micro-scale.

$$\frac{u_\eta}{U} \sim \frac{1}{Re^{\frac{1}{2(n+1)}}} \tag{11}$$

Now, it is easy to show that the friction can be expressed as

$$f = g(n)\,\frac{1}{Re^{\frac{1}{2(n+1)}}} \tag{12}$$

The coefficient $g(n)$ can be obtained using experimental results (fitting). Such a process has been done in Anbarlooei *et al.* (2015b) and the resulting equation is

$$f = 1.0178\left(0.1 - 0.0322\,n + \frac{0.00982}{n}\right)\frac{1}{Re_{MR}^{\frac{1}{2(n+1)}}} \tag{13}$$

for Metzner-Reed Reynolds number and Fanning friction factor.

It can be seen that the results of the new equation diverge form DM equation at higher Reynolds. The source of this discrepancy lies in the assumption made here to calculate τ. Comparing with the theory developed in Gioia and Chakraborty (2006) for Newtonian fluids, it reveals that here the effects of energetic range on friction is neglected (which is accurate only for low Reynold numbers). Thus, the new equation act as Blasius-type equation for PVNN fluids.

2.3 Corrected friction equation

As mentioned in the previous sub-section, the introduced explicit equation is restricted to low Reynolds numbers. The idea is to use this equation to obtain (calibrate) correction coefficient (C) in equation (9). If this correction is independent or weakly dependent on the Reynolds number, then the corrected equation must be have wider range of applicability rather than explicit equation.

Present study shows that this dependency on Reynolds number is weak. So, the constant correction assumption is used. To obtain the value of this constant, averaging has been done on local values of C over the range of Re=$3000 - 10000$ (where explicit equation seems accurate). Applying the least square fitting to the correction coefficients results to the following correlation (with $R^2 = 0.99908$).

$$C = 4.23\,n - 3.8 \tag{14}$$

Combining Equ. (13) into (9) and using correction coefficient (14) will result in following explicit equation,

$$f = \frac{2}{\left(A_n \ln(Re_{MR}) + B_n\right)^2} \tag{15}$$

where A_n, B_n and C_n are defined as

$$A_n = \frac{25n+10}{8n(n+1)} \tag{16}$$

$$B_n = 2.5\,ln\left(C_n^{\frac{1}{n}-\frac{1}{2}}\left(3+\frac{1}{n}\right)2^{\frac{1}{2}-\frac{4}{n}}\right) - \frac{15}{4} + 5.0 - 5.44\,ln(n) + 4.23n - 3.7987 \tag{17}$$

$$C_n = 1.0178\left(0.1 - 0.0322n + \frac{0.00982}{n}\right) \tag{18}$$

It is more convenient for the engineering tasks to simplify B_n. Using a curve fitting, one can find following approximation for that (with less than 1% deviation).

$$B_n \approx 50.136\,e^{-2.5165n} \tag{19}$$

In the next section, the new corrected equation is compared with both equations (9) and (13). Also, comparison with well-known Dodge and Metzner friction equation is provided.

3. RESULTS

In this section, the results of developed explicit equation is compared with both previous implicit and explicit formulations. Also, comparison with well-known empirical equation such as Dodge and Metzner has been done.

To start, the result of equation (14) is compared with both Colebrook-White (Prandtl) and Dodge and Metzner friction equations (Dodge and Metzner, 1959) for the Newtonian case (n=1). The analysis shows that the difference between our results and both other equations is less than %1 for this case.

Next step is to compare results for non-Newtonian fluids. The results of such comparison is shown in figures 2 and 3 for n=0.7 and 0.5. In these figures, the result of current work is compared with both implicit and explicit equations. Also, as a reference, results of Dodge and Metzner equation is provided. As can be seen and discussed before, the explicit equation has very good accuracy in low-Reynolds number. However, by increasing the Reynolds it start to deviate sharply from DM reference curve in both figures. On the other hand, the implicit equation shows very similar behavior as DM curve, but with an error (due to omitting the effects of viscous, buffer and outer regions). Figures 2 and 3 show that correction introduced in previous section can remove this difference and extend the application range of both implicit and explicit equations.

Finally, the comparison with experimental results are shown in figure 4. Also, the relative errors of predictions obtained by equation 14 to the experimental results are shown in figure 5. As can be seen, the total error is less than 10% for entire range of study and less than 5% for higher Reynolds numbers (the purpose of this work). Also, it is obvious that the current formulation can represent functional dependency of friction on both Reynolds number and flow index very well, while the accuracy of predictions are in acceptable range.

It should be mentioned that in all of our comparisons, the difference between predictions of equation (14) and Dodge and Metzner equation (1) is less than 5%. While the results of DM equation is accepted in scientific and engineering communities as a reference value, it can be concluded that our corrected explicit equation has acceptable accuracy to be used in engineering applications. However, it must noticed that due to lack of experimental data on high-Reynolds flows of non-Newtonian fluids, the accuracy of D.M. equation (and ours) in this range is unknown.

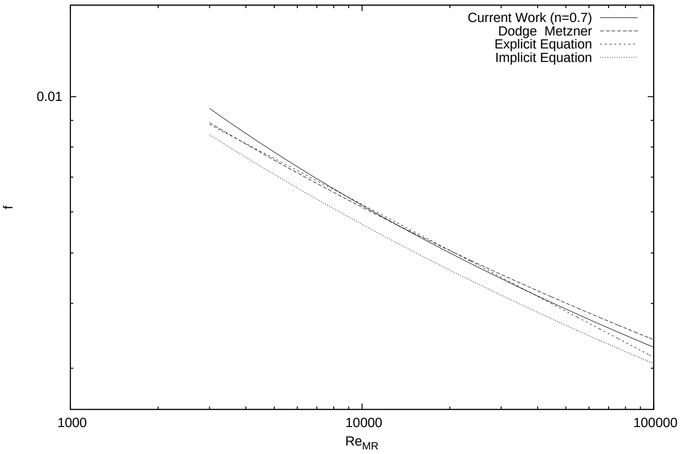

Figure 2. Comparison of corrected friction equation with implicit (9), explicit (13) and Dodge and Metzner friction (1) equations for n=0.7.

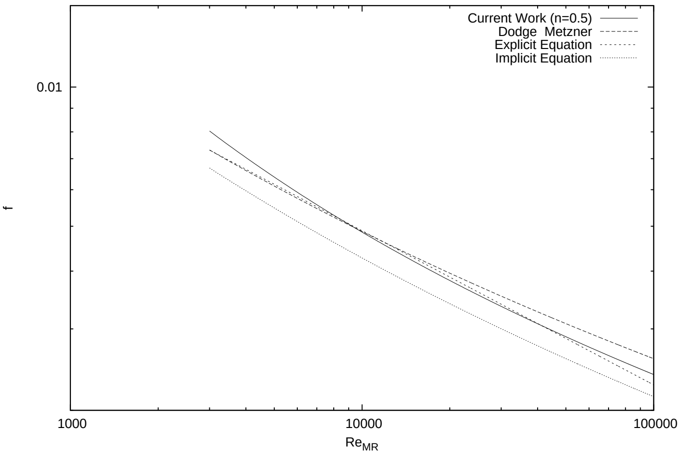

Figure 3. Comparison of corrected friction equation with implicit (9), explicit (13) and Dodge and Metzner friction (1) equations for n=0.5.

4. SUMMARY

The friction factor of the turbulent flow of purely viscous non Newtonian fluids is examined. A new explicit type friction equation is developed by combining an implicit equation (obtained by integrating velocity profile) and an explicit equation (obtained using phenomenological theory relating micro-scale turbulence structures to macro-scale properties). It is shown that the new corrected equation is as accurate as well-known Dodge and Metzner friction equation, while due to it's explicit formulation is easier to use specially in engineering tasks. The idea of correcting implicit equation resulted from integrating velocity profile has a great potential to be extended to incorporate roughness effects. This topic is ongoing.

5. REFERENCES

Anbarlooei, H.R., Cruz, D.O.A. and Silva Freire, A.P., 2014. "logarithmic solution of purely viscous non-Newtonian fluids". In *10th International ERCOFTAC Symposium on Engineering Turbulence Modelling and Measurements (ETMM10)*, Spain.

Anbarlooei, H.R., Cruz, D.O.A. and Silva Freire, A.P., 2015. "Fully turbulent mean velocity profile for purely viscous non-Newtonian fluids". In *Proceedings of the 15th European Turbulence Conference*. Delft, Netherlands.

Anbarlooei, H.R., Cruz, D.O.A. and Silva Freire, A.P., 2015. "Friction equation for purely viscous non-Newtonian fluids from Kolmogorov's theory". In *Proceedings of Turbulence, Heat and Mass Transfer 8*. Sarajevo, Bosnia and

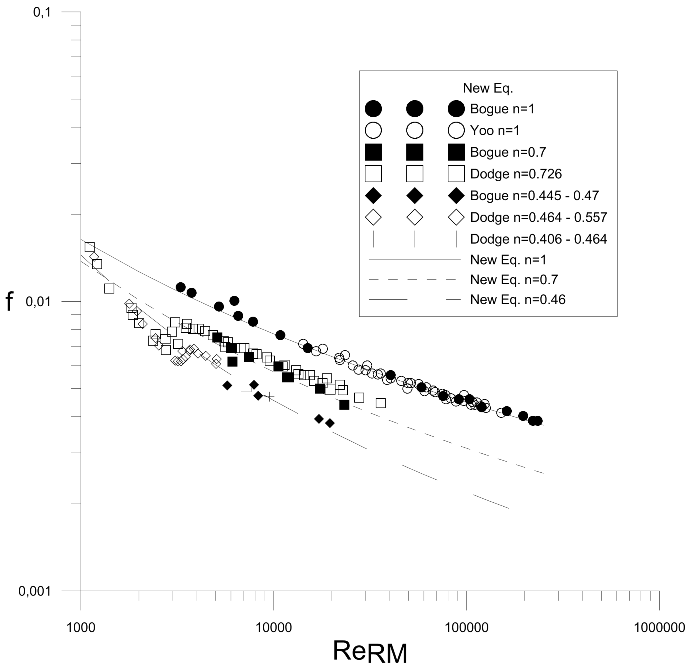

Figure 4. Comparison of the results of corrected friction equation with experimental results.

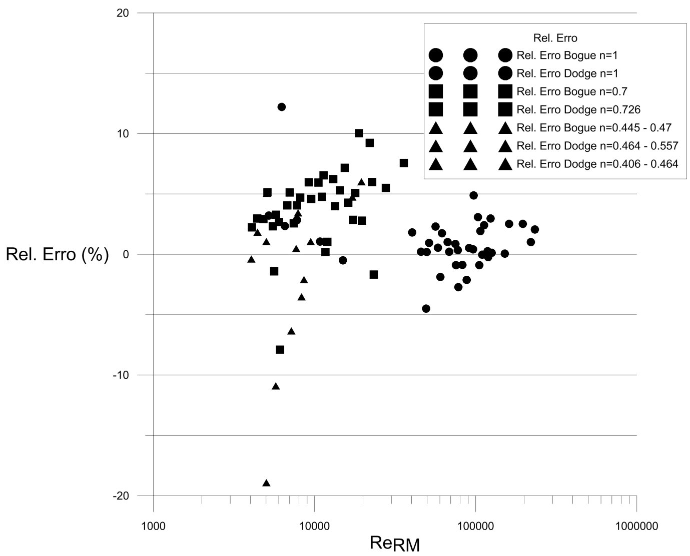

Figure 5. Relative errors of the results of corrected friction equation to the experimental results.

Herzegovina.

Clapp, R. M., 1961. “Turbulent heat transfer in pseudoplastic non-newtonian fluids”. In *Conference of International Developments in Heat Transfer*, pp. 652-662.

Dodge, D.W., and Metzner, A.B., 1959. “Turbulent flow of non-newtonian systems”. *AIChE Journal* 5:189-204.

Peng Gao and Jin-jun Zhang, 2007. “New assessment of friction factor correlations for power law fluids in turbulent pipe flow: A statistical approach”. *Journal of Central South University of Technology*, Vol. 14, pp. 77–81.

Gioia, G. and Chakraborty, P., 2006. “Turbulent Friction in Rough Pipes and the Energy Spectrum of the Phenomenological Theory”. *Physical Review Letters*, Vol. 96.

Hanks, R.W. and Ricks, B.L., 1975. “Transitional and Turbulent Pipeflow of Pseudoplastic Fluids”. *Journal of Hydronautics*, Vol. 9.

Kolmogorov, A.N., 1941. The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers. *Dokl. Akad. Nauk.SSSR 30, 299-303, 1941, Reprinted in 1991 Proc. R. Soc. London A*. Vol. 434, pp. 9–13.

Rudman, M., Blackburn, H.M., Graham, L.J.W. and Pullum, L., 2004. “Turbulent pipe flow of shear-thinning fluids”. *Journal of Non-Newtonian Fluid Mechanics*. Vol. 118,pp. 33–48.

El-emam, N., Kamel, A.H. and El-shafei, M., 2003. “New equation calculates friction factor for turbulent flow of non-

Newtonian fluids". *Oil & Gas Journal*, Vol. 101(36), pp. 74–83.

6. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.