OPTIMAL CONTROL OF A 3D CRANE USING DIRECT TRANSCRIPTION

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Abstract. *Cranes keep to be employed in a variety of ways in the freight transport industry. Increasing the performance of the operation may result in a reduction of the operation time, an improvement of the maneuvers and a reduction of the consumed energy. A strategy to ensure these improvements can be found through the mathematical optimization problem that has the formulation and solution of the optimal control problem of its trajectory. It is a challenge for a 3D crane due to its nonlinear dynamics. Optimal control problems can be converted into nonlinear discrete optimization problems that include the dynamic equations of the system as constraints of the problem. Other constraints are given by the limits of the actuator, the initial and final conditions and the limit of the payload oscillation. The optimal control problem of minimal time of a 3D crane was studied in this project. The continuous optimal control problem was manually formulated and then automatically converted into the equivalent nonlinear discrete optimization problem through the compiler Tomlab/PROPT. The simulations show solutions that would not be easily thought without a rigorous formulation of the problem.*

Keywords: *Crane; Optimal Control; Optimization.*

1. INTRODUCTION

Cranes keep being employed in a variety of ways in the freight transport industry. Automated systems can help the operator to control the load swing or even replace the operator, (Inteco, 2008). Once an automatic control system for the crane is available, it is possible to optimize the crane trajectory, considering the constraints of the system (Puglia, 2011). This implies, for instance, in reducing the time of the operation.

There are two major classes of control for cranes: the optimal control of the trajectory and the closed loop control. The closed loop control is commonly adopted when the controlled variables are likely to be modified by considerable disturbances. The feedback system can be used, for example, to reduce the oscillation during a maneuver through measuring the oscillation of the payload. This method can be used even if the trajectory is not previously known.

The optimal control of the trajectory is usually an open loop control problem. In this case, the signal related to the manipulated variable is determined in order to minimize, for example, the trajectory time, given the constraints of the system, such as the maximum angle of oscillation, the limits of the actuator and the initial and final states.

There are many papers dealing with crane control. Beeston (1969), used boundary conditions to deal with the optimal control problem. Sakawa and Shindo (1982) derived the model using Lagrange's equation and a solution for the optimal control problem was obtained. Increasing accuracy by gain scheduling was proposed by Corriga *et al.* (1998). Noakes and Jansen (1992) proposed to use zero angular velocity at the target point.

Puglia (2011) obtained the control signal and the optimal trajectory of discrete time for a cart pendulum system with one degree of freedom that can be considered a simplified model of a 2D crane. The problem was solved through Linear Programming. The control effort problem was incorporated into the objective function and the minimal time problem was solved indirectly by sequentially reducing the final time while the constraints are satisfied.

The governing state equations for a 3D crane system with varying pendulum length are nonlinear and highly coupled (Hongjun, 2005). The optimal control for a 3D crane trajectory cannot be solved by using Linear Programming due to its nonlinear dynamics. However, in such problems it is possible to use Nonlinear Programming (Nocedal, 2000). The optimal control problem has to be transformed into a nonlinear optimization problem which the dynamics of the plant is included as a constraint of the problem. The states cannot be considered as linear combination of the control. Given that, in such problems it is usual to include the states as variables of the problem. Two methods for doing so are the Direct Collocation and the Shooting. The optimal control for the 3D crane is solved here through this method.

2. DIRECT COLLOCATION

Nonlinear Programming problems are optimization problems where at least one of the functions of the models, objective function or the constraints, are nonlinear. This problem usually consists in finding X (x_1, x_2, x_3, …, x_4), to obtain Max Z(X), subjected to

$$g_i(X) \leq b_i \qquad i = 1, 2, 3, \ldots, m \tag{1}$$

where Z(X) is the objective function (or cost function) and $g_i(X)$ represents the constraint function of the model (Lachtemacher, 2009).

The Direct Collocation problem is descripted here as the nonlinear optimization problem of a dynamic system where both the control and the state variables are included in the formulation (Betts, 2001).

Let p be the vector that contains the discretized controls u and the states y. The vector p of parameters that satisfies the given optimal control problem is called optimal vector. Notice that the dynamics of the plant are included as constraint of the optimization problem in each discretization time. By writing these constraints in the matrix form, it is possible to notice that the matrix is sparse. If the optimization method is ready to deal with this sparsity, the null elements of the matrix do not overload the optimization. The solver SNOPT/Tomlab is proper to deal with this sparsity, but the user needs to provide the pattern of sparsity (Holmstrom *et al.,* 2008). In order to automate this process, the solver PROPT/Tomlab can be used (Rutquist and Edvall, 2010). In this paper, the solver PROPT/Tomlab is used to generate the nonlinear optimization problem equivalent to the optimal control problem of a 3D crane.

PROPT is a solver developed to solve dynamic optimization problems (Rutquist and Edvall, 2010). The solver uses pseudo-spectral methods. Such methods provide a solution mathematically equivalent to the solution that would be obtained through the maximum principle of Pontryagin.

The formulation of optimal control problems using the PROPT is relatively easy, since the solver automates the necessary steps to generate the nonlinear optimization problem equivalent to the optimal control problem.

3. METHOD

The model developed by Pauluk, M., *et al* (s.d) was used here to formulate the solution of the optimal control problem. Models considering an obstacle and drag were also studied.

3.1 Model

The 3D crane model considered in this paper consists of a cart, and a payload hanging on a lift-line. The cart moves in a horizontal plane: it can be shifted along a rail, which also can be shifted in a perpendicular direction. The length of the lift-line is independently controlled. This leads to a tenth-order system of state equations with three controls. It is represented in the Fig. 3. The notation used for the model can be seen in Tab. 1 and the notation used for the symbols can be seen in Tab. 2 and by formulae defined as in Tab. 4 (see Appendix).

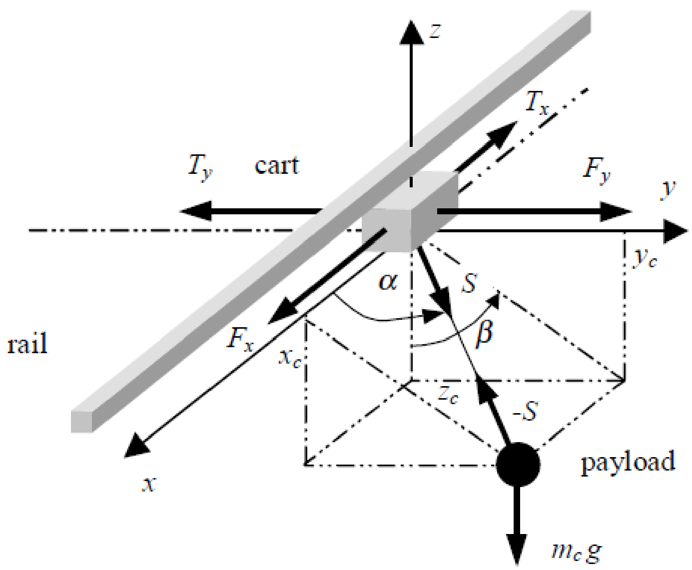

Figure 3. Representation of the 3D crane.

Table 1. Notation used for the model.

M_c	Mass of payload
m_w	Mass of cart
m_s	Mass of moving rail
x_c, y_c, z_c	Coordinates of payload
S	Traction on lift-line
F_x	Force driving the cart
F_y	Force driving the moving rail
F_r	Force controlling the lift-line
T_x, T_y, T_r	Friction forces
x_w	Distance to the center of the rail
y_w	Distance of the cart to the center of y axis
R	Length of the lift-line
α	Angle between x axis and the lift-line
β	Angle between the z axis and the projection of the lift-line on the yz plane

The position of the payload is described by

$$x_c = x_w + Rcos(\alpha) \tag{2}$$

$$y_c = y_w + Rsen(\alpha)\,\mathrm{sen}(\beta) \tag{3}$$

$$z_c = -Rsen(\alpha)\cos(\beta) \tag{4}$$

The dynamics of the crane are described by

$$m_c\ddot{x}_c = -S_x \tag{5}$$

$$m_c\ddot{y}_c = -S_y \tag{6}$$

$$m_c\ddot{z}_c = -S_z - m_c g \tag{7}$$

$$m_w\ddot{x}_w = F_x - T_x + S_x \tag{8}$$

$$(m_w + m_s)\ddot{y}_w = F_y - T_y - S_y \quad , \tag{9}$$

where S_x, S_y and S_z are the components of S

$$S_x = Scos(\alpha) \tag{10}$$

$$S_y = Ssen(\alpha)sen(\beta) \tag{11}$$

$$S_z = -Ssen(\alpha)cos(\beta) \tag{12}$$

Assuming that the lift-line is always stretched

$$S_x(x_c - x_w) + S_y(y_c - y_w) + S_z z_c > 0. \tag{13}$$

The length of the lift-line is controlled by the force F_r, then

$$S = F_r - T_r \quad . \tag{14}$$

Making the proper substitutions

$$\ddot{x}_c = -N_3\cos(\alpha) \tag{15}$$

$$\ddot{y}_c = -N_3 sen(\alpha) cos(\beta) \tag{16}$$

$$\ddot{z}_c = N_3 \text{sen}(\alpha)\cos(\beta) - g \tag{17}$$

$$\ddot{x}_w = N_1 + \mu_1 N_3 cos(\alpha) \tag{18}$$

$$\ddot{y}_w = N_2 + \mu_2 N_3\, sen(\alpha)\, sen(\beta). \tag{19}$$

Deriving twice

$$\ddot{x}_c = \ddot{x}_w + \left(\ddot{R} - R\dot{\alpha}^2\right) cos(\alpha) - \left(2\dot{R}\dot{\alpha} + R\ddot{\alpha}\right) sen(\alpha) \tag{20}$$

$$\ddot{y}_c = \ddot{y}_w + \left(\ddot{R} - R\dot{\alpha}^2 - R\dot{\beta}^2\right)\sin(\alpha)\, sen(\beta) + 2R\dot{\alpha}\dot{\beta}\cos(\alpha)\cos(\beta) + \left(2\dot{R}\dot{\alpha} + R\ddot{\alpha}\right)\cos(\alpha)\, sen(\beta) + \left(2\dot{R}\dot{\beta} + R\ddot{\beta}\right) sen(\alpha)\cos(\beta) \tag{21}$$

$$\ddot{z}_c = \left(-\ddot{R} + R\dot{\alpha}^2 + R\dot{\beta}^2\right)\sin(\alpha)\, cos(\beta) + 2R\dot{\alpha}\dot{\beta}\cos(\alpha)\,\text{sen}(\beta) + - \left(2\dot{R}\dot{\alpha} + R\ddot{\alpha}\right)\cos(\alpha)\, cos(\beta) + \left(2\dot{R}\dot{\beta} + R\ddot{\beta}\right) sen(\alpha)\text{sen}(\beta) \tag{22}$$

As notation, assume that

$$x_1 = x_w \tag{23}$$

$$x_2 = \dot{x}_1 = \dot{x}_w \tag{24}$$

$$x_3 = y_w \tag{25}$$

$$x_4 = \dot{x}_3 = \dot{y}_w \tag{26}$$

$$x_5 = \alpha \tag{27}$$

$$x_6 = \dot{x}_5 = \dot{\alpha} \tag{28}$$

$$x_7 = \beta \tag{29}$$

$$x_8 = \dot{x}_7 = \dot{\beta} \tag{30}$$

$$x_9 = R \tag{31}$$

$$x_{10} = \dot{x}_9 = \dot{R} \tag{32}$$

$$s_n = sen(x_n) \tag{33}$$

$$c_n = cos(x_n) \tag{34}$$

$$V_5 = c_5 s_5 {x_8}^2 x_9 - 2x_{10}x_6 + g c_5 c_7 \tag{35}$$

$$V_6 = 2x_8(c_5 x_6 x_9 + s_5 x_{10}) + g s_7 \tag{36}$$

$$V_7 = {s_5}^2 {x_8}^2 x_9 + g s_5 c_7 + {x_6}^2 x_9 \tag{37}$$

Assuming that the friction force is proportional to the velocity

$$N_1 = u_1 - k_1 x_2 \tag{38}$$

$$N_2 = u_2 - k_2 x_4 \tag{39}$$

$$N_3 = u_3 - k_3 x_{10} \tag{40}$$

where k_1, k_2 and k_3 are constants. Finally, the model is

$$\dot{x}_1 = x_2 \tag{41}$$

$$\dot{x}_2 = N_1 + \mu_1 c_5 N_3 \ \ ) \tag{42}$$

$$\dot{x}_3 = x_4 \tag{43}$$

$$\dot{x}_4 = N_2 + \mu_2 s_5 s_7 N_3 \tag{44}$$

$$\dot{x}_5 = x_6 \tag{45}$$

$$\dot{x}_6 = (s_5 N_1 - c_5 s_7 N_2 + (\mu_1 - \mu_2 s_7^2) c_5 s_5 N_3 + V_5)/x_9 \tag{46}$$

$$\dot{x}_7 = x_8 \tag{47}$$

$$\dot{x}_8 = -(c_7 N_2 + \mu_2 s_5 c_7 s_7 N_3 + V_6)/(s_5 x_9) \tag{48}$$

$$\dot{x}_9 = x_{10} \tag{49}$$

$$\dot{x}_{10} = -c_5 N_1 - s_5 s_7 N_2 - (1 + \mu_1 c_5^2 + \mu_2 s_5^2 s_7^2) N_3 + V_7 \tag{50}$$

3.2 Constraints

The boundary constraints were have adopted as shown in Tab. 3 (see Appendix). For the maximum angle of oscillation, an absolute value of 0.05π rad was adopted

$$-0.05\pi \leq \beta \leq 0.05\pi \ . \tag{51}$$

For the control efforts, an absolute value of 10 m/s^2 was adopted as bounds

$$-10 \leq u_1, u_2\, , u_3 \leq 10 \ . \tag{52}$$

3.3 Model considering an obstacle

Considering that the obstacle is a cylinder, and the payload has to avoid this obstacle, the equation that represents the constraint is

$$(x_c - 0.5)^2 + (y_c - 0.5)^2 \geq 0{,}1 \tag{53}$$

3.4 Model considering drag

The effect of drag over the trajectory and the drag on the moving rail and cart were neglected. The only force considered in this problem was the drag on the payload on the *x* axis. A value of 8 m/s was adopted for the wind speed.

Supposing that the payload is a sphere of mass of 1 kg, made of Aluminum (ρ=2.7 g/cm³), the diameter would be D=0.0446 m. The drag coefficient for a smooth sphere in these conditions is approximately C_d=0.5 (Fox, 2011).

After the proper substitutions and including the drag force into the Eq. 10

$$N_1 = u_1 - k_1 x_2 - 0{,}5 \ . \tag{54}$$

4. RESULTS AND ANALYSIS

4.1 Optimal Control

The Optimal control problem was solved for a number of nodes equal to 100 and an initial guess for the final time of 5 seconds. The model revealed to be very sensitive to changes in the initial guess for *tf*. The optimal time found was approximately *tf = 3.62* seconds. As it can be seen in the Fig. 4, the most used component to make the maneuvers was the moving rail, since its displacement and velocity were hanged significantly during the trajectory. The displacement and velocity of the car were maintained constant along the path. The control variables oscillating to the extremities are features of *a bang-bang* control, typical in minimal time optimal control problems.

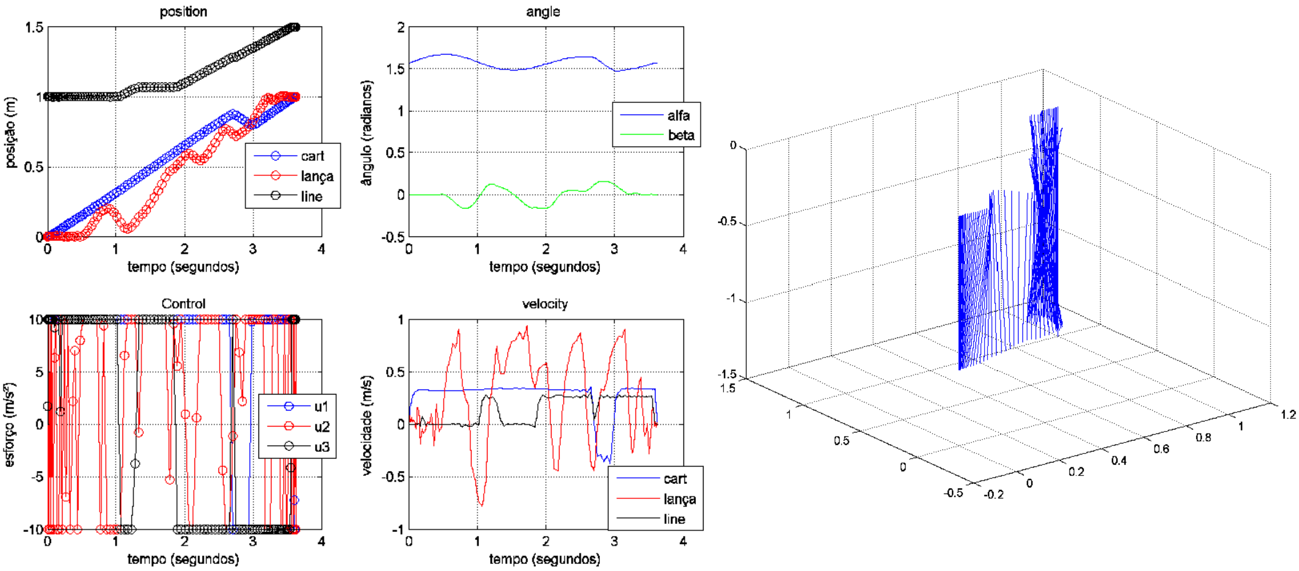

Figure 4. Solution of the optimal control problem.

4.2 Optimal Control considering an obstacle

For the model considering an obstacle, the solution converged for 127 nodes and an initial guess for the final time of 5 seconds. The time for the optimal trajectory was approximately *tf = 3.79* seconds. As it can be seen in the Fig. 5, the swing of the mass was used to avoid the obstacle. Again, the most used component was the moving rail.

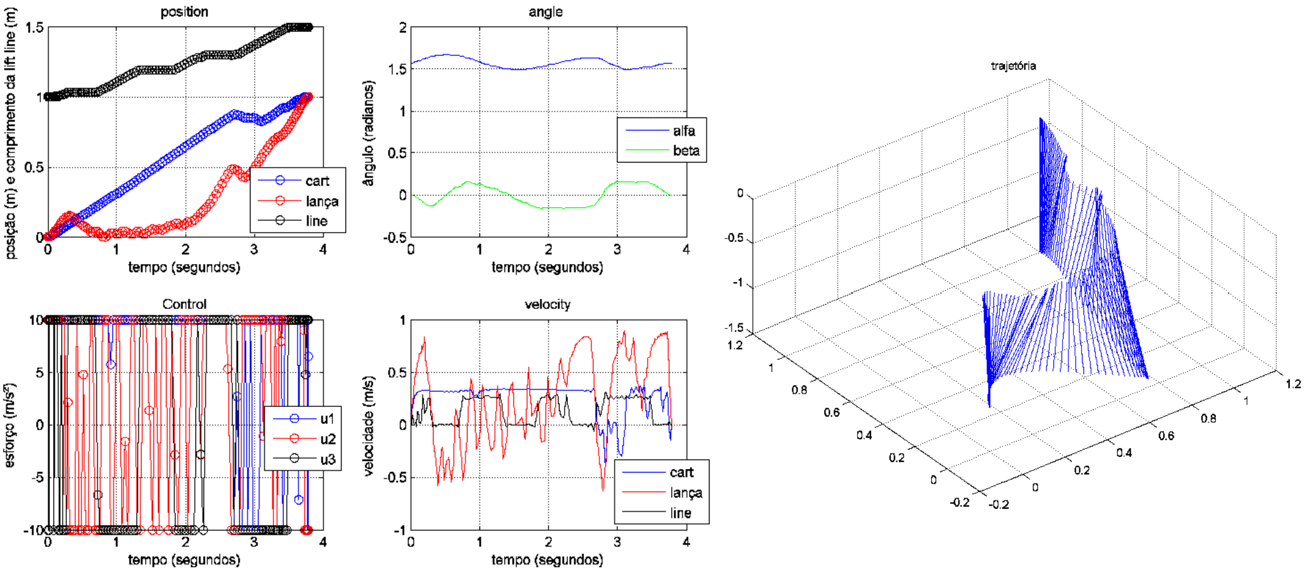

Figure 5. Solution of the optimal control problem with an obstacle.

4.3 Optimal Control considering drag

The solution for the optimal control considering the drag in the x axis direction was obtained for 100 nodes and an initial guess for the final time of 6 seconds. The time for the optimal trajectory obtained was approximately *tf = 3.98* seconds. It can be noticed that the states showed a certain change compared to the problem without drag, even if the drag is applied only in the payload in the *x* direction. This suggests that the drag is an important factor in the study of 3D cranes.

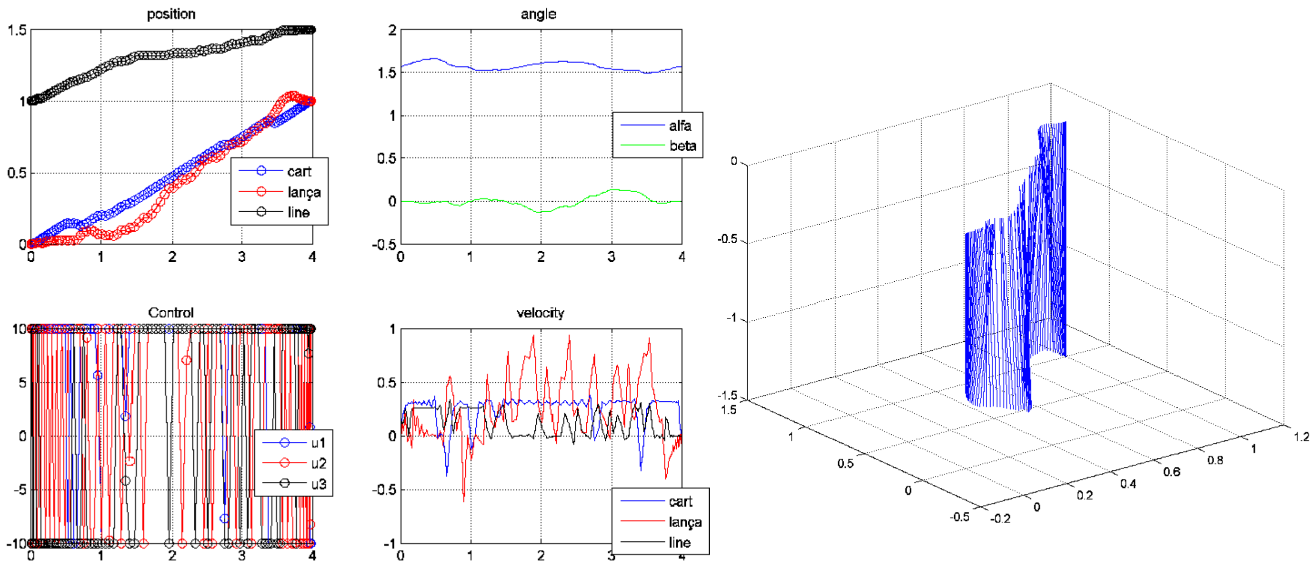

Figure 6. Solution of the optimal control problem considering drag.

5. CONCLUSIONS

The objective of this project was to study the optimal control problem of a 3D crane by using the solver PROPT/Tomlab that generates the nonlinear optimization problem equivalent to the optimal control problem. As the efficiency of the operation depends on the time and the safety that the transport of the freight is done, the maneuvers should be done as fast as possible without violating the constraints of the system, such as the maximum angle of oscillation.

The optimal control problem of minimal time was solved for the 3D crane. It was also presented the solution for the cases with an obstacle and the effect of drag. It can be noticed that the solution is sensitive to the initial guesses, especially for the guess of final time. In some cases, a slight change in the initial guess for the final time was enough to make the solution not converging. The number of points chose in the problem is also important for the final result.

In this paper the time of the optimal trajectory found was similar to the time found by the author of the 3D crane model in this paper. The control reach its limits exhibiting a *bang-bang* action, a common feature of minimum time optimal control problems. For the problem considering an obstacle, the time of the trajectory found was just two tenths more than the problem without obstacle. The swing of the mass was used to avoid the obstacle. For the problem considering drag in the *x* axis, the final time was slightly greater and the trajectory was different, even the drag being small. It suggests that, in a real model, the drag s an important factor. Ultimately, based on the results of the computational simulation, the use of a prototype or application in laboratory would be very interesting to continue the research.

6. ACKNOWLEDGEMENTS

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8. RESPONSIBILITY NOTICE

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APPENDIX

Table 2. Notation used for the symbols.

μ_1	Friction coefficient at lift-line
μ_2	Friction coefficient between the cart and the rail
u_1,u_2,u_3	Control efforts
T_1,T_2,T_3	Friction force by unit of mass
N_1,N_2,N_3	Resultant forces by unit of mass

Table 3. Constraints.

state	Initial value	Final value
x_1	0	1
x_2	0	0
x_3	0	1
x_4	0	0
x_5	0.5π	0.5π
x_6	0	0
x_7	0	0
x_8	0	0
x_9	1	1.5
x_{10}	0	0

Table 4: Formulae

Symbol	Definition
μ_1	$\frac{m_c}{m_w}$
μ_2	$\frac{m_c}{m_w + m_s}$
u_1	$\frac{F_x}{m_w}$
u_2	$\frac{F_y}{m_w + m_s}$
u_3	$\frac{F_r}{m_c}$
T_1	$\frac{T_x}{m_w}$
T_2	$\frac{T_y}{m_w + m_s}$
T_3	$\frac{T_r}{m_c}$
N_1	$u_1 - T_1$
N_2	$u_2 - T_2$
N_3	$u_3 - T_3$