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BACKSTEPPING CONTROLLER FOR ATTITUDE CONTROL OF A QUADROTOR UAV

Joana D'arc Dias Costa
Davi A. dos Santos
Instituto Tecnológico de Aeronáutica, DCTA/ITA/IEM, 12228-900, São José dos Campos, SP, Brazil
joana@ita.br, davists@ita.br

Abstract. *This work deals with the attitude control problem for a quadrotor unmanned aerial vehicle (UAV). To solve the problem a backstepping attitude controller was designed using quaternion as attitude parametrization that guarantees the asymptotic stability of the system. The result was a control law similar to a P-D law with an additional nonlinear term in order to eliminate the nonlinear term from the attitude dynamics. The system was simulated with 6 degrees of freedom (3 translational and 3 rotational) and a P-D controller with constraints on the total thrust and on the inclination angle was used for the position control and to provide the desired attitude for the attitude controller. Propeller dynamics and perturbations acting on the quadrotor were modelled to obtain a more realistic result. In order to validate the designed controller, simulation tests were made using Monte Carlo simulations and the results were effective for a way point trajectory.*

Keywords: *Attitude control, Backstepping, Quadrotor, Lyapunov*

1. INTRODUCTION

A quadcopter or quadrotor consists of a rigid cross structure equipped with rotors in its ends capable of propelling it. It has two sets of counter-rotative propellers that produce the pitch, roll and yaw movements (Figure 1) and allows vertical take-off and landing (VTOL).

Multirotors, specially quadrotors, have become objects of great interest for commercial as well as educational purposes, due to factors as its capability of hovering and its high stability. Applications include fire fighting, surveillance, environmental reconnaissance and filming.

To guide a quadrotor, a position control (for translational motion) as well as an attitude control (for rotational motion) are needed. Usually, these control laws are separately developed and a cascade configuration is adopted. In this work, the focus will be the attitude control.

The attitude control problem for rigid bodies was widely discussed in the past decades (Wen and Kreutz-Delgado, 1991; Tayebi, 2008), in particular for spacecrafts and satellites (Joshi *et al*, 1995; Kristiansen *et al.*, 2009). Recently, this problem has been studied for applications on multirotors using several control techniques such as $\mathcal{H}_\infty$ control (Raffo *et al.*, 2010), geometric control (Lee *et al.*, 2010), $\mathcal{L}_1$ adaptive control (Mallikarjunan *et al.*, 2012) and backstepping (Huo *et al.*, 2014).

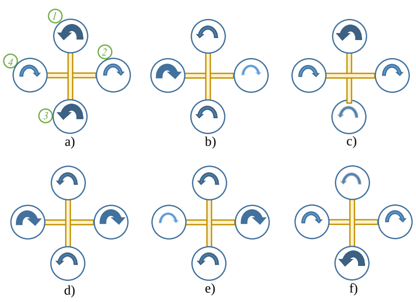

Figure 1. Movements of a quadrotor UAV: a) Yaw (clockwise direction), b) Roll (positive), c) Pitch (up), d) Yaw (anti-clockwise direction), e) Roll (negative), f) Pitch (down).

This work uses backstepping control design, which is a recursive procedure that interlaces the choice of a Lyapunov function with the design of feedback control. It breaks a design problem for the full system into a sequence of design problems for lower order (even scalar) systems, eventually solving stabilization, tracking and robust control problems under conditions less restrictive than those encountered in other methods (Khalil, 1996).

Even using the same control strategy as in (Huo *et al.*, 2014), here the approach of the quaternion parametrization is quite different once an error attitude, called *quaternion error*, is defined. This kind of error also appears in (Fritsch *et al.*, 2012), where a nonlinear controller based on an energy approach is designed in order to align the body-fixed thrust direction with a constant desired direction, and in (Guerrero-Castellanos *et al.*, 2011), where a bounded control law based on nested saturation is developed. Similar application of this concept appears in (Fresk and Nikolakopoulos, 2013), but in this case the attitude control uses only the complex term of the quaternion error.

The present paper proposes a new *backstepping* attitude controller for quadrotor vehicles. The remaining text is organized as follows. Section 2 shortly introduces the quaternion attitude parametrization. Section 3 states the problem and presents the plant model used on the simulations. Section 4 shows the design of the position and attitude controllers. Section 5 presents the simulation results and some considerations. Lastly, Section 6 concludes the work.

2. QUATERNION ATTITUDE PARAMETRIZATION

The attitude control law presented in this work was designed based on the quaternion attitude parametrization. Such parametrization is directly obtained from an axis, $a = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}^T$, and from the rotation angle, Φ, around this axis (Euler axis and angle parametrization).

A quaternion can then describe the rotation from a coordinate system X to a coordinate system Y and is defined as:

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} a_1 \sin \frac{\Phi}{2} \\ a_2 \sin \frac{\Phi}{2} \\ a_3 \sin \frac{\Phi}{2} \\ \cos \frac{\Phi}{2} \end{bmatrix} \tag{1}$$

such that $q^T q = 1$. Additionally, the conjugate or inverse of q, which describes the rotation from a coordinate system Y to a coordinate system X, is defined as:

$$\bar{q} = \begin{bmatrix} -q_1 \\ -q_2 \\ -q_3 \\ q_4 \end{bmatrix} \tag{2}$$

The multiplication of two quaternions, on the other hand, is defined as $q \otimes p = Q(p)q$, with

$$Q(p) = \begin{bmatrix} p_4 & p_3 & -p_2 & p_1 \\ -p_3 & p_4 & p_1 & p_2 \\ p_2 & -p_1 & p_4 & p_3 \\ -p_1 & -p_2 & -p_3 & p_4 \end{bmatrix} \tag{3}$$

which corresponds to the consecutive execution of the rotations parametrized by q and p.

Lastly, the quaternion attitude kinematics is given by $\dot{q} = -0.5 S(q)\omega$, with

$$S(q) = \begin{bmatrix} -q_4 & q_3 & -q_2 \\ -q_3 & -q_4 & q_1 \\ q_2 & -q_1 & -q_4 \\ q_1 & q_2 & q_3 \end{bmatrix} \tag{4}$$

where $\omega = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix}^T$ denotes the angular velocity of X with respect to Y.

3. PROBLEM DESCRIPTION AND PLANT MODEL

Consider the quadrotor illustrated in Figure 2. At this point, three Cartesian coordinate systems are defined: the inertial system $\{x_i, y_i, z_i\}$, the reference system $\{x_r, y_r, z_r\}$ and the body system $\{x_b, y_b, z_b\}$.

The first one is fixed on the ground and its z_i axis points towards the direction of the local vertical. The reference system is aligned with the inertial system and its origin coincides to the mass center of the vehicle. Analogously, the body system is centered in the mass center of the quadrotor, but its axis x_b and y_b are on the plane of the rotors as illustrated in Figure 2, with x_b pointing towards the rotor number 1.

The quadrotor attitude is defined as the orientation of the body coordinate system relative to the reference coordinate system. As mentioned earlier, the attitude parametrization adopted in this work will be quaternion due to its convenient product rule for successive rotations and its absence of trigonometric functions and singularities (Wertz, 1978).

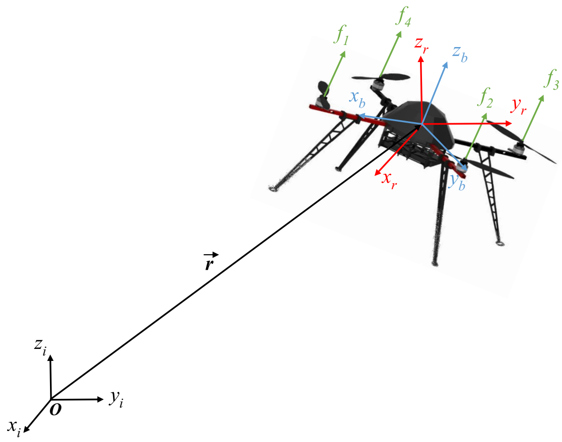

Figure 2. Quadrotor and the Cartesian coordinate systems adopted.

Thereby, the error attitude of the quadrotor is here defined as:

$$e = q_b \otimes \overline{q_d} \tag{5}$$

where q_b describes the rotation from the body system to the reference system, and $\overline{q_d}$ describes the rotation from the reference system to the desired system. Note that e is also a quaternion.

So, the problem consists in obtaining a control law in the form:

$$\tau_b = f(e, \omega_b) \tag{6}$$

where $\tau_b \in \mathbb{R}^3$ is the control moment in the body-fixed frame and such that $e \to \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$ and $\omega_b \to 0$ as $t \to \infty$.

It's important to notice that the problem could consist to do $e \to \begin{bmatrix} 0 & 0 & 0 & -1 \end{bmatrix}^T$, since this attitude corresponds to $\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$. However, this equilibrium point corresponds to an unstable point and the proof can be found in (Wen and Kreutz-Delgado, 1991).

Additionally, to help on the evaluation of the attitude tracking, the following error function is defined:

$$\Phi(D_d, D) = \arccos\left(\frac{1}{2}[tr(D_d^T D) - 1]\right) \tag{7}$$

where D denotes the Direction Cosine Matrix (DCM) representing the actual attitude of the vehicle (from the body frame to the reference frame) and D_d denotes the desired rotation matrix (from the desired frame to the reference frame).

The kinematics and dynamics equations of motion (translational and rotational) of the quadrotor can therefore be written as:

$$\dot{r} = v \tag{8}$$

$$m\dot{v} = fD^T\hat{k} - mg + D^T F_p \tag{9}$$

$$\dot{q_b} = -0.5S(q_b)\omega_b \tag{10}$$

$$\tau_b + \tau_p = J\dot{\omega_b} + \omega_b \times J\omega_b \tag{11}$$

where:

$r \in \mathbb{R}^3$	position of the quadrotor center of mass in the inertial system
$v \in \mathbb{R}^3$	velocity of the quadrotor center of mass in the inertial system
$m \in \mathbb{R}$	quadrotor mass
$f \in \mathbb{R}$	quadrotor total thrust
$\hat{k} \in \mathbb{R}^3$	unitary vector along the z axis $= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$
$g \in \mathbb{R}^3$	gravitational acceleration
$F_p \in \mathbb{R}^3$	total disturbance force acting on the quadrotor in the body-fixed frame
$\tau_p \in \mathbb{R}^3$	total disturbance torque acting on the quadrotor in the body-fixed frame
$J \in \mathbb{R}^{3\times3}$	inertia matrix

The total thrust f, that is, the sum of the thrust generated by each propeller, $f_i, i \in \{1, 2, 3, 4\}$ is the other control input to the system considered in this work.

Moreover, in this paper disturbance force and torque are also modelled to prove the controller robustness. The model is based on the Gauss-Markov model and is represented in the body system as:

$$\dot{F}_p(t) + \beta_F F_p(t) = w_F(t) \tag{12}$$
$$\dot{\tau}_p(t) + \beta_\tau \tau_p(t) = w_\tau(t) \tag{13}$$

where β_F and β_τ are constant parameters, and w_F and w_τ are zero-mean white Gaussian noises with covariances $\alpha_F I_{3\times3}$ and $\alpha_\tau I_{3\times3}$, respectively, with α_F and α_τ constant parameters.

Considering that the commands sent to the vehicle are the velocities in each rotor, it's necessary to convert the control inputs (τ_b and f) into these quantities. For this, it's considered that the thrust, $f_i, i \in \{1,2,3,4\}$, and the reaction torque, $\tau_i, i \in \{1,2,3,4\}$, generated by each propeller are proportional to the square of the angular velocity and the motor dynamics is modelled as a first order system. The equations modelling this behavior are showing bellow:

$$f_i = k_f \omega^2 \tag{14}$$
$$\tau_i = k_\tau \omega^2 \tag{15}$$
$$\frac{\Omega(s)}{\overline{\Omega}(s)} = \frac{k_m}{\tau_m s + 1} \tag{16}$$
$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} 1/4 & 0 & -1/2l & 1/4k \\ 1/4 & -1/2l & 0 & -1/4k \\ 1/4 & 0 & -1/2l & 1/4k \\ 1/4 & -1/2l & 0 & -1/4k \end{bmatrix} \begin{bmatrix} f \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \tag{17}$$

where k_f and k_τ are rotor constants (depending on the air density, the propeller geometry, the angle of attack and the air flow regime), ω is the propeller angular velocity, $\Omega(s) = \mathcal{L}\{\omega(t)\}$ is the Laplace transform of the angular velocity, $\overline{\Omega}(s) = \mathcal{L}\{\overline{\omega}(t)\}$ is the Laplace transform of the commanded angular velocity for the motor, k_m and τ_m are motor constants, l is the distance between the quadrotor center of mass to each rotor and $k = k_\tau / k_f$.

4. CONTROLLER DESIGN

This section intend to develop a controller capable of following a prescribed trajectory, $r_d(t)$, such that the quadrotor preserves a desired attitude, D_d, based on the direction of the desired *heading*, ψ, which is the orientation relative to the rotor plane.

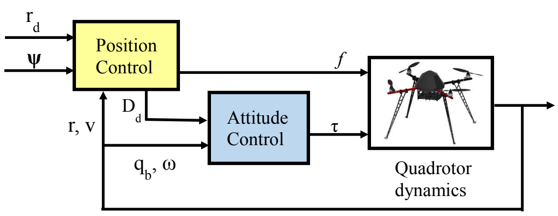

Figure 3. Block diagram showing the controller structure.

4.1 Position Control

The control input provided to the vehicle from the position control is the total thrust, f, as can be seen in Figure 3. Defining the unitary vector normal to the rotor plane as:

$$n = \begin{bmatrix} n_x & n_y & n_z \end{bmatrix}^T \tag{18}$$

it's possible to obtain the P-D control law with constraints on the total thrust as:

$$f = \begin{cases} f_{min}, & \gamma_z < f_{min} \\ \gamma_z, & f_{min} < \gamma_z < f_{max} \\ f_{max}, & \gamma_z > f_{max} \end{cases} \tag{19}$$

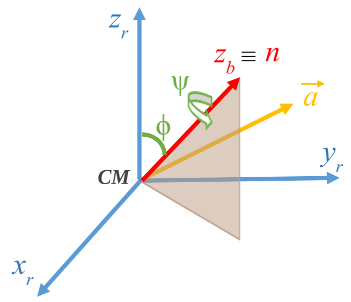

Figure 4. Illustrative scheme of vectors and angles used on the attitude reference generation.

where:

$$\gamma_z = \frac{m}{n_z}(g - k_1(r_z - r_{d,z}) - k_2 v_z) \tag{20}$$

and k_1, k_2 are controller constants.

The other components of the vector n are obtained as:

$$n_{xy} = \begin{bmatrix} n_x \\ n_y \end{bmatrix} = \begin{cases} \frac{\gamma_{xy}}{\|\gamma_{xy}\|} \sin\phi_{max}, & \|\gamma_{xy}\| > \sin\phi_{max} \\ \gamma_{xy}, & \|\gamma_{xy}\| \leq \sin\phi_{max} \end{cases} \tag{21}$$

where:

$$\gamma_{xy} = \frac{m}{f}(-k_3(r_{xy} - r_{d,xy}) - k_4 v_{xy}) \tag{22}$$

and k_3, k_4 are controller constants, and $r_{xy} = \begin{bmatrix} r_x & r_y \end{bmatrix}^T$, $v_{xy} = \begin{bmatrix} v_x & v_y \end{bmatrix}^T$.

Thus the equations above are used on the horizontal position control with constraints on the inclination angle. Additionally, the vector a and the angle ϕ can be obtained from:

$$a = \hat{k} \times n \tag{23}$$

$$\phi = \arccos n_z \tag{24}$$

So the desired attitude matrix provided to the attitude controller is given by:

$$D_d = D_3(\psi) D_a(\phi) \tag{25}$$

where $D_3(\psi)$ indicates the rotation matrix from the rotation of ψ towards the body z axis and $D_a(\phi)$ denotes the rotation matrix from the rotation of ϕ towards the vector a.

For more details about this controller, see (Santos and Saotome, 2013).

4.2 Attitude Control

As mentioned earlier, the backstepping control design uses Lyapunov functions breaking the problem in smaller ones that can be solved more easily. So, the first step to stabilize the vehicle attitude is to define a function containing just the quaternion error:

$$V_1 = \frac{1}{2} z_1^T z_1 \tag{26}$$

where $z_1 = \begin{bmatrix} e_1 & e_2 & e_3 & 1 - e_4 \end{bmatrix}^T$ is the first modified state vector and V_1 is a positive-definite function of z_1.

The time derivative of this candidate function is then given by:

$$\dot{V}_1 = z_1^T \dot{z}_1 = -0.5 z_1^T Q(\overline{q_d}) P(q_b) \omega_b + z_1^T (q_b \otimes \dot{\overline{q_d}}) \tag{27}$$

where:

$$P(q_b) = \begin{bmatrix} -q_{b4} & q_{b3} & -q_{b2} \\ -q_{b3} & -q_{b4} & q_{b1} \\ q_{b2} & -q_{b1} & -q_{b4} \\ -q_{b1} & -q_{b2} & -q_{b3} \end{bmatrix} \tag{28}$$

Here it is considered that there are small variations on the desired attitude of the quadrotor, in other words, $\dot{\overline{q_d}} \simeq 0$. Thus, choosing the new state as $z_2 = \omega_b - \alpha_1$, results:

$$\begin{aligned}\dot{V}_1 &= -0.5 z_1^T Q(\overline{q_d}) P(q_b) \omega_b \\ &= -0.5 z_1^T Q(\overline{q_d}) P(q_b) z_2 - 0.5 z_1^T Q(\overline{q_d}) P(q_b) \alpha_1\end{aligned} \tag{29}$$

Thus, designing the virtual control $\alpha_1 = k_5 P(q_b)^T Q(\overline{q_d})^T z_1$, with $k_5 > 0$:

$$\dot{V}_1 = -0.5 z_1^T Q(\overline{q_d}) P(q_b) z_2 - 0.5 k_5 \underbrace{z_1^T Q(\overline{q_d}) P(q_b) P(q_b)^T Q(\overline{q_d})^T z_1}_{\|z_1^T Q P\|^2 = \|P^T Q^T z_1\|^2} \tag{30}$$

So, in order to guarantee system stability ($\dot{V}_1 \leq 0$) , it is necessary to have $z_2 = 0$.
At this time, the second Lyapunov candidate is chosen to be:

$$V_2 = \frac{1}{2} z_1^T z_1 + \frac{1}{2} z_2^T J z_2 \tag{31}$$

where J is the quadrotor inertia matrix.

Once J is symmetric positive-definite, the new candidate, V_2, is also positive-definite. Knowing the nonlinear plant dynamic is governed by the equation $\tau_b = J\dot{\omega_b} + \omega_b \times (J\omega_b)$, the time derivative of V_2 is then given by:

$$\begin{aligned}\dot{V}_2 &= z_1^T \dot{z_1} + z_2^T J \dot{z_2} = z_1^T \dot{z_1} + z_2^T J(\dot{\omega_b} - \dot{\alpha_1}) \\ &= -0.5 k_5 \|P(q_b)^T Q(\overline{q_d})^T z_1\|^2 + z_2^T [(J\omega_b) \times \omega_b + \tau_b - J\dot{\alpha_1} - 0.5 P(q_b)^T Q(\overline{q_d})^T z_1]\end{aligned} \tag{32}$$

Replacing $\dot{\alpha_1} = k_5 \dot{P}(q_b)^T Q(\overline{q_d})^T z_1 + k_5 P(q_b)^T \dot{Q}(\overline{q_d})^T z_1 + k_5 P(q_b)^T Q(\overline{q_d})^T \dot{z_1}$ in equation 32, it can be concluded that a control torque which stabilizes the quadrotor is given by:

$$\begin{aligned}\tau_b = \omega_b \times (J\omega_b) &+ [(0.5 + k_5 k_6) P(q_b)^T Q(\overline{q_d})^T + k_5 J \dot{P}(q_b)^T Q(\overline{q_d})^T] z_1 - [k_6 I_{3\times3} \\ &+ 0.5 k_5 J P(q_b)^T Q(\overline{q_d})^T Q(\overline{q_d}) P(q_b)] \omega_b\end{aligned} \tag{33}$$

with $k_6 > 0$, $I_{3\times3}$ the identity matrix and P, Q and S are given by Equations 28, 3 and 4, respectively.

5. SIMULATION RESULTS

This section shows the simulation results that corroborates the effectiveness of the designed controllers. Table 1 and Table 2 show the quadrotor and simulation parameters used, respectively.

In all simulations, was used the following parameters for the position control: f_{min} = 2N, f_{max} = 20N, $\psi = 0°$ and ϕ_{max} = 25°. Also, the quadrotor had to follow a way point trajectory with different linear velocities (v = 0.5, 1 and 2m/s). The points of the trajectory are: $P_1 = (0, 0, 0)$, $P_2 = (0, 0, 8)$, $P_3 = (8, 0, 8)$, $P_4 = (8, 8, 8)$ and $P_5 = (8, 8, 0)$. Moreover, 100 Monte-Carlo simulations were performed for each desired velocity.

Additionally, the angular velocity of each rotor is set to be between 0 and 7000 rpm. Also a modelling error was taken into account through the variation of $\pm 1\%$ on the inertia matrix and the thrust, k_f, and torque, k_τ, coefficients. Finally, measurement errors were simulated adding zero-mean white Gaussian noise as well as bias to the attitude states in the output of the quadrotor dynamics block.

Figure 5 shows the mean of the Euler angle function (Equation 3) and three times the standard deviation for the 100 realizations for each velocity. Analysing this figure is possible to notice that as the linear velocity increase, the attitude controller has difficulty to stabilize the quadrotor attitude, specially near the sudden changes of trajectory.

Furthermore, one can note, evaluating Figure 6, that the deviation from the prescribed path is augmented as the velocity increase. Actually, there is a time delay for the vehicle to reach the desired point once the position control does not prioritize reaching the desired velocity because it does not compute the velocity error in the control law.

6. CONCLUSION

This paper presents a novel attitude control in combination with a pre existent position control. To approximate the simulation from reality, disturbances, propeller dynamics, saturation, measurement noise and modelling errors were taken into account. Simulation showed satisfactory results such the robustness of the controller and its asymptotic stability. Thus, future work includes experimental tests for recreate this result and deal with some other aspects such as delay and sampling issues provoked by the use of a digital microprocessor.

Symbol	Description	Value	Unit
l	Distance between the quadrotor center of mass to each motor	0.277	m
m	Quadrotor mass	1.200	kg
J_{xx}	Equivalent moment of inertia about the x axis	22.595×10^{-3}	kg.m^2
J_{yy}	Equivalent moment of inertia about the y axis	22.185×10^{-3}	kg.m^2
J_{zz}	Equivalent moment of inertia about the z axis	37.615×10^{-3}	kg.m^2
J_{xy}	Equivalent product of inertia about the x and y axes	0.284×10^{-6}	kg.m^2
J_{xz}	Equivalent product of inertia about the x and z axes	-8.114×10^{-6}	kg.m^2
J_{yz}	Equivalent product of inertia about the y and z axes	-70.249×10^{-6}	kg.m^2
k_f	Thrust coefficient	1.57×10^{-5}	N.s^2
k_τ	Torque coefficient	7.60×10^{-6}	N.m.s^2

Table 1. GYRO-200ED-X4 parameters

Symbol	Description	Value	Unit
β_F	Disturbance force parameter	3×10^{-1}	s$^-$1
β_τ	Disturbance torque parameter	3×10^{-1}	s$^-$1
α_F	Parameter of the covariance disturbance force noise	1×10^{-22}	N^2/s^2
α_τ	Parameter of the covariance disturbance torque noise	1×10^{-24}	N^2.m^2/s^2
k_1	Proportional constant of the altitude control	60	s^{-2}
k_2	Derivative constant of the altitude control	30	s^{-1}
k_3	Proportional constant of the horizontal control	20	s^{-2}
k_4	Derivative constant of the horizontal control	10	s^{-1}
k_5	First constant of the attitude control	12	-
k_6	Second constant of the attitude control	4	-
-	Bias on the angular velocity sensor	0.02	rad/s
-	Noise variance on the angular velocity sensor	0.005	rad^2/s^2
-	Bias on the Euler angles sensor	0.01	degrees
-	Noise variance on the Euler angles sensor	0.001	degrees^2

Table 2. Simulation parameters

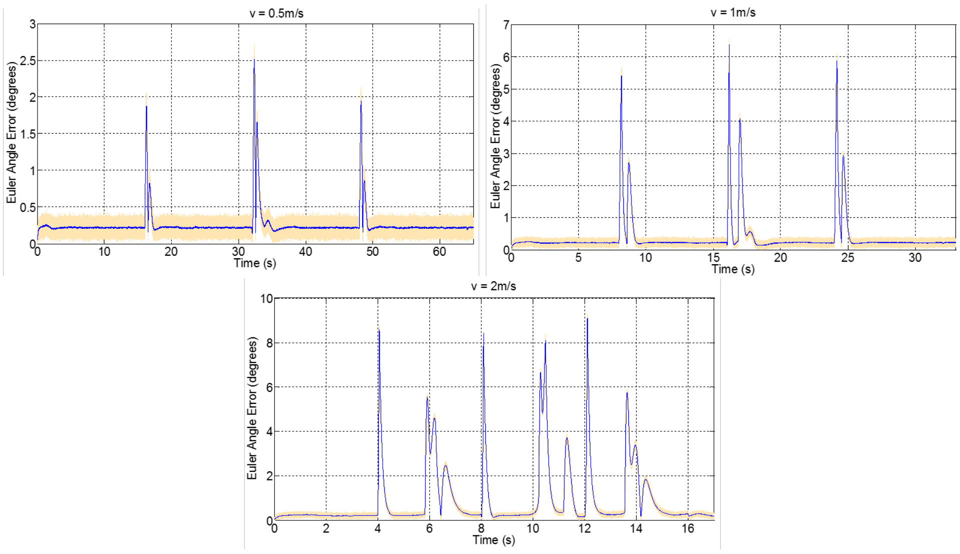

Figure 5. Euler angle function for 100 Monte Carlo simulations for each quadrotor velocity. The blue line corresponds to the mean of all realizations.

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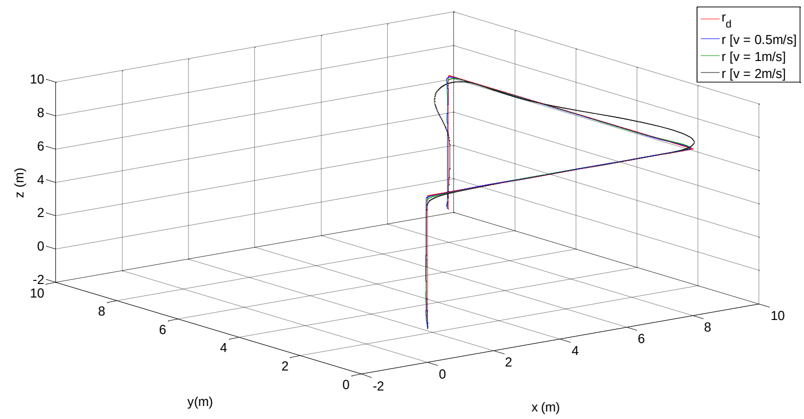

Figure 6. System response to the way point trajectory.

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