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ENERGY HARVESTING SYSTEM IN THE SUSPENDED PNEUMATIC PENDULUM DEVICE EXCITED BY NON-IDEAL ENERGY SOURCE

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Abstract. *Numerical investigations on the parameters of an inverted pendulum based system, designed to harvest energy from the ambient are presented. The system is composed by a pendulum, a main body (the platform) connected to a serial spring and damper. The platform is excited by an EDS (Electro-Dynamical Shaker) excitation device, and for power generation two pneumatic pistons are introduced in the system in order to convert the oscillatory movement of the pendulum into compressed air. Pistons parameters were determined in order to obtain the highest kinetic energy generated by the movement of the pistons. The system presents nonlinearities in both, electrical (shaker system) and mechanical (torsional spring of the pendulum) parts. The results of the numerical investigation shows that these nonlinear parameters have direct effect, on energy harvesting as well as on the dynamic behavior of the system*.

Keywords: *Mechanical Vibrations, Nonlinear Systems, Energy Harvesting, Numerical Investigation*.

1. INTRODUCTION

In times of energy shortage, associated to a massive consumption and deep exploitation of oil and its derivatives, a solution to the global energy crisis would be to identify and make use of new energy sources. Then, the importance of the energy harvesting concept has been growing gradually and, therefore, is related to various applications. Many of these applications are related to devices that usually try exploiting thermal, kinetic, vibrational and electromagnetic energy.

The use of vibrational energy provided by pendulum systems can be a good application in energy research captured by the tidal movement. Energy extraction from a base excited pendulum has been studied by several authors, and the dynamic behavior of the damped vertically excited pendulum has been studied extensively (Xu et. al. 2005; Xu and Wiercigroch 2007; Horton et. al. 2008 and Lenci et. al. 2008). The optimization of harvested energy by a parametrically excited pendulum was considered using the control of damping and variations of the excitation source (Nandakumar et. al. 2012).

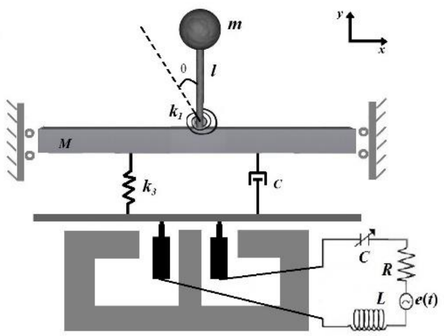
A proposed approach, Wiercigroch (2005), is to extract energy by converting vertical oscillations into rotary motion using a pendulum. Another described proposed approach, Ogai et. al. (2010), is to use a system to generate compressed air using a pendulum oscillation energy converter. The principle of a pendulum oscillation energy converter was introduced, Lin et. al. (2013), and the designs of the dual-medium pressurizer and the dual-stroke hydraulic system are emphasized. An experimental apparatus has been built to simulate the production of energy from pendulum oscillations (Lenci and Rega 2011). Pendulum systems have also been used as a passive control of electro-mechanical systems, Iliuk et. al. 2013, and for energy harvesting (Wiercigroch 2005; Ogai et. al. 2010 and Lin et. al. 2013). The results for the latter application showed a greater uptake of energy for the cases where the behavior is chaotic.

In this work, the inverted pendulum dynamics is analyzed regarding its ability to generate energy through the oscillations of its platform. The excitation source considered in this system is a non-ideal one, represented by an EDS (Xu 2005 and Xu et. al. 2007). To generate energy through the movements of the pendulum, two pneumatic pistons are introduced in the system with the goal of storing compressed air to generate electricity. This strategy is similar to the one proposed by Ogai et. al. (2010). In order to obtain the optimal compressed air generation (Nandakumar et. al. 2012 and Iliuk et al. 2013), the system parameters were analyzed via numerical simulations.

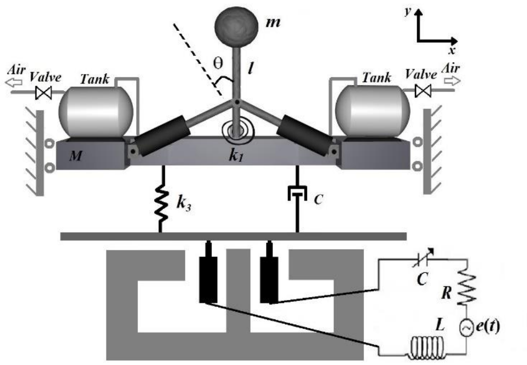
The rest of this paper is organized as follows. Section 2 contains the mathematical modelling. In Section 3 numerical simulations necessary to analyze the dynamics of the system are performed. Section 4 contains the proposed and implemented strategy for generating energy. Conclusions are given in Section 5.

2. MATHEMATICAL MODELING AND DERIVATION OF THE DYNAMIC EQUATIONS

In this work, we consider two pendulum systems excited via an electro-dynamical shaker (EDS) (Figures 1.a and 1.b), in which the shaker was considered as a RLC circuit with a sinusoidal voltage input, as illustrated below. Figure 1.b shows a schematic model of the proposed system to generate pressurized air using the pendulum oscillations. The strategy proposed in this work attempting to harvest energy is similar to that used by [8], and consists of coupling two pneumatic pistons to a pendulum. The air is pressurized and stored in cylinders for later use, such as in pneumatic motor to generate electricity.



(a)



(b)

Figure 1. (a) Pendulum system excited by electro-dynamical shaker. (b) Pendulum system excited by electro-dynamical shaker with pneumatic pistons.

Analyzing the mathematical model presented in (Fig. 1.a), we observe that it can be described as an inverted pendulum, fixed to a platform, which is represented by a rigid body of mass M. The pendulum is connected to a torsional spring, which provides a static unstable equilibrium at the upright position, so that even small oscillation amplitudes of its base would be enough to result in pendulum motion, oscillating or even rotating movements. The pendulum has mass m, connected to the platform with a rod of length l; θ is the angle of the pendulum, which is stabilized by a nonlinear torsional spring stiffness k_1 is given by $k_1 = k_2 + h\theta$ and a linear damper, c_θ. The spring that connects the Suspended Pendulum Platform (SPP) has stiffness known as k_3, the linear damper of the SPP is c and the excitation source is given by an electrical dynamics shaker (EDS).

The coupling between the SPP and EDS is performed by an electromagnetic force due to the permanent magnet, $F_B = K\dot{q}$, in which $K = l_c B$ with l_c and B being the length of the conductor and the magnetic field, respectively. This coupling generates a Laplace force on the mechanical part and the Lenz electromotive voltage in the electrical part. The electrical system consists of a resistor R, an inductor L, a capacitor C and a source of sinusoidal voltage $e(t) = e_0 \cos(\Omega\, \mathrm{t})$, all three connected in series, where e_0 and Ω are the amplitude and frequency, respectively and t is time. The nonlinear capacitance C is defined by $C = C_0(1 + \alpha_3 \mathrm{C}_0 \mathrm{q}^2)^{-1}$, in which C_0 is the linear parameter of C and α_3 is the nonlinear coefficient, depending on the type of the capacitor that will be used. In this model, the voltage at the capacitor is a nonlinear function of instantaneous electric charge q, defined as:

$$V_C = \frac{1}{C_0} q + \alpha_3 q^3 \tag{1}$$

In the EDS' electric circuit, it was made an analogy voltage-force based on Kirchhoff's laws, obtaining the kinect energy *(T)* and the potential energy *(V)* m:

$$T = \frac{1}{2}(\mathrm{M+m})\,\dot{\mathrm{q}}_y^{\,2} + ml\,\dot{\mathrm{q}}_y\,\dot{\theta}\sin(\theta) + \frac{1}{2}ml^2\dot{\theta}^2 + \frac{1}{2}L\dot{q}^2 \tag{2}$$

$$V = (\mathrm{M+m})(-\mathrm{g})\,\mathrm{q}_y - mgl(1-\cos(\theta)) + \frac{1}{2}k_3 q_y^{\,2} + \frac{1}{2}k_2\theta^2 + \frac{1}{2}h\theta^4 + K\dot{q}q_y + \frac{1}{2}\left(\frac{q}{C_0} + \alpha_3 q^3\right)q \tag{3}$$

Using equations (2) and (3), and considering:

$$D = \frac{1}{2}c_\theta\dot{\theta}^2 + \frac{1}{2}c\dot{q}_y^{\,2} + \frac{1}{2}R\dot{q}^2 \tag{4}$$

The dynamic equations of the system are given by:

$$\begin{aligned}\ddot{\mathrm{q}}_y = &-\frac{m^2 l\cos(\theta)\sin^2(\theta)}{\mathrm{Mt}\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\dot{\theta}^2 - \frac{mk_3\sin^2(\theta)}{\mathrm{Mt}\left(\mathrm{Mt}-m\sin^2(\theta)\right)}q_y - \frac{mK\sin^2(\theta)}{\mathrm{Mt}\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\dot{q} - \frac{mc\sin^2(\theta)}{\mathrm{Mt}\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\dot{q}_y + \\ &+\frac{k_2\sin(\theta)}{l\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\theta + \frac{2h\sin(\theta)}{l\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\theta^3 + + \frac{c_\theta\sin(\theta)}{l\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\dot{\theta} - \frac{ml\cos(\theta)}{\mathrm{Mt}}\dot{\theta}^2 + \mathrm{g} - \frac{k_3}{\mathrm{Mt}}q_y - \frac{K}{\mathrm{Mt}}\dot{q} - \frac{c}{\mathrm{Mt}}\dot{q}_y\end{aligned} \tag{5}$$

$$\begin{aligned}\ddot{\theta} = &\frac{m\cos(\theta)\sin(\theta)}{\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\dot{\theta}^2 + \frac{k_3\sin(\theta)}{\left(\mathrm{Mt}-m\sin^2(\theta)\right)l}q_y + \frac{K\sin(\theta)}{\left(\mathrm{Mt}-m\sin^2(\theta)\right)l}\dot{q} + \frac{c\sin(\theta)}{\left(\mathrm{Mt}-m\sin^2(\theta)\right)l}\dot{q}_y + \\ &-\frac{\mathrm{Mt}\,k_2}{ml^2\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\theta - \frac{2\,\mathrm{Mt}\,h}{ml^2\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\theta^3 - \frac{\mathrm{Mt}\,c_\theta}{ml^2\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\dot{\theta}\end{aligned} \tag{6}$$

$$\ddot{q} = \frac{e_0}{L}\cos(\Omega\,\mathrm{t}) + \frac{K}{L}\dot{q}_y - \frac{q}{LC_0} - \frac{2\alpha_3}{L}q^3 - \frac{R}{L}\dot{q} \tag{7}$$

3. NUMERICAL SIMULATIONS

The dynamical behavior of both systems is analyzed through numerical simulations using a 4th order Runge Kutta solver (ODE45), and the algorithms were made in Matlab®. The analyzes occurred through Phase-Plane; Time domain response of the system; FFT; Bifurcation Diagrams and Poincaré Section. We consider 10^3 seconds of integration time on both systems and the 25% final vectors were considered the steady state of the system.

3.1 Pendulum system excited by electro-dynamical shaker without the pneumatic piston

The values of parameters used for the model illustrated in figure 1-(a), and the system of equations (5) are: $M = 6kg$; $m = 1.2kg$; $g = 9.81m/s^2$; $l = 0.5m$; $k_2 = 4Nm/rad$; $h = 9Nm/rad$; $k_3 = 10N/m$; $c = 0.003Ns/m$;

$c_\theta = 0.001 Ns/rad$; $\alpha_3 = 374.5$; $\Omega = 1.1 rad/s$; $e_0 = 18V$; $C_0 = 5.7F$; $K = 12.6 N/A$; $L = 0.002626H$; $R = 0.3\Omega$.

In figure 2, we can observe the dynamic behavior of this system.

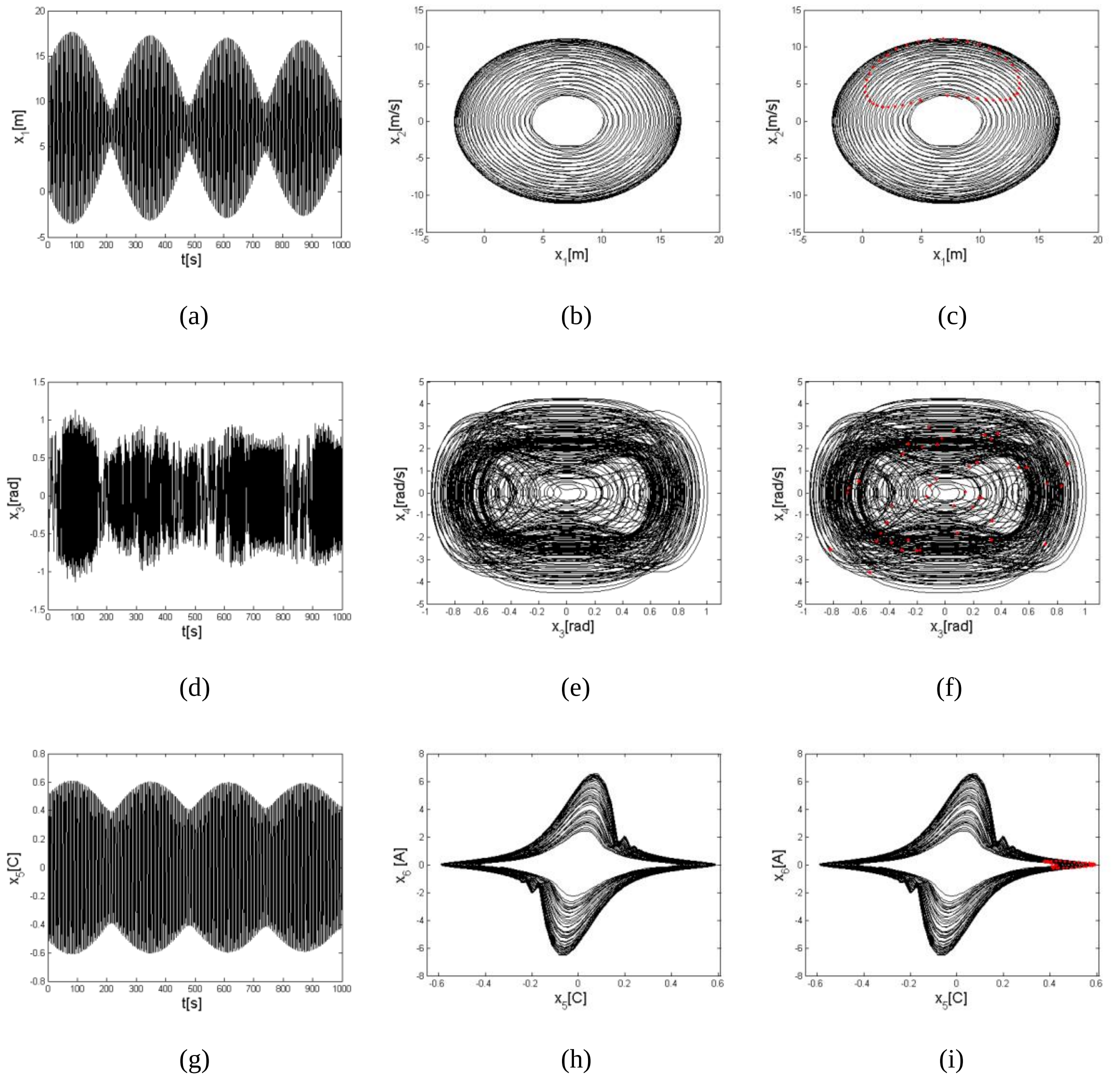
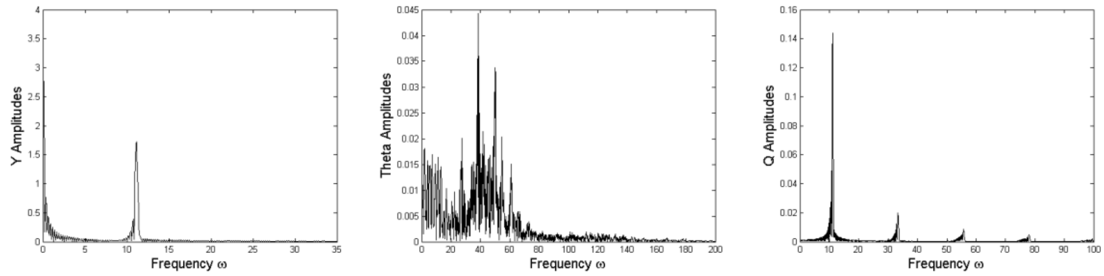


Figure 2. (a) Time response of the vertical displacement. (b) Phase plane of the vertical displacement. (c) Poincaré Map of the vertical displacement (d) Time response of the angular displacement. (e) Phase plane of the angular displacement. (f) Poincaré Map of the angular displacement. (g) Time response of the instantaneous electric charge. (h) Phase plane of the instantaneous electric charge. (i) Poincaré Map of the instantaneous electric charge.

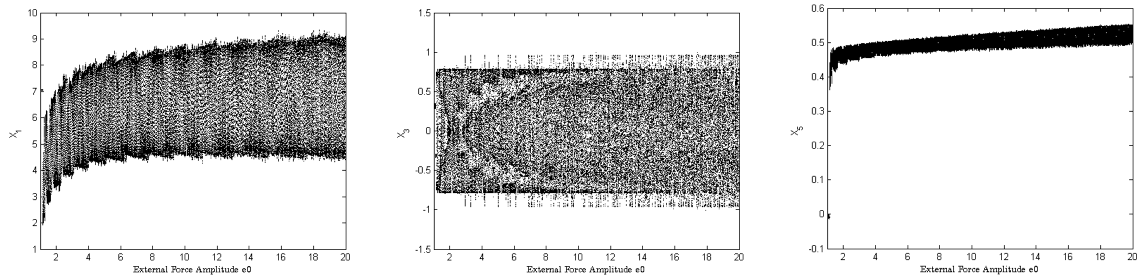
In Fig. 2 we can observe that the system has a quasiperiodic behavior in both SPP and EDS parts of the system, illustrated by Poincaré maps in red, see (Fig. 2-c and Fig.2-i), illustrating a single quasiperiodic attractor for the vertical displacement and the electric circuit of the system, respectively. As can be seen in Fig.2-f the angular displacement of the pendulum presents a chaotic behavior. In Fig. 3 we can observe the FFTs confirming chaotic behavior of the pendulum system, Fig.3-b, and the quasiperiodic behavior of the SPP and the EDS, Fig.3-a and Fig.3-c.



(a) (b) (c)

Figure 3.(a) FFT of the vertical displacement. (b) FFT of the angular displacement. (c) FFT of the instantaneous electric charge.

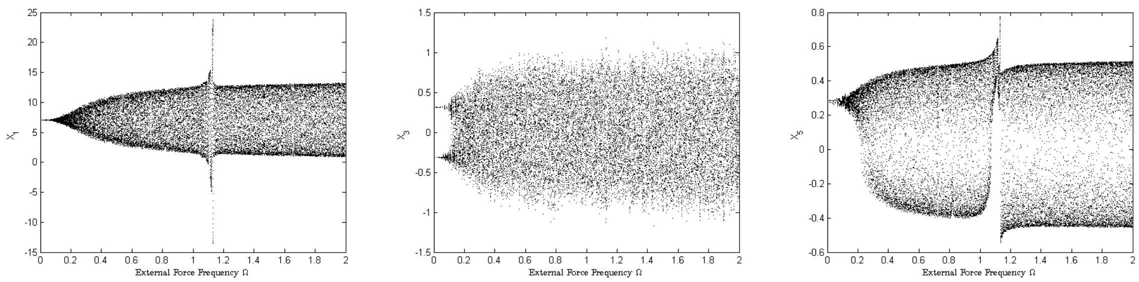
As can be seen in Fig. 4, when the amplitude of the voltage source of the shaker evolve from e_0 =1 V to 20 V the system maintains a quasiperiodic behavior in both SPP and EDS sub-systems. On the other hand, the pendulum system grows from quasiperiodic solutions, 1 V ~ 3.5 V, to chaotic behavior and back to quasiperiodic solutions intermittently until near 10.5V, then the pendulum system maintain chaotic behave up to 20 V.



(a) (b) (c)

Figure 4. Bifurcation diagram. (a) Vertical displacement to variation (e_0). (b) Angular displacement to variation (e_0). (c) Instantaneous electric charge to variation (e_0).

In Fig. 5, when the frequency of the shaker evolve from $\omega = 0.01$ rad/s to 0.065 rad/s the system shows periodic behavior in vertical displacement (Fig.5-a), then the system grows to quasiperiodic behavior and approximately at $\omega = 1.121$ rad/s evolve to chaotic behavior up to 1.138 rad/s and then back again to quasiperiodicity till the end of the analysis. On the other hand, the vertical displacement (Fig.5-b) starts, $\omega = 0.01$ rad/s, at a double periodic attractor and grows to double quasiperiodic solutions from $\omega = 0.067$ rad/s to $\omega = 0.095$ rad/s, and then evolve to chaotic solutions till the end of the analysis. The instantaneous electric charge of the shaker system (Fig.5-c) shows quasiperiodicity from the beginning to the end of the analysis.



(a) (b) (c)

Figure 5. Bifurcation diagram. (a) Vertical displacement to variation (ω). (b) Angular displacement to variation (ω). (c) Instantaneous electric charge to variation (ω).

4. PROPOSED DESIGN FOR ENERGY HARVESTING AND STORAGE

The strategy proposed in this work is similar to that used by Ogai et. al. (2010), and consists of coupling two pneumatic pistons to a pendulum. The air is pressurized and stored in cylinders for later use, such as in pneumatic motor to generate electricity.

As the pneumatic piston only generates resistance in compression, the dissipative force of the pneumatic system can be represented by:

$$Fp = c_p \cdot \dot{\theta} \tag{8}$$

in which c_p is the viscous damping of the pneumatic damper and $\dot{\theta}$ the angular velocity of the pendulum.

With the objective of determining the most suitable value of c_p, the following objective function is considered:

$$Z = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\tau^2} \tag{9}$$

in which (Z) represents the rms kinetic energy generated by the movement of the pendulum, and τ is the pneumatic piston kinetic energy, which can be calculated by:

$$\tau = \frac{1}{2}c_p\dot{\theta}^2 \tag{10}$$

4.1 Generation and Energy Harvesting System through pneumatic system

Now, analyzing the system illustrated in figure (1.b), it is easy to observe that is the same system studied at first, with the inclusion of coupling pneumatic system, so we have the same equations of kinect and potential energies that we had before in (2) and (3), but it is necessary consider a new Rayleigh's dissipation function for this new system.

$$D = \frac{1}{2}c_\theta\dot{\theta}^2 + \frac{1}{2}c_p\dot{\theta}^2 + \frac{1}{2}c\dot{q}_y^{\,2} + \frac{1}{2}R\dot{q}^2 \tag{11}$$

Considering the equations (2), (3) and (11) we obtain the dynamic equations of the system represented in figure (1.b), as follows:

$$\begin{aligned}\ddot{q}_y = &-\frac{m^2 l\cos(\theta)\sin^2(\theta)}{\mathrm{Mt}\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\dot{\theta}^2 - \frac{mk_3\sin^2(\theta)}{\mathrm{Mt}\left(\mathrm{Mt}-m\sin^2(\theta)\right)}q_y - \frac{mK\sin^2(\theta)}{\mathrm{Mt}\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\dot{q} + \\ &-\frac{mc\sin^2(\theta)}{\mathrm{Mt}\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\dot{q}_y + \frac{k_2\sin(\theta)}{l\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\theta + \frac{2h\sin(\theta)}{l\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\theta^3 + \\ &+\frac{c_\theta\sin(\theta)}{l\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\dot{\theta} + \frac{c_p\sin(\theta)}{l\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\dot{\theta} - \frac{ml\cos(\theta)}{\mathrm{Mt}}\dot{\theta}^2 + \mathrm{g} - \frac{k_3}{\mathrm{Mt}}q_y - \frac{K}{\mathrm{Mt}}\dot{q} - \frac{c}{\mathrm{Mt}}\dot{q}_y\end{aligned} \tag{12}$$

$$\begin{aligned}\ddot{\theta} = &\frac{m\cos(\theta)\sin(\theta)}{\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\dot{\theta}^2 + \frac{k_3\sin(\theta)}{\left(\mathrm{Mt}-m\sin^2(\theta)\right)l}q_y + \frac{K\sin(\theta)}{\left(\mathrm{Mt}-m\sin^2(\theta)\right)l}\dot{q} + \frac{c\sin(\theta)}{\left(\mathrm{Mt}-m\sin^2(\theta)\right)l}\dot{q}_y \\ &-\frac{\mathrm{Mt}\,k_2}{ml^2\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\theta - \frac{2\,\mathrm{Mt}\,h}{ml^2\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\theta^3 - \frac{\mathrm{Mt}\,c_\theta}{ml^2\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\dot{\theta} - \frac{\mathrm{Mt}\,c_p}{ml^2\left(\mathrm{Mt}-m\sin^2(\theta)\right)}\dot{\theta}\end{aligned} \tag{13}$$

$$\ddot{q} = \frac{e_0}{L}\cos(\Omega\, t) + \frac{K}{L}\dot{q}_y - \frac{q}{LC_0} - \frac{2\alpha_3}{L}q^3 - \frac{R}{L}\dot{q} \tag{14}$$

In Fig.6 we can observe the variation of the objective function (7) for a variation of the parameter c_p in the range $0.001 \le c_p \le 10$.

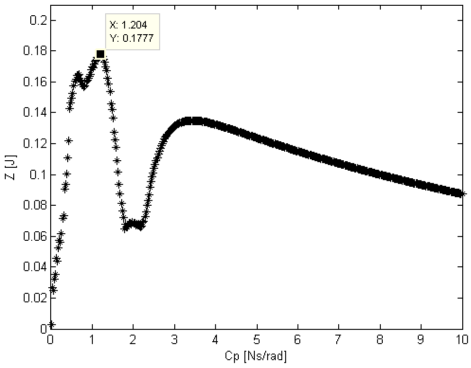
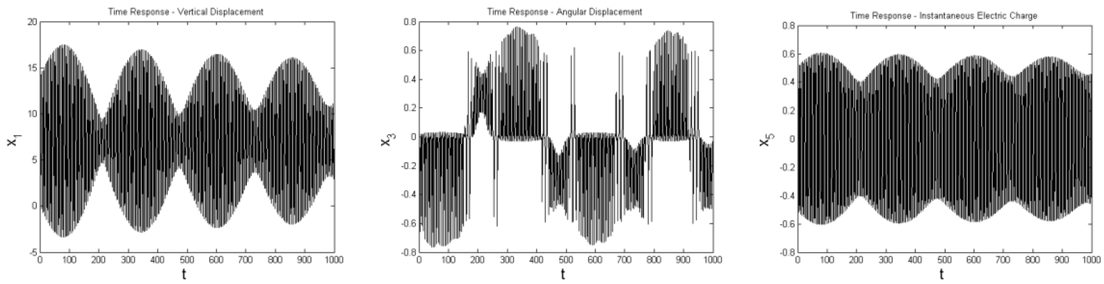


Figure 6.Variation of the value of Z.

It can be seen from Fig. 6 that for a coefficient $c_p = 1.204$ Ns/rad, the maximum value for the objective function (13), ($Z = 0.1777$ J) is obtained.

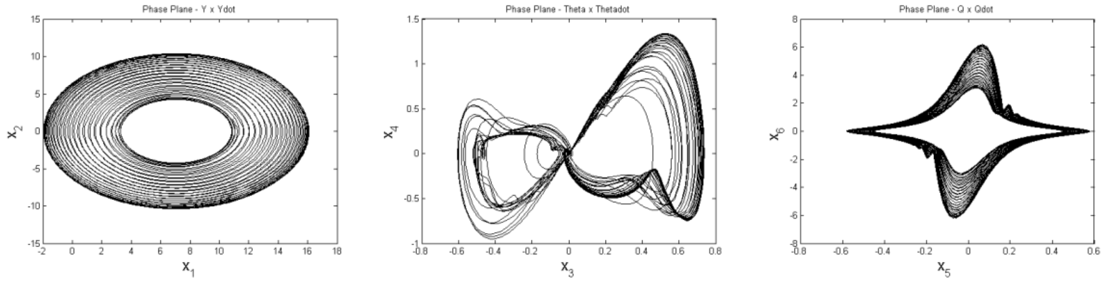
In Figures 7 and 8, we can observe the dynamic behavior of the system given by equations (12), (13) and (14) considering the same parameters of the system illustrated in Fig.1-a, and considering $c_p = 1.204$ Ns/rad.



(a) (b) (c)

Figure 7. (a) Time response of the vertical displacement. (b) Time response of the angular displacement. (c) Time response of the instantaneous electric charge.

According to Figures 7 and 8, we can observe that the system maintains the quasiperiodic behavior for the vertical displacement (7-a; 8-a) and for the instantaneous electric charge of the EDS system (7-c; 8-c). On the other hand, we can observe a strong irregular behavior for the pendulum system illustrated by Figures (7-b) and (8-b).



(a) (b) (c)

Figure 8. (a) Phase plane of the vertical displacement. (b) Phase plane of the angular displacement. (c) Phase plane of the instantaneous electric charge.

5. CONCLUSIONS

The results of the dynamic analysis show that the system has chaotic and quasi-periodic behavior for this non-ideal excitation source (EDS). With the introduction of pneumatic pistons in the system it was possible to verify the possibility to capture and store energy from the pendulum motion. In order for this to occur, the optimal parameter range was determined through the kinetic energy analysis of the pendulum motion. This has been a purely theoretical study, and practical realization of these motions in actual energy generation devices may be taken up in future work.

6. ACKNOWLEDGEMENTS

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