23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

# COMPUTATIONAL TOOL FOR FATIGUE ANALYSIS IN COMPOSITE MATERIAL

**Ricardo de Medeiros**
**Giovani Sacchetto Daniel**
**Marcelo Leite Ribeiro**
**Volnei Tita**
EESC/USP - University of São Paulo, São Carlos School of Engineering, Department of Aeronautical Engineering
Av. João Dagnone, 1100 São Carlos, SP, Brazil
medeiros@sc.usp.br, gdaniel@sc.usp.br, malribei@usp.br, voltita@sc.usp.br

***Abstract.*** *The failure of composite laminates under fatigue loading is usually characterized by a combination of various damage modes such as delamination, matrix cracking, debonding, ply failure, and fiber breakage. These damage modes have been known to cause changes in material properties such as the residual strength, stiffness and life. Thus, it is possible to obtain a measure of the damage in composite laminates by measuring the changes in these properties. The mechanics of fatigue failure of laminated composite materials are completely different from that of conventional materials. The mechanics of fatigue failure of conventional materials involve the initiation and propagation of a single crack, which causes the material to fail. Laminated composite material can arrest the propagating crack to within a single lamina, thus avoiding immediate failure. Only after accumulation of cracks, the laminate fails. The fatigue life of the laminate is best described by S-N curves. Unlike conventional materials, in which failure occurs due to a single crack, in composite materials there is a process of accumulation of many cracks. Therefore, fatigue life is expressed by loading history and residual strength is determined subsequently by the load which will cause eventual failure. Given the aforementioned context, the present study aimed to develop a computational tool that was able to consistently predict both the fatigue life of composite structures like the residual strength of these structures under the action of cyclic loads. Thus, a model based on the residual resistance, which involves numerical integration, is implemented in a computer program called SAFECOM System (Fatigue Analysis of Composite Structures), which is evaluated for some case studies from literature. There is a great convergence between the predicted results by computational tool and published in the literature, and the difference there between is less than 2.0% for most cases.*

***Keywords:*** *fatigue, composite materials, laminates, life prediction, residual strength.*

## 1. INTRODUCTION

Composite materials are widely used in numerous aerospace, automotive, and construction application because of their high specific stiffness and strength, which benefits the material and provides potential energy savings (Mao and Mahadevan, 2002). Some of these applications involve components that are subjected to cyclic loading. Cyclic loading causes damage and material property degradation in a cumulative manner. It is desirable that the composites and the constituents have sound fatigue-resistant properties during the service time of the component and the structure. It is important to evaluate the damage and degraded properties accurately to ensure that the structures operate with high reliability during their lives. Based on the literature, it is well known that the structural fatigue is a process of damage accumulation and subsequent failure of a component under load cyclic (repetitive), which the maximum stress values are below the material ultimate strength (Dowling, 1999). In recent decades, extensive works have been implemented in the fatigue field and damage tolerance of composites reinforced by fibers. During this period, a range of model for such purpose has been proposed and experimentally evaluated.

A consistent literature review on the subject can be found in Degrieck and Paepegem (2001). In this review, the fatigue models have been classified in three major categories. First, fatigue life models, which do not take into account the actual degradation mechanisms but use S-N curves or Goodman-type diagrams and introduces some sort of fatigue failure criterion. Second, phenomenological models for residual stiffness/strength. Third, finally progressive damage models, which use one or more damage variables, related to measurable manifestations of damage (transverse matrix cracks, delamination size). Others important reviews on fatigue behavior of fiber reinforced composites have been published in by Goetchius (1987), Reifsnider (1990), Sendeckyj (1990) and by Saunders and Clark (1993). These papers intend to focus on the existing modeling approaches for the fatigue behavior of fiber-reinforced polymer.

Moreover, in the literature, there are a number of contributions to determine experimentally, computational and/or hybrid fatigue life of structures made of composite material. Since these works have strengths and limitations. Talreja (1981) proposed the classification of the composite damage mechanisms and suggested a fatigue ratio definition, showing the fatigue limit in terms of strain. The analyses were limited to unnotched continuous fiber laminate with polymeric matrix materials. Reifsnider and Jamison (1982) exanimated the complexity of the estimating strength (or residual strength), when pre-fracture damage state is present, and attempts to assess the manner in which pre-fracture fatigue damage affects residual strength and the fracture event. It is found that the large strength reductions observed prior to failure at low load levels can be accounted for by internal stress redistribution and material degradation events. In addition, the damage mechanism of a laminate according to modifies different tracks cyclic amplitude applied which introduces serious difficulties in analysis when the material is subjected to spectral amplitude variable load (Barnard *et al.*, 1985; Daniel and Charewicz, 1986). And, the growth of damage is different for each loading sequences (for example, in blocks), or the load history is important (Hwang and Han, 1986).

Ye (1989) presented correlation in the damage state, stiffness and fatigue life, and thereby developed models for predicting the residual properties of a composite that has undergone a known loading and environmental history. The residual strength and fatigue life (durability) decrease more sharply when the blocks of a loading sequence are small (Farrow, 1989). This phenomenon is known as "Effect of Mixed Cycle" and its consequence is, for the same number of cycles, the same shall laminate residual strength and lower average life when subjected to the sequence with smaller blocks. The effect of the frequency of the fatigue load was assessed rolled fiberglass. In addition, it is concluded that there is a significant influence of this parameter, particularly for laminated less resilient and less rigid, i.e., the response controlled by the polymer matrix. Sendeckyj (1990) addressed the main models and methodologies for the fatigue analysis in composites. However, some errors are usually committed in the development of mathematical models for predicting the durability of composites, for example, misleading generalizations, lack of rigor in the acquisition/data analysis and oversimplifications (Fong, 1982). Irving and Thiagarajan (1998) analyzed of the electrical response of composite laminates to static and fatigue damage. The objective was better definition of electrical changes resulting from damage to the composite. The electrical potential technique was used as a tool to investigate damage mechanisms in composite materials. The changes in resistance have been correlated both with damage mechanisms in the composite and with the test conditions maintained during the mechanical tests.

Post *et al.* (2008) presented a review and discussion of the prior art in experimental research, as well as models for fatigue and residual strength fiber reinforced polymer composites under load variable amplitude. Reifsnider e Majumdar (2011) used a new experimental technique called "Electrical Impedance Spectroscopy" to study the initiation and propagation of a dominant crack, which will take a particular laminate to its ultimate failure under fatigue loading and quasi-static (Fazzino and Reifsnider, 2008). The aim of the study was to obtain accurate interpretations and quantifications of damage evolution in materials with large deformations before rupture. Mao and Mahadevan (2002) developed a mathematical model for fatigue damage evolution in composite materials. The characteristics of damage growth in composite materials are studied and compared with those of damage growth in homogeneous materials. Continuum damage mechanics concepts are used to evaluate the degradation of composite materials under cyclic loading.

Haojie *et al.* (2014) proposed a synergistic damage mechanic model for the stiffness properties. The model included the microcosmic responses of the physical damage and macroscopic performance of the material's stiffness. In micro-level, mesoscopic RVE(representative volume element) model was established to obtain crack opening displacement and crack sliding displacement, which were used to define the damage tensor. In macro- level, through homogenizing the material strain and the surface displacement of the damage, the relationship of the stiffness matrix of unidirectional laminate or laminates in damage statue and damage tense was set up. Devakki and Khan (2015) reviewed nine papers where fatigue analysis has been carried out on different composite materials structures. Fatigue analysis of an aircraft includes determination of fatigue life, damage factor, safety factor, SN curve etc. Lately lot of research has been made on improving composite materials' mechanical properties by the addition of rubber, e-waste. Stens and Middendorf (2015) implemented a simple progressive damage model including strength and stiffness degradation into finite element (FE) software. To reduce computational time, the major part of stress calculations is carried out by classical lamination theory. The simplified model is tested against a reference model using FEA after each load cycle.

Given what has been exposed, among numerous scientific contributions, it was decided in the present work based on the approach suggested by Reifsnider (1986), Case (1996) and by Reifsnider and Case (2002). This choice is due to the fact that this approach uses a mathematical model of cumulative damage versatile enough to represent the residual strength of global laminates. This representation can be explained in terms of information micromechanical (fiber, matrix and interface) or meso-mechanical (blade homogenized). The general formulation of the model can be individualized for any problems involving stress fields uniform or non-uniform, provided they know the failure mode, the function fails the process of accumulation of damage and the effect of this on the distribution of voltages involved. The methodology can be applied for the calculation of fatigue laminate under uniform tension or containing regions of stress concentration as, for example, discontinuities mechanically joined or glued together, provided they have the necessary information. Based on the above observations, the present study aimed to develop a computational tool that was able to consistently predict both the fatigue life of composite structures like the residual strength of these structures under the action of cyclic loads. Thus, a model based on the residual resistance, which involves numerical integration, is implemented in a computer program called SAFECOM System (Fatigue Analysis of Composite Structures) (Daniel, 2011), which is evaluated for some case studies from literature.

## 2. MODEL IMPLEMENTATION

In order to develop the solution of the residual strength integral equation for discontinuous fail functions, it will considered a loading block as described in Case (1996). In addition, the works present by Case (1996), Reifsnider *et. al* (2000) and Reifsnider and Case (2002) showed a great theoretical explanation about the residual strength methodology, however, they do not detail about how the residual strength is integrated and used in a practical case calculation. Therefore, it was necessary to establish an integration strategy in the SAFECOM program (Daniel, 2011). In this work, the methodology can be summarized in the following steps:

- ✓ Fail function – Fa(n):
  - o Step 1: Get the overall external loading $\sigma_a(n)$, which can be constant or in blocks (Fig. 1(a)).
  - o Step 2: Calculate the curve Fa(n) in the Critical Element due to changes in materials and the applied load (Fig. 1(b)).
  - o Step 3: Discretize Fa(n) for the element critic in order to obtain its approach into blocks, the same applied load points (Fig. 1(c)).

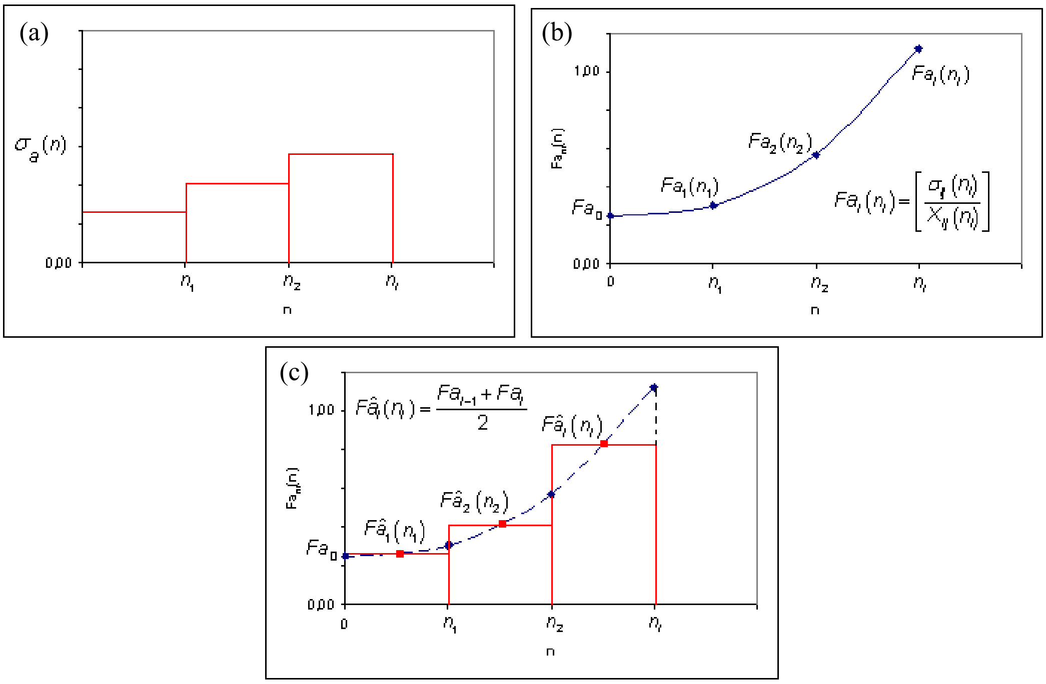


Figure 1. (a) Loading amplitude block (b) Fa evolution in the critical element (c) Fa discretization in the critical element

- ✓ Resistance curve – Fr(n): Once known the discrete form Fa(n), the resistance curve can be determined by direct application of the following method:
  - o The residual resistance curve can be obtained by calculating successively the accumulated damage during each time interval and subtracting the previous value (known) resistance.

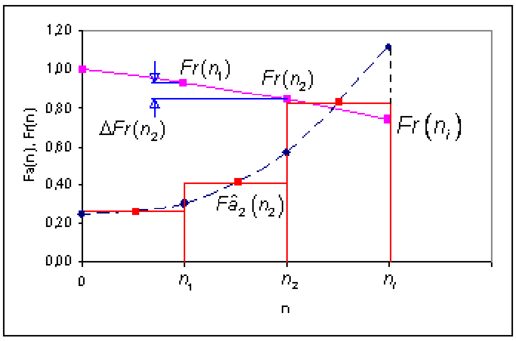


Figure 2. Fr(n) calculation

In the Figure 2, each new Fr(n) point is determined by:

$$Fr\left(n_i\right) = Fr\left(n_{i-1}\right) - \Delta Fr\left(n_i\right) \tag{1}$$

This process is repeated iteratively, refining the solution with each new iteration by reducing the range until reaching the desired convergence.

Considering $\Delta n$ an interval of any number of fatigue cycles (or any range of time), dividing this range into a positive integer K with subintervals equally spaced $(n_{i-1}, n_i)$, with a length $\delta n$ and numbering latter by a positive integer parameter $\lambda$, the Fig. 3 shows this situation.

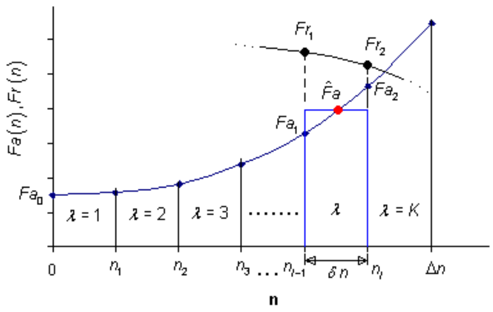


Figure 2. Fr(n) calculation

## 3. SAFECOM: CASE STUDIES

The SAFECOM program is a computer tool that has been developed based on the proposed approach by Reifsnider (1986), Case (1996), Reifsnider and Case (2002). It was implemented in the VBA (Visual Basic for Applications) language (Daniel, 2011). In order to evaluate the potential and limitations of this tool, five case studies were investigated, which will be detailed below.

### 3.1 Case 1: Laminate without notch

This case study considers the fatigue failure of a polymeric laminate without discontinuity and it is detailed by Reifsnider and Case (2002). In addition, the redistribution of internal stresses is not considered, therefore, is the simplest possible case of using the residual strength equation. In this case, failure of the laminate $[45/0/-45/90]_s$ is considered when the layers oriented at 0° fail. Figure 3 shows the variation of residual resistance (Rf) and the fail function (Fa) versus number of cycles (n). The damage is measured by the normalized residual strength at critical element, Fr. Physically, the Fr degradation in this case is due to the progressive increase in the fractured fibers numbers. The fatigue failure occurs when Fr is reduced to a level of maximum stress acting in the normalized critical element, *i.e.*, $F_a = \sigma_1 / X_T$, which in this case is constant as can be seen in the Fig. 3.

Table 1 shows the comparison between the literature results and the results obtained in this study for the Case 2.

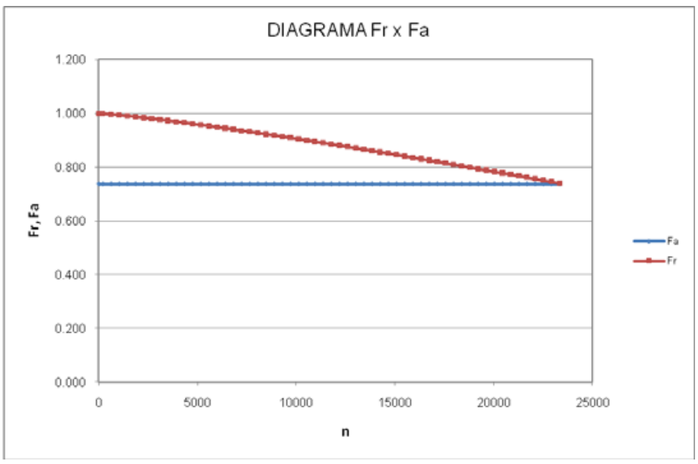


Figure 3. Residual strength and Failure Function *vs.* Number of Cycle: Case 1

Table 1. Results for Case 1.

| Properties | Reifsnider and Case (2002) | Present Work | Relative difference |
|---|---|---|---|
| $\sigma_a$ (MPa) | 600 | 600 | - |
| $F_a$ (GPa) | 0.7377 | 0.7377 | - |
| N (cycles) | 23550 | 23343 | 0.88% |

It is possible to observe in the Tab. 1 that the number of cycles calculated by SAFECOM code is 23343 cycles, and 23550 cycles obtained by Reifsnider and Case (2002). Therefore, there is a difference of 0.88% in the results.

### 3.2 Case 2: Laminate with micro-cracks

This case study considers the same laminate and loading from Case 1 (Reifsnider and Case, 2002), but the fatigue failure takes into account the redistribution of internal stresses caused by the propagation of micro cracks in the matrix of the layers at different angles of 0°. This effect is accounted by degrading the elastic modulus $E_2$ and $G_{12}$.

Figure 4 shows the nonlinear damage evolution represented by the decay resistance of the critical element (layers 0°). In this case, there are two damage modes: the progressive increase in the number of fractured fibers, and the nucleation and growth of micro cracks in the matrix density of the layer at different angles of 0°. These micro cracks imply the reduction of stiffness of these layers $E_2$ and $G_{12}$, which induces a stress transfer to the critical element. This effect can be seen in the Fig. 4 by the gradual growth of the normalized stress (or fail function), Fa.

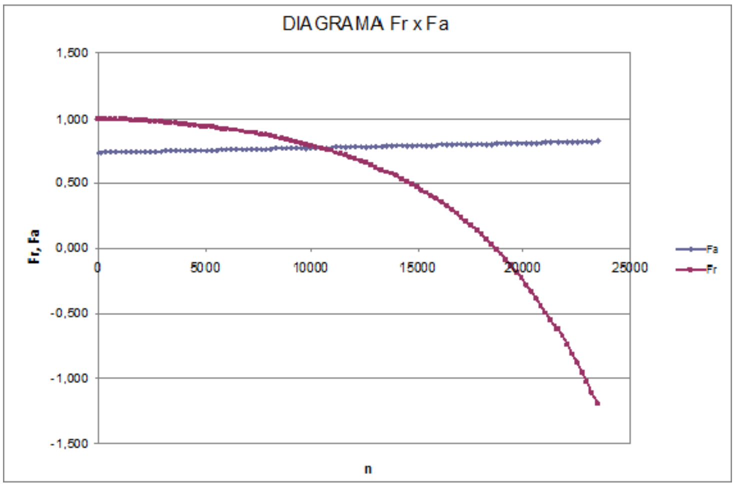


Figure 4. Residual strength and Failure Function *vs.* Number of Cycle: Case 2

Table 2 shows the comparison between the literature results and the results obtained in this study for the Case 2.

Table 1. Results for Case 2.

| Properties | Reifsnider and Case (2002) | Present Work | Relative difference |
|---|---|---|---|
| $\sigma_a$ (MPa) | 600 | 600 | - |
| $F_a$ (GPa) | - | - | - |
| N (cycles) | 10320 | 10480 | 1.55% |

It was observed that the calculated life is 10480 cycles by SAFECOM and 10320 cycles obtained by Reifsnider and Case (2002). Therefore, there is a difference of 1.55% in the results.

### 3.3 Case 3: Laminate without notch

As in Case 1, this study considers the fatigue failure results from polymeric laminate without discontinuity, for a stacking sequence $[0/90]_s$, which were obtained by Curtis (1986). The redistribution of internal stresses is not considered. In this case, fatigue lives were calculated for the thirteen levels of overall axial stress. The dominant damage mechanism is the progressive increase of fractured fibers. For each level of applied axial stress, a pair of Fr and Fa curves was obtained, similarly to Fig. 3. In each case, then, the fatigue life has been obtained as the intersection point of the curves, i.e. the value of n for which Fr = Fa. It is possible to observe in Fig. 5 that very good agreement between the values of life calculated by SAFECOM and literature.

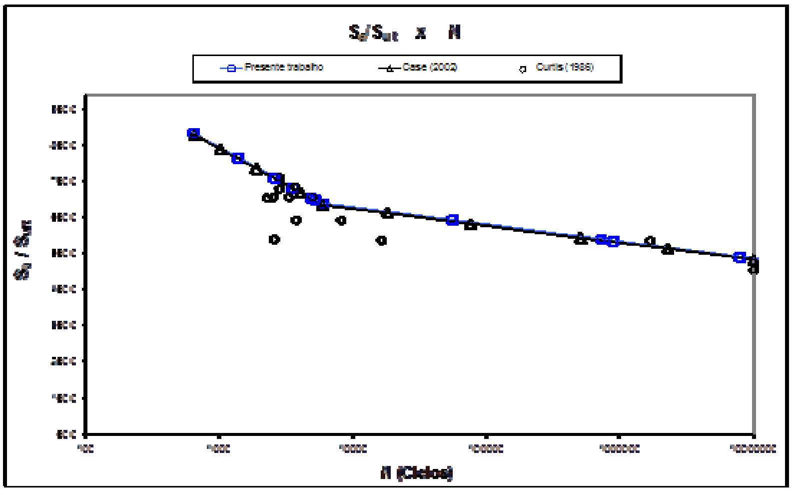


Figure 5. Fatigue curve for the Case 3

### 3.4 Case 4: Laminate with circular notch under compression

This analysis considers a laminate fatigue failure with a central circular hole as data obtained by Simonds and Stinchcomb (1989). In this study was considered the laminate $[(0/45/90/-45)_s]_4$. The fail occurs in the compression process zone defined by the characteristic length b. This parameter is calculated from the static load failure obtained by experimental tests. Therefore, Xn is equal to 289.6 MPa. In the simulation, it was used 30 points and b equal to 4,76mm, while b is equal to 4.52 mm, according to literature data. Fatigue life were calculated for 10 different levels of axial load and the results are shown in Figure 6. For each applied stress level, it was obtained a Fa and Fr diagram similar to the Fig. 3 and Fig. 4, and the life was obtained as the value of "n" at the crossing point of these quantities. The results obtained by SAFECOM show very good agreement with literature values, because there is a maximum variation of 0.81% in the results.

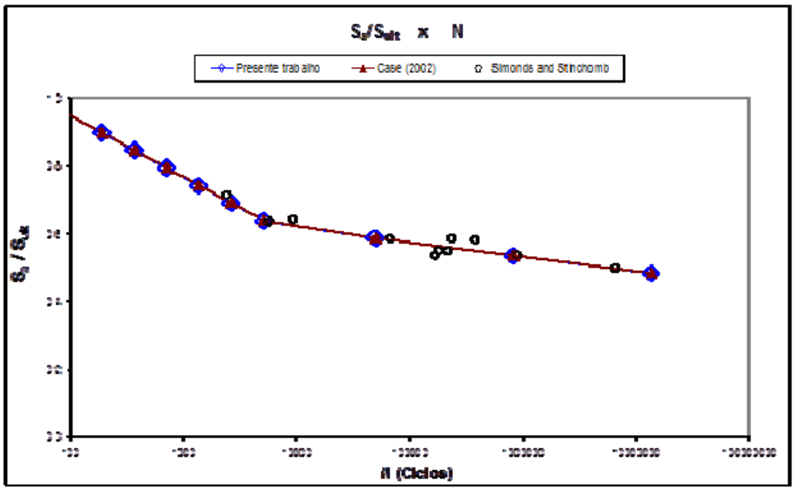


Figure 4. Fatigue curve for the Case 4

### 3.5 Case 5: Laminate with circular notch under traction

This case study considers fatigue data by Vure (1993), who investigated the behavior of a laminate $[45/0/-45/90]_s$ with a central hole under cyclic loading only for traction. In this case, the damage accumulation near the hole is accelerated. Thus, before the occurrence of the fail, this phenomenon will imply a relaxation of stress distribution near of the hole due to the increase of the number of applied load cycles. Case and Reifsnider (2002) evaluate the damage based on the concept of effective ellipticity. With the increasing number of cycles, the first circular hole tends to assume an oval shape. This geometry change is considered in the stress analysis after each applied cycle.

Figure 5 shows a good agreement between the values calculated by SAFECOM and the literature values. However, due to the reduced quantity of available data concerning the relaxation phenomenon of tensions near of the hole, it is concluded that future investigations and improvements in this respect are necessary.

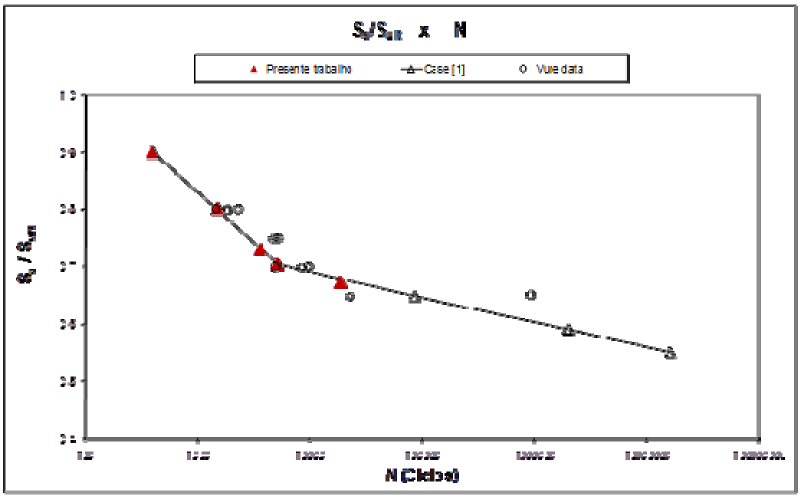


Figure 5. Fatigue curve for the Case 5

## 4. CONCLUSION

In this work a unique computer code, SAFECOM, was developed with the initial goal of implementing the methodology of damage analysis by the residual strength of the critical element. This program employs the integral equation of residual strength in case of polymeric laminates of fibers subjected to cyclic mechanical loads of constant amplitude for the fatigue life determination. With respect to stress concentrator, the problem has been implemented a laminate with a central elliptical hole. In order to evaluate the functionality, case studies were conducted and compared with literature data. The vast majority of cases investigated showed an excellent correlation between the results obtained by SAFECOM with those presented in the literature. However, for a single case, it is necessary more information and any experimental investigations to improve and make additional checks of that program's functionality.

Therefore, based on the results obtained for the case studies we have concluded that the SAFECOM program is an extremely computational strategic tool for life prediction of structures made of composite material. Furthermore, it has validated for some number of case studies, requiring further evaluation for other cases. A very attractive advantage of the SAFECOM is that it allows other important problems can be easily evaluated. To do this, just add the necessary subroutines. So, if you know for a given problem, the Critical Element, its failure mode, the fail function and the mechanisms that can cause an increase (or decrease) the value, it is possible to develop solutions for fatigue analysis of a laminate with a through-crack in a mechanically joined or glued joint, for example. However, it is still necessary to assess the potential and limitations of this computational tool, based on experimental results.

## 5. ACKNOWLEDGEMENTS

The authors are thankful to FAPESP (process number: 2009/00544-5, 2012/01047-8 and 2015/13844-8), as well as, CNPq (process number: 135652/2009-0 and 208137/2012-2) for partially funding the present research work.

## 6. REFERENCES

Barnard, P.M., Butler, R.J. and Curtis P.T., 1985. “Fatigue scatter of UD glass epoxy, a factor or fiction.” In: *Marshall IH (ed), Composite Structures 3, Proc. of 3rd Int. Conf. on Composite Structures (ICCS-3)*, 9–11 September 1985, Scotland, Elsevier, pp. 69–82.

Case, W.S., 1996. “*Mechanics of Fiber-Controlled Behavior in Polymeric Composite Materials*”. PhD Thesis, Virginia Polytechnic Institute and State University.

Curtis, P.T. and Dorey, G. “Fatigue of composite materials.” *Fatigue of Engineering Materials and Structures*, Vol. 2, pp. 297-306.

Daniel, G.S., 2011. "Dano em Materiais Compósitos: SAFECom (Sistema de Análise de Fadiga de Estruturas em Compósito)." Dissertation (Master of Science) – School of Engineering of São Carlos, University of São Paulo, São Carlos, SP, Brazil, (in portuguese).

Daniel, I.M. and Charewicz, A., 1986. "Fatigue damage mechanisms and residual properties of graphite/epoxy laminates". *Engineering Fracture Mechanics*, Vol. 25, pp. 793-808.

Degrieck, J and Van Paepegem, W., 2001. "Fatigue damage modeling of fiber-reinforced composite materials: Review". *Applied Mechanics Reviews*, Vol. 54, no. 4, pp. 279 – 300.

Devakki, B and Khan, I.A., 2015. "Review on fatigue analysis of mechanical structures made up of composite materials." Research and Reviews: Journal of Space Science and Technology, Vol. 4, no. 1, pp. 15-19.

Dowling, N.E., 1999. *Mechanical Behaviour of Materials: Engineering Methods for Deformation, Fracture and Fatigue*. Prentice Hall, 2nd edition.

Farrow, I.R., 1989. "*Damage accumulation and degradation of composite laminates under aircraft service loading assessment and prediction*", Volumes I and II. Cranfield Inst. of Tech., PhD thesis.

Fazzino, P. and Refsnider, K.L., 2008. "Electrochemical impedance spectroscopy detection of damage in out of plane fatigued fiber reinforced composite materials", *Applied Composite Materials*. Vol. 15, pp. 127-138.

Fong, J.T., 1982. "What is fatigue damage", In *K.L. Reifsnider, Damage in composite materials, ASTM STP 775, Am. Soc. For Testing and Materials*, pp. 243-266.

Goetchius, G.M., 1987. "Fatigue of composite materials". In A*dvanced Composite III. Expanding the Technology. Proceedings of the Third Annual Conference on Advanced Composites*, 15-17 September 1987, pp. 289-298.

Haojiea, S., Weixingb, Y. and Yitaoa, W., 2014. "Synergistic damage mechanic model for stiffness properties of early fatigue damage in composite laminates." *Procedia Engineering*, Vol. 74, pp. 199-209.

Hwang, W. and Han, K.S., 1986. "Cumulative damage models and multi-stress fatigue life prediction", *Journal of Composite Materials*, Vol. 20, pp. 125–153.

Irving, P.E. and Thiagarajan, C., 1998. "Fatigue damage characterization in carbon fibre composite materials using an electrical potential technique", Smart Material and Structure, Vol. 7, pp. 456–466.

Mao, H. and Mahadevan, S., 2002. "Fatigue damage modelling of composite materials". *Composite Structures*, Vol. 58, pp. 405 – 410.

Post, N.L., Case, S.W., Lesko, J.J. Modeling the variable amplitude fatigue of composite materials: A review and evaluation of the state of the art for spectrum loading. *International Journal of Fatigue* 30, 2064-2086, 2008.

Reifsnider, K.L. and Jamison, R., 1982. "Fracture of fatigue-loaded composite laminates". International Journal of Fatigue, Vol. 4, no. 4, pp. 187–197.

Reifsnider, K.L., 1986. "The critical element model: a modeling philosophy", *Engineering Fracture Mechanics*. Vol. 25, pp. 739–749.

Reifsnider, K.L., 1990. *Fatigue of composite materials*. Composite Materials Series 4, Elsevier.

Reifsnider, K.L. and Case, S.W., 2002. *Damage Tolerance and Durability of Material Systems*, Wiley & Sons, USA.

Reifsnider, K.L. and Majumdar, P., 2011, "Material state change relationships to fracture path development for large-strain fatigue of composite materials". *Mechanics of Composite Materials*, Vol. 47, no. 1.

Saunders, D.S. and Clark, G., 1993. "Fatigue damage in composite laminates". *Materials Forum*, Vol. 17, pp. 309-331.

Sendeckyj G.P., 1990. "Life Prediction for resin-matrix composite materials", In *K.L. Reifsnider, Fatigue of composite materials, Material series: Elsevier*, pp. 431-483.

Simonds, R.A. and Stinchcomb, W.W., 1989. "Response of Notched AS4/PEEK Laminates to Tension/Compression Loading, Advances in Thermoplastic Matrix Composite Materials", *ASTM STP 1044, G.M. Newas, Ed., ASTM, Philadelphia*, pp. 133-145.

Talreja, R., 1981. "Fatigue of composite materials: Damage mechanisms and fatigue-life diagrams". *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, Vol. 378, pp. 461-475.

Vure, N.R.S., 1993. "*Effect of Cooling Rate and Stacking Sequence on the Fatigue Behavior of Notched Quasi-Isotropic APC-2 Laminates*". M.S. Thesis, Virginia Polytechnic Institute and State University.

Ye, L., 1989. "On fatigue damage accumulation and material degradation in composite materials". *Composites Science and Technology*, Vol. 36, pp. 339-350.

## 7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.