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PERTURBATIVE ANALYSIS OF THE NEUTRON POINT KINETICS EQUATIONS WITH AND WITHOUT TEMPERATURE FEEDBACK EFFECTS BY POLYNOMIAL APPROACH METHOD

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Abstract. *In this paper, we report a perturbative analysis of the Neutron Point Kinetics Equations with and without temperature feedback effects by Polynomial Approach Method. The idea of the method is to expand the neutron density, delayed neutron precursors concentration and temperature as a power series considering the reactivity as an arbitrary function of time in a short time span interval around an ordinary point. In the first interval one applies the initial conditions of the problem and the analytical continuation is used to determine the solutions of the next intervals. The objective of this analysis is to verify the stability of the method before perturbations included both in reactivity as in the initial condition to study the behavior of the linear and non-linear stiff differential equations. For the perturbation in reactivity, one considers different functions types: constant, ramp, quadratic, zig-zag, sinusoidal and pulsed source. Furthermore, one varies the time step size of the Polynomial Approach Method and performs an analysis of variance and one calculates the relative errors between the solutions with and without perturbation*.

Keywords: *Perturbation Analysis, Neutron Point Kinetics Equations, Polynomial Approximation Method, Temperature Feedback*

1. INTRODUCTION

The Neutron Point kinetics Equations (NPKE) are of furthest importance in reactor physics, as they allow a prediction of reactor power in real time, providing information on the dynamic reactor where, for example, the adjustment of the control rods occurs in the reactor core during startup or shutdown. These equations take into account prompt neutrons emitted immediately after fission (in the order of 10^{-4} to 10^{-5} seconds - time scales for a thermal reactor) and delayed neutrons from the decay of fission products (in the order of 10^{-1} to 10 seconds). This gives the stiffness characteristic for equations, making them difficult to resolve and demanding, to solve numerically, the very small time step increments. Therefore, the solution can be only obtained by specific methods, known as A-stable. (Petersen et al., 2011). However, when one considers the reactivity dependent on time and temperature the set of equations becomes nonlinear.

The system of Neutron Point Kinetics Equations remains the subject of study by its importance in physic reactors. For illustration, we can mention: (Nahla, 2011) presents the method *New Analytic Method* (NAM) based on the roots of the Inhour Equation using the method of Gaussian elimination to solve the NPKE with the following types of reactivities: constant, ramp and temperature feedback; the decomposition method of (Petersen et al., 2011a) for time-dependent reactivities is an analytical method that separating the matrix of coefficients into two, one with time dependence and the other constant, turning the equations in one set of recursive problems similar to equations with constant reactivity; (Aboanber; Nahla; Al-Malki, 2012) use the *Analytical Perturbation* method for one group of delayed neutrons precursors with temperature feedback based on approaches with small parameters for the neutron density; in (Ganapol, 2009; Ganapol; Picca, 2010; Ganapol et al.,2012) shows the *A High Accurate Technique* based on constant approach by parts (Kinard; Allen, 2004) and it is enhanced by explicitly accounting for the feedback and the reactivity variation within a time step through an iterative cycle, the high accuracy is achieved by introducing a sub-mesh for the numerical evaluation of integrals correcting the source term.

In this paper, we obtain the solution of the NPKE using the Polynomial Approach Method (PAM). The main idea is to verify the stability of the method before perturbations included both in reactivity as in the initial condition to study the behavior of the linear and non-linear stiff differential equations. The idea of PAM is to expand the variables of interest in power series, searching a solution around a ordinary point t_0 centered on a range I_0. So, replacing these series in the original equations finds a recurrence relation for the generation of coefficients of the series. Whereas the reactivity as a constant function in a short time, applies the initial conditions in the first interval and making use analytic continuation for other. Therewith we are able to find the coefficients simply by resolution a linear system of fixed size.

It is considered one and six groups of delayed neutron precursors with and without temperature feedback effects. For the perturbation in reactivity, one considers different functions types: constant, ramp, quadratic, zig-zag, sinusoidal and pulsed source. Furthermore, one varies the time step size of the PAM and performs an analysis of variance and one calculates the relative errors between the solutions with and without perturbation.

2. METODOLOGY

The starting point is the initial value problem given by NPKE with six groups of delayed neutron precursors, given by:

$$\begin{aligned} &\frac{dn(t)}{dt} - \frac{\rho(t)-\beta}{\Lambda} n(t) - \sum_i \lambda_i C_i(t) = 0 \\ &\frac{dC_i(t)}{dt} - \frac{\beta_i}{\Lambda} n(t) + \lambda_i C_i(t) = 0 \end{aligned}, \tag{1}$$

where $n(t)$ is the neutron density, t is time, $\rho(t)$ is the reactivity, β is the total fraction of delayed neutrons, Λ is the mean generation time, λ_i is the radioactive decay constant of group i of precursors, $C_i(t)$ is the delayed neutrons precursors concentration and β_i is the fraction of delayed neutron group i .With the following initial conditions: $n(0) = n_0$ and $C_i(0) = \beta_i n_0 / (\lambda_i \Lambda)$.

In order to apply the PAM, we expand the neutron density and delayed neutron precursor concentration in a power series around an ordinary point t_0 centered in I_0 in the form:

$$\begin{aligned} n(t) &= a_0 + a_1(t-t_0) + \ldots + a_r(t-t_0)^r + \ldots = \sum_{r=0}^{\infty} a_r (t-t_0)^r \\ C_i(t) &= b_{i,0} + b_{i,1}(t-t_0) + \ldots + b_{i,r}(t-t_0)^r + \ldots = \sum_{r=0}^{\infty} b_{i,r}(t-t_0)^r \end{aligned} . \tag{2}$$

Replacing the Eq. (2) and its first order derivative in Eq. (1), changing the index of summation of the terms of the derivatives to have the same general term, we obtain:

$$\begin{aligned} &\sum_{r=1}^{\infty} (r+1) a_{r+1} (t-t_0)^r - \left(\frac{\rho(t)-\beta}{\Lambda} \right) \sum_{r=0}^{\infty} a_r (t-t_0)^r - \sum_i \lambda_i \sum_{r=0}^{\infty} b_{i,r} (t-t_0)^r = 0 \\ &\sum_{r=1}^{\infty} (r+1) b_{i,r+1} (t-t_0)^r - \frac{\beta_i}{\Lambda} \sum_{r=0}^{\infty} a_r (t-t_0)^r + \lambda_i \sum_{r=0}^{\infty} b_{i,r} (t-t_0)^r = 0 \end{aligned} . \tag{3}$$

For the equations to be satisfied for all t is necessary that the coefficient of each power of t is zero, so solving these algebraic equations for the term of highest index in each equation, we can conclude that:

$$a_{r+1} = \frac{\left(\frac{\rho(t)-\beta}{\Lambda} \right) a_r + \sum_i \lambda_i b_{i,r}}{(r+1)}, \qquad b_{i,r+1} = \frac{\frac{\beta_i}{\Lambda} a_r - \lambda_i b_{i,r}}{(r+1)}, \tag{4}$$

for $r = 0,1,2\ldots$. The Eq. (4) is known as the recurrence relation and to initiate the generation of a_r and $b_{i,r}$ the initial conditions for $r = 0$ are needed.

The idea for solving the NPKE by PAM around a point t_0 is to consider the reactivity as a constant function in a relatively short time interval (constant approximation reactivity) together with the analytic continuation. That is, the solution around a point t_0 in time interval $I_0 = [0, 2\Delta t]$, where $\Delta t = t - t_0$ is the time step chosen, is the initial condition for the next interval, and so on.

Without loss of generality, assuming $\rho(t) = \rho$ and that the neutron density and delayed neutron precursor concentration are, for example, a local linear approximation around t_0 to I_0 we have that the series from Equation (2), with a_1 and $b_{1,i}$ provided by the recurrence formula from the Eq. (4) for $r = 0$, are:

$$n(t) \cong a_0 + \left[\left(\frac{\rho(t)-\beta}{\Lambda} \right) a_0 + \sum_i \lambda_i b_{i,0} \right] (t-t_0), \qquad C_i(t) \cong b_{i,0} + \left(\frac{\beta_i}{\Lambda} a_0 - \lambda_i b_{i,0} \right) (t-t_0), \tag{5}$$

for I_0 interval around t_0. Applying the initial conditions, one finds the a_0 and $b_{1,0}$ coefficients solving the linear system given by Eq. (5). Once found the coefficients a_0 and $b_{1,0}$ just replace in the Equation (4) for to get the solution around t_0 to I_0. The same idea is used for all intervals. We are able to find the solution for all I_{r+1} around t_0 making use of analytic continuation. That is, the solution in the end of an interval is the initial condition for the next one.

Considering the temperature feedback effects, one considers the following equations:

$$\begin{aligned}
&\frac{dn(t)}{dt} = \frac{\rho_0 - \alpha[T(t)-T(0)] - \beta}{\Lambda} n(t) + \sum_i \lambda_i C_i(t) \\
&\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) \\
&\frac{dT(t)}{dt} = Hn(t)
\end{aligned} \quad , \tag{6}$$

where $\mathrm{i} = 1:6$, $T(t)$ is the temperature, $T(0)$ is the temperature at the initial time, α is the reactivity temperature coefficient, H is the constant of proportionality between temperature and neutron density. The Eq. (6) has the same initial conditions adding $T(0) = T_0$.

The expansion in power series for neutron density and delayed neutrons precursors concentration are the same as that presented in Eq. (2). Whereas for temperature, we obtain:

$$T(t) = c_0 + c_1(t-t_0) + c_2(t-t_0)^2 + ... + c_r(t-t_0)^r + ... = \sum_{r=0}^{\infty} c_r (t-t_0)^r , \tag{7}$$

where $\mathrm{i} = 1:6$. Replacing the Eq. (7) and its first order derivative in Eq. (6), changing the index of summation of the terms of the derivatives to have the same general term, we have:

$$\begin{aligned}
&\sum_{r=0}^{\infty} (r+1) a_{r+1} (t-t_0)^r - \left(\frac{\rho_0 - \alpha T(0) - \beta}{\Lambda} \right) \sum_{r=0}^{\infty} a_r (t-t_0)^r + \frac{\alpha}{\Lambda} \sum_{r=0}^{\infty} c_r (t-t_0)^r \sum_{r=0}^{\infty} a_r (t-t_0)^r - \sum_{i=1}^{6} \lambda_i \sum_{r=0}^{\infty} b_{i,r} (t-t_0)^r = 0 \\
&\sum_{r=0}^{\infty} (r+1) c_{r+1} (t-t_0)^r - H \sum_{r=0}^{\infty} a_r (t-t_0)^r = 0
\end{aligned} ,\tag{8}$$

where $\mathrm{i} = 1:6$. The development of methodology for the delayed neutrons precursors concentration is the same as presented above.

Likewise, for the equations be satisfied for all t is required that the coefficient of each power of t be zero. Thus, isolating the terms of highest index of each equation, the recurrence relation considering the temperature feedback effects becomes:

$$a_{r+1} = \frac{\left(\frac{\rho_0 - \alpha T(0) - \beta}{\Lambda} \right) a_r - \frac{\alpha}{\Lambda} \sum_{k=0}^{r} c_r a_{r-k} + \sum_{i=1}^{6} \lambda_i b_{i,r}}{(r+1)}, \quad b_{i,r+1} = \frac{\frac{\beta_i}{\Lambda} a_r - \lambda_i b_{i,r}}{(r+1)}, \quad c_{r+1} = \frac{H a_r}{(r+1)}, \tag{9}$$

with $\mathrm{i} = 1:6$ and where $\sum_{k=0}^{r} c_r a_{r-k}$ arises from the third term of Equation (8) through the multiplication property of infinite series.

In this case, when one considers a local linear approximation around t_0 in I_0, with a_1, $b_{i,1}$ and c_1 determined by recurrence relation (Eq. (9)) for $r = 0$, we have:

$$\begin{aligned}
&n(t) \cong a_0 + \left[\left(\frac{\rho_0 - \alpha T(0) - \beta}{\Lambda} \right) a_0 - \frac{\alpha}{\Lambda} c_0 a_0 + \sum_{i=1}^{6} \lambda_i b_{i,0} \right] (t-t_0) \\
&C_i(t) \cong b_{i,0} + \left(\frac{\beta_i}{\Lambda} a_0 - \lambda_i b_{i,0} \right) (t-t_0) \\
&T(t) \cong c_0 + H a_0 (t-t_0)
\end{aligned} \quad , \tag{10}$$

with $i = 1:6$, to the I_0 interval around t_0. Applying the initial conditions, one finds the coefficients for the resolution of the system. It should be noted that the system becomes nonlinear due the product $c_0 a_0$. Thus, with analytical continuation we determine the next coefficients, as explained above.

3. PERTURBATIVE ANALYSIS

In this section, we analyze the behavior of the NPKE by PAM including perturbations in the initial condition and reactivity. The objective of this analysis is to verify the stability of the method against these disturbances in the system.

In this work, we cover reactivities that characterize the system of differential equations in linear and non-linear. In linear systems a perturbation must cause in their variable proportional reactions. When we apply two separate disturbances at the same time they behave as if they were applied independently so that the sum of individual reactions of each disturbance is the answer of the system. However for non-linear systems, a small perturbation may result in a large reaction, in other words, perturbations applied do not result in proportional reactions and distinct perturbations applied in the system generate totally unexpected and unpredictable results, and they are not additives as in the linear case.

The perturbation is inserted in the form of a random function present in SciLab Software (*http://www.scilab.org/*) in each time step. The "rand" function is a random number generator showing values between zero and one. For the perturbation have both positive and negative values, we add a factor of -0.5 thus taking values between -0.5 and 0.5. Then, the perturbation is given by $\delta = \tau(rand(t_0) - 0{,}5)$ where the constant τ controls the magnitude of the disturbance, for example, 10^{-2}.

First, we introduce a perturbation generated in the system (internal perturbation) that tends to affect the value of the output variable. Thus, the NPKE with disturbance in reactivity has the form:

$$\begin{aligned} \frac{dn(t)}{dt} &= \frac{(\rho(t)+\delta)-\beta}{\Lambda} n(t) + \sum_i \lambda_i C_i(t) \\ \frac{dC_i(t)}{dt} &= \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) \end{aligned}, \tag{11}$$

where $i = 1:6$. Likewise, the NPKE with temperature feedback including a perturbation in reactivity are given by:

$$\begin{aligned} \frac{dn(t)}{dt} &= \frac{[\rho_0 - \alpha(T(t)-T(0)) + \delta] - \beta}{\Lambda} n(t) + \sum_i \lambda_i C_i(t) \\ \frac{dC_i(t)}{dt} &= \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t) \\ \frac{dT(t)}{dt} &= Hn(t) \end{aligned}, \tag{12}$$

where $i = 1:6$. For the perturbation in the initial condition of the neutron density we consider $n(0) = n_0 + \gamma$, where γ is the perturbation in the initial condition in cm^{-3}.

The results for the density in cm^{-3} with perturbation for both the linear as non-linear case, will be presented in the next section.

4. NUMERICAL RESULTS

4.1 Perturbation in reactivity

The kinetic parameters used in the simulations are shown in Tab. 1. It is considered as initial neutron density $n_0 = 1 cm^{-3}$ and the concentration of delayed neutron precursors $C_i(0) = \beta_i / \lambda_i \Lambda$ for $i = 1:6$. The reactivities constant, ramp, quadratic, zig-zag and sinusoidal respectively, including the perturbation are given by: $\rho = -1\beta + \delta$, $\rho(t) = 0.1\beta t + \delta$, $\rho(t) = at + bt^2 + \delta$, $\rho(t) = 0.00073 sin(t) + \delta$, $\rho(t) = \begin{cases} 0.0075t + \delta, & 0 \le t \le 0.5 \\ -0.0075(t-0.5) + 0.00375 + \delta, & 0.5 \le t \le 1 \\ 0.0075(t-1) + \delta, & 1 \le t \le 1.5 \\ 0.00375 + \delta, & 1.5t \end{cases}$,

where $\delta = \tau(rand(t_0) - 0{,}5)$ and we assume $\tau = 10^{-2}$. The responses of the neutron density with and without perturbation in cm^{-3} are shown in Fig. 1 for the ramp and zig-zag cases. In all cases to be presented, it is used as a time step $\Delta t = 0.0001 s$.

Table 1. Reactivities with theirs respectively kinetics parameters used in the simulations.

Reactivities	Mean generation time (Λ)	Group i	Fraction of delayed neutron of group i precursors (β_i)	Radioactive decay constant of group i precursors $\lambda_i(s^{-1})$
• Constant without temperature feedback • Quadratic[1] • Zig-Zag	$5\cdot10^{-4}$	1	0.000285	0.0127
		2	0.0015975	0.0317
		3	0.001410	0.115
		4	0.0030525	0.311
		5	0.00096	1.40
		6	0.000195	3.87
			β = 0.0075	
• Ramp	$2\cdot10^{-5}$	1	0.000266	0.0127
		2	0.001491	0.0317
		3	0.001316	0.115
		4	0.002849	0.311
		5	0.000896	1.40
		6	0.000182	3.87
			β = 0.007	
• Sinusoidal	$3\cdot10^{-5}$	1	0.000214	0.0124
		2	0.001423	0.0305
		3	0.001247	0.111
		4	0.002568	0.301
		5	0.000748	1.14
		6	0.000273	3.01
			β = 0.006473	
• Constant with temperature feedback	$5\cdot10^{-5}$	1	0.00021	0.0124
		2	0.00141	0.0305
		3	0.00127	0.111
		4	0.00255	0.301
		5	0.00074	1.13
		6	0.00027	3.0
			β = 0.00645	

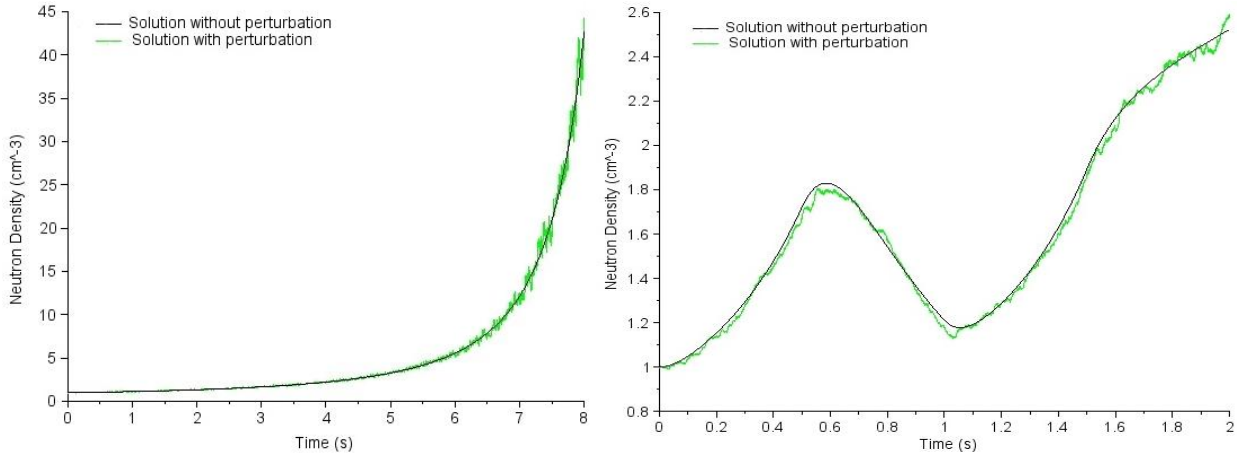

Figure 1. Neutron density with and without perturbation in cm^{-3} for the ramp and zig-zag reactivities

In order to study the behavior of neutron density with perturbation, we vary the size of the time steps and one presents in Tab. 2 the values of the variance of the perturbed solutions compared to the PAM. It worth mentioning that are made five simulations of perturbed solutions to calculate the variance and is used $\tau = 10^{-2}$ to the constant case and $\tau = 10^{-3}$ in other cases. In the same table are presents the relative errors between perturbed and non perturbed solutions,

[1] In the quadratic reactivity was used $\Lambda = 1\cdot10^{-4}$.

with $\tau = 10^{-3}$ for the reactivities above mentioned. We observe that according we decrease the step time, the relative error between perturbed and non perturbed solutions and the value of variance decreases, in other words, a smaller step time controls more efficiently perturbations included in the system. The PAM behaves stably to perturbations included in reactivity and not diverging with progression of iterations.

For the solution in the nonlinear case, it perturbs the reactivity dependent on time and temperature. One presents the answers of neutron densities with and without perturbation in cm^{-3} with temperature feedback for two cases constants with one and six groups of delayed neutron precursors in Fig. 2. The neutron density is in logarithmic scale for better viewing for the second sub case. For the case with one group ($\rho = 0,25\beta + \delta$) , we assume the following data: $\beta = 0.0065$, $\Lambda = 10^{-4}$, $\lambda = 0.07741 s^{-1}$, $n_0 = 10\, cm^{-3}$, $T_0 = 300\, K$, $H = 0.05\, K/MWs$ and $\alpha = 5{\cdot}10^{-5} K^{-1}$. For the case of constant reactive $\rho = 1\beta$, it was used the kinetics parameters of Tab. 1 with $T_0 = 300\, K$, $H = 2,5{\cdot}10^{-6}\, K/MWs$ and $\alpha = 1 K^{-1}$. For the temperature Fig. 3 shows the answer to the perturbed solutions in K in the same order of reactivity cases cited above. For each graph, we show the result of the neutron density using different constants that control the order of magnitude of the perturbation, assuming the values $\tau = 0.001$ and $\tau = 0.01$. In all cases to be presented, is used as a time step $\Delta t = 0.001 s$. The reactivity of constant type with one and six groups of delayed neutrons precursors, respectively, including the perturbation are given by:

$$\rho = 0.25\beta + \delta, \tag{13}$$

$$\rho = 1\beta + \delta, \tag{14}$$

where $\delta = \tau(rand(t_0) - 0,5)$.

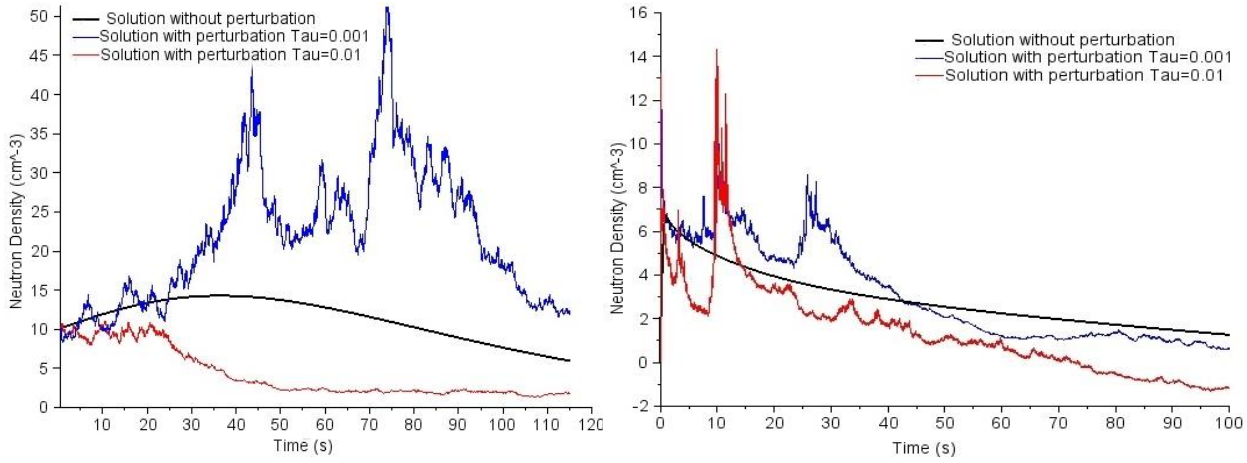

Figure 2. Neutron density with and without perturbation in cm^{-3} for the constant reactivities ($\rho = 0,25\beta + \delta$ and $\rho = 1\beta + \delta$) with temperature feedback for one and six groups of delayed neutron precursors, respectively

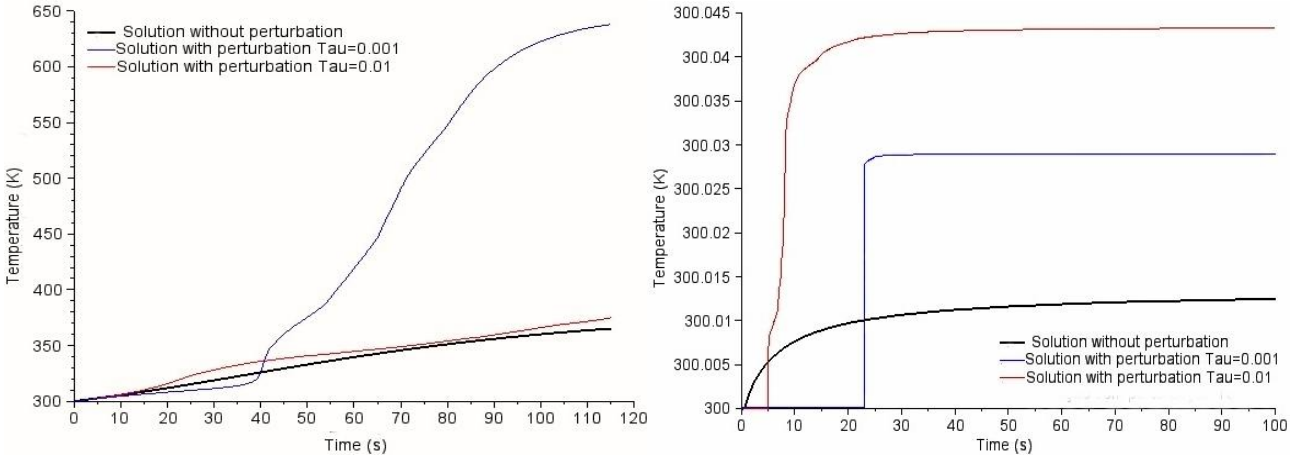

Figure 3. Temperature with and without perturbation in K for the constant reactivities ($\rho = 0,25\beta + \delta$ and $\rho = 1\beta + \delta$) with temperature feedback for one and six groups of delayed neutron precursors, respectively

To analyze the behavior of neutron density with perturbation in the non-linear system, the size of the time steps is varied and the values of the variance and the relative errors of solutions perturbed in comparison to PAM solution are presented in Tab. 2. In the non-linear case were made five simulations of perturbed solutions to perform calculations cited as in the linear cases, and it was used $\tau = 10^{-4}$. It may be noted that according we decrease the step time the variance and the relative errors not decrease. It could be justified because perturbations inserted in non-linear systems have unpredictable results, in other words, a small δ may generate uncontrolled reactions. Thus, nothing can be said about the control of disturbance by decreasing the time step, because as the disturbances are generated randomly, now one disturbance generated using $\Delta t = 0,001s$ is greater than another disturbance with $\Delta t = 0.01s$, now is smaller.

Table 2. Variance and relative errors between the solutions with and without perturbation.

Reactivity	$\Delta t(s)$	Variance between the solutions with and without perturbation	Relative errors between the solutions with and without perturbation
Constant $\rho = -1\beta + \delta$	0.01	0.007123474	0.079969609
	0.001	0.002504951	0.066024328
	0.0001	0.001281068	0.043545677
Ramp $\rho(t) = 0.1\beta t + \delta$	0.01	150.5499258	0.401990267
	0.001	12.51383315	0.139054403
	0.0001	1.954420755	0.055313195
Quadratic $\rho(t) = at + bt^2 + \delta$	0.01	0.466520886	0.145195153
	0.001	0.443411726	0.044186332
	0.0001	0.427750580	0.023436942
Zig-Zag	0.01	0.005764137	0.137926853
	0.001	0.001194258	0.068589737
	0.0001	0.000073805	0.009785104
Sinusoidal $\rho(t) = 0.00073sin(t) + \delta$	0.01	0.019039469	0.144815155
	0.001	0.002739065	0.068589737
	0.0001	0.000364706	0.026092893
Constant with temperature feedback $\rho = 0.25\beta + \delta$	0.01	240.0889684	0.6411726
	0.001	19524.37852	12.899263
	0.0001	39906429	408.029
Constant with temperature feedback $\rho = 1\beta + \delta$	0.01	22235673.46	112.975294
	0.001	3023706683	5254.887482
	0.0001	1.64040D+12	3297.550237

4.2 Perturbation in the initial condiction

To analyze the behavior of the PAM solution we include a perturbation in the initial condition. It is used as illustration the sinusoidal case for the linear system. In the first graphic of Fig. 4 presents the neutron density in cm^{-3} with the following values for the initial condition: $n_0 = 1$, $n_0 = 1.01$, $n_0 = 1.05$, $n_0 = 1.1$, $n_0 = 0.99$, $n_0 = 0.95$ and $n_0 = 0.9$. In all simulations are used as time step $\Delta t = 0,0001s$.

For the non-linear case is used as an illustration the case constant $\rho_0 = 0.25\beta$ with one group of delayed neutrons precursors and one ramp case. In the second graphic of Fig. 4 the neutron density is presented in cm^{-3} of constant case with the following values for initial condition: $n_0 = 11$, $n_0 = 10.5$, $n_0 = 10$, $n_0 = 9.5$ and $n_0 = 9$.

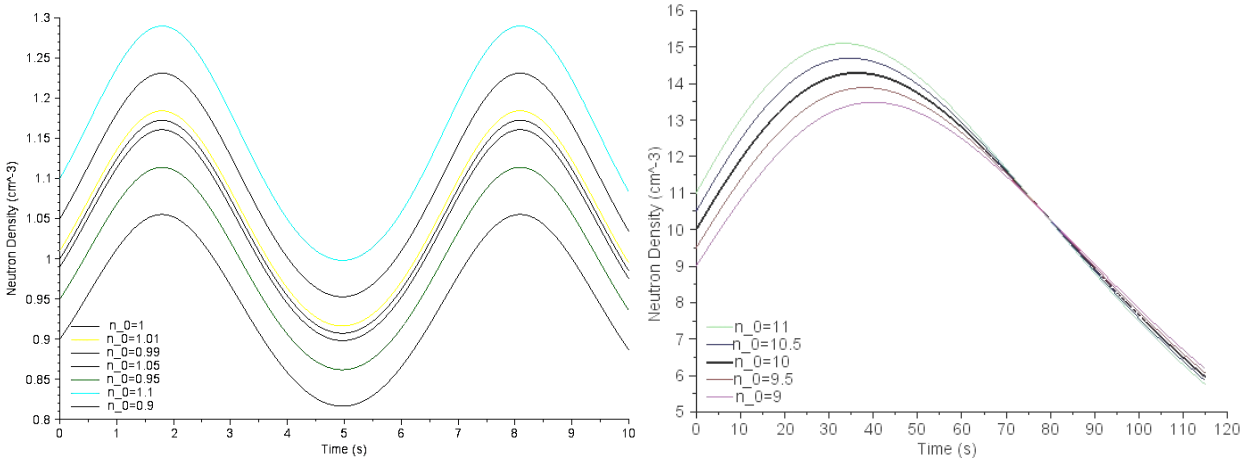

Figure 4. Neutron density in cm^{-3} with and without perturbation in the neutron initial condition for the sinusoidal (linear system) and constant (non-linear system) temperature feedback reactivities

5. CONCLUSIONS

In this work, we report the perturbative analysis of the Point Neutron Kinetics Equations with and without temperature feedback effects by Polynomial Approach Method, considering one and six groups of delayed neutron precursors and insertions of constant, ramp, quadratic, sinusoidal and zig-zag reactivity. In such a way, one varies the time step size of the Polynomial Approach Method and performs an analysis of variance and one calculates the relative errors between the solutions with and without perturbation.

The tests of the system response of the perturbations have shown that, using PAM and analytical continuation, as the iterations progress the solution does not diverge, suggesting the stability of the method. As expected, the reactivity of linear systems the perturbation generated in their variable proportional reactions. Whereas, also as it was predicted, for the reactivities of the non-linear systems, one small perturbation results in a large reaction and distinct perturbations applied in each time step generate totally unpredictable results.

The simplicity of this method together with the unusual direct handling of the stiffness problem with and without temperature feedback effects makes PAM a good option for finding accurate results to simulate the behavior of a nuclear reactor. Moreover, since it is polynomial, are easy to handle both analytically and numerically, that is, a simple and easy computational implementation method and in addition it has stable behavior against perturbations.

6. ACKNOWLEDGMENTS

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