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ANALYSIS OF A CYLINDER SUBMITTED TO ACOUSTIC EXCITATION DUE TO TURBULENT BOUNDARY LAYER

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Abstract. *A launching rocket and its payload are submitted to strong acoustic loads in some moments of its flight like lift off, during transonic flight and in the instant of maximum dynamic pressure. These loads could affect the payload and other rocket internal equipment. It must be taken into account that in the higher stages, where delicate control equipment and the payload are placed, the acoustic excitations are dominant. The knowledge of the acoustic loads is of great importance in order to provide estimated loads for the design phase, define intensity levels to specify qualification and acceptation acoustic tests to which subsystems and elements of the launcher and its payload shall be submitted and to design an acoustic protection system. In the three types of acoustic loads oscillating pressures are applied to all vehicle external surfaces. These pressure oscillations behave randomly and need to be analyzed by evaluating its statistical properties as: power spectrum and correlation coefficients along the principal axes of the structure. In this work the structural vibrations of a module of the IAE Microsatellite Launcher Vehicle (VLM) caused by the Turbulent Boundary Layer (TBL) during flight are evaluated numerically. A reference power spectrum and the correlation coefficients corresponding to a specific Mach number obtained from an IAE internal report were used to define the acoustic random load due to TBL. The structure modal parameters (natural frequencies and mode shapes) were obtained by using the finite element method. A MATLAB routine use the already calculated structure modal basis to evaluated the structure frequency response functions. By combining the system frequency response functions and the load power spectrum the routine calculates the structure response.*

Keywords: *Microsatellite launching vehicle, Turbulent Boundary Layer, Random vibrations*

1. INTRODUCTION

A launching rocket and its payload are submitted to strong acoustic loads during its lift off and atmospheric flight. When a re-entrance procedure is part of the mission it must be considered also in the acoustic loads analysis since thermo-acoustic excitations will be present. These loads could affect the payload and other rocket internal equipment (STAVRINIDIS et al, 2001). It must be taken into account that in the higher stages, where delicate control equipment and the payload are placed, the acoustic excitations are dominant.

The knowledge of the acoustic loads and the payload dynamic acoustic behavior are of great importance in order to: provide estimated loads for the design phase, define intensity levels to specify qualification and acceptation acoustic tests to which subsystems and elements of the launcher and its payload shall be submitted and to design an acoustic protection system. A system's response to an acoustic field can be calculated by using mathematical models (MIRANDA, 1995) but a precise description of the acoustic loads generated by the launching vehicle remains a difficult task. Therefore, field measurements and dynamic tests in acoustic reverberant chambers are always strongly recommended.

The acoustic load consists of oscillating pressures applied to all vehicle external surfaces. The pressure oscillations behave randomly and need to be analyzed by evaluating its statistical properties as (COCKBURN, & ROBERTSON, 1974): power spectrum and correlation coefficients along the principal axes of the structure.

During the vehicle lift off the acoustic excitation is caused by the high velocity exhaust gases. During the vehicle atmospheric flight the acoustic excitation is caused by the turbulent boundary layer. Both kinds of excitations can be approximated by mathematical or semi-empirical models for its power spectrum and correlation coefficients.

In this work the structural response of a module of an IAE launcher vehicle due to the turbulent boundary layer at a Mach number of 0.8527 and an altitude of 3910.2 m was estimated.

2. SYSTEM STUDIED

The VLM-1 rocket is a vehicle developed by IAE in conjunction with German Aerospace Center (DLR). The vehicle is still in its early design stages and no prototype was already built. It will be used to launch suborbital re-entry payloads (like the German experiment SHEFEX) or microsatellites (up to 150kg) in LEO (Low Earth Orbit). Its basic configuration consists of three solid-propellant stages: the first two stages with the S50 engine with 12 tons solid propellant and a third stage with the S44 engine as can be seen in Fig.(1).

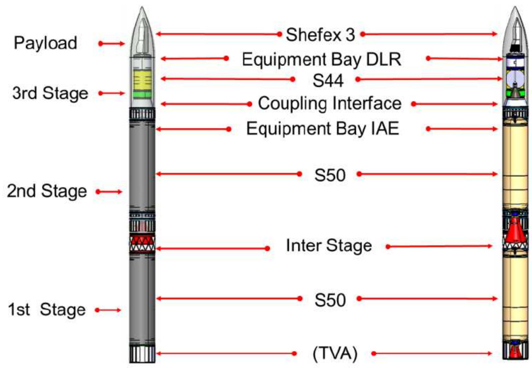

Figure 1. VLM-1 initial configuration

This work focused on an important component of the launch vehicle, the equipment bay, whose function is to house the electronic equipment used by the vehicle during the flight. Typically the structure of this kind of module is a cylindrical shell. An example of this kind of structure can be seen in Fig.(2), the Satellite Launch Vehicle (VLS) equipment bay used in vibration tests.

Figure 2. Satellite Launch Vehicle (VLS) equipment bay structure

3 RANDOM DISTRIBUTED PRESSURE LOADING

Many acoustic loads in the aerospace industry are spatially distributed and behave randomly.

Acoustic pressure oscillations in a launcher external surface caused by aerodynamic effects (like a turbulent boundary layer) and the acoustic diffuse field generated by hot gases exhaust during a launcher lift-off have this characteristics.

According to (NIEUWENHOF, 2008) the acoustic excitations cited above can be described as a homogeneous random process (if the pressure auto-power spectrum is spatially invariant) or a heterogeneous random process (if the pressure auto-power spectrum is dependent of the position of the point considered).

A homogeneous random process is characterized by a reference power spectrum $S_{ref}(\omega)$ and a correlation function f_c dependent on ς_a and ς_b (positions of points *a* and *b*) and the frequency ω. The cross-power spectrum of points *a* an *b* have the form:

$$S_{p_a p_b}(\omega) = S_{ref}(\omega) f_c\left(\left|\varsigma_a - \varsigma_b\right|, \omega\right) \quad (1)$$

A heterogeneous random process is characterized by a map of the auto-power spectrum $S_p(\varsigma, \omega)$ and a correlation function f_c dependent on ς_a and ς_b (positions of points *a* and *b*) and the frequency ω. The cross-power spectrum of points *a* an *b* have the form:

$$S_{p_a p_b}(\omega) = \sqrt{S_p(\varsigma_a, \omega) S_p(\varsigma_b, \omega)}\, f_c(\varsigma_a, \varsigma_b, \omega) \quad (2)$$

The acoustic excitations analyzed in this report are heterogeneous process.

In next sections the estimation of $S_p(\varsigma, \omega)$ and the calculation of f_c are described for turbulent boundary layer and for the diffuse acoustic field.

4 - STRUCTURAL RESPONSE TO RANDOM LOADS

Considering a discretized structure submitted to external pressures, the vector of global nodal forces $\{f_S\}$ and the vector of nodal pressures on the structure external surface $\{p\}$ are related by (NIEUWENHOF, 2008):

$$\{f_S(\omega)\} = [E][L]\{p(\omega)\} \quad (3)$$

Where [*L*] is a coupling matrix which converts local nodal pressures into a consistent nodal force vector and [*E*] is a binary matrix that splits these contributions into the global forces vector $\{f_S(\omega)\}$.

Therefore the matrix of power spectrum of the applied nodal forces ($[S_{ff}(\omega)]$) and the matrix of power spectrum of nodal acoustic pressures ($[S_{pp}(\omega)]$) are related by:

$$[S_{ff}(\omega)] = [E][L][S_{pp}(\omega)][L]^T [E]^T \quad (4)$$

The matrix of power spectrum of the structural responses can then be calculated [26] :

$$[S_{uu}(\omega)] = [\overline{H}(\omega)][S_{ff}(\omega)][H(\omega)]^T = [\overline{H}(\omega)][E][L][S_{pp}(\omega)][L]^T [E]^T [H(\omega)]^T \quad (5)$$

Where the structure receptance matrix is:

$$[H(\omega)] = (-\omega^2 [M] + i\omega[C] + [K])^{-1} \quad (6)$$

And its complex conjugate is denoted by $[\overline{H}(\omega)]$.

If the number of degrees of freedom (*n*) is quite large, $[H(\omega)]$ can be converted to the modal space [26]:

$$[H(\omega)] = [\psi](-\omega^2 [m] + i\omega[c] + [k])^{-1} [\psi]^T \quad (7)$$

Where:

$$[m]=[\psi]^T[M][\psi]$$
$$[c]=[\psi]^T[C][\psi]$$
$$[k]=[\psi]^T[K][\psi]$$

$[\psi]$ is a matrix containing the first *m* structural eigenvectors (*m*<<*n*) as its columns.

Equation (5) then became:

$$\begin{aligned}[S_{uu}(\omega)]&=[\psi](-\omega^2[m]+i\omega[c]+[k])^{-1}[\psi]^T[S_{ff}(\omega)]\left([\psi](-\omega^2[m]-i\omega[c]+[k])^{-1}[\psi]^T\right)^T=\\&=[\psi](-\omega^2[m]+i\omega[c]+[k])^{-1}[\psi]^T[S_{ff}(\omega)][\psi](-\omega^2[m]-i\omega[c]+[k])^{-T}[\psi]^T=\\&=[\psi](-\omega^2[m]+i\omega[c]+[k])^{-1}[J(\omega)](-\omega^2[m]-i\omega[c]+[k])^{-T}[\psi]^T\end{aligned}\tag{8}$$

As pointed out in (Petyt, 1990), non-dimensional forms of the elements of $[J(\omega)]$ are in some texts referred as joint acceptances.

In the numerical point of view is preferable to evaluate the product $[J(\omega)]=[\psi]^T[S_{ff}(\omega)][\psi]$ first and then calculate $(-\omega^2[m]+i\omega[c]+[k])^{-1}[J(\omega)](-\omega^2[m]-i\omega[c]+[k])^{-T}$. By doing so, one will obtain an *m x m* matrix. Finally the power spectral density of the degree of freedom *r* is given by:

$$[S_{u_ru_r}(\omega)]=[\psi_r](-\omega^2[m]+i\omega[c]+[k])^{-1}[J(\omega)](-\omega^2[m]-i\omega[c]+[k])^{-T}[\psi_r]^T\tag{9}$$

Where $[\psi_r]$ is the r^{th} line of the matrix $[\psi]$.

By introducing eq. (4) in the expression of $[J(\omega)]$ we obtain:

$$[J(\omega)]=[\psi]^T[E][L][S_{pp}(\omega)][L]^T[E]^T[\psi]\tag{10}$$

5 –PRESSURE TO FORCE CONVERSION

The external pressures applied to the structure boundary Γ can be converted to a vector of structural nodal forces {F} by using a transformation matrix, $[L_{ext}]$ as described in Eq. (11) (SOUTO, 2013):

$$\{F\}_{n_{gdl}x1}=[L_{ext}]_{n_{gdl}xn_\Gamma}\{\bar{p}\}_{n_\Gamma x1}\tag{11}$$

Where n_Γ is the number of structural nodes in the boundary Γ, n_{gdl} is the number of the whole structure degrees of freedom and $\{\bar{p}\}$ is a vector which contains external pressures applied to the structure boundary Γ.

6 –ACOUSTIC EXCITATION DUE TO TURBULET BOUNDARY LAYER

In the atmospheric flight the turbulence of the airflow in contact with the launcher external structure is a source of acoustic excitation.

According to the semi-empirical model developed by (CORCOS, 1963), the spatial correlation function of an excitation caused by turbulent boundary layer is given by Eq. (12):

$$\Gamma(\xi,\eta,\omega)=A(\xi,\omega)B(\eta,\omega)e^{-i\theta(\xi,\omega)}\tag{12}$$

Where $A(\xi,\omega)$, $B(\eta,\omega)$ and $\theta(\xi,\omega)$ are given by:

$$A(\xi,\omega) = e^{-\frac{\xi}{L_x}} \tag{13}$$

$$B(\eta,\omega) = e^{-\frac{\eta}{L_y}} \tag{14}$$

$$\theta(\xi,\omega) = -\frac{\omega\xi}{U_c} \tag{15}$$

In the expressions shown above the correlation lengths in directions x and y $L_x(\omega)$ e $L_y(\omega)$ respectively, are calculated by the following equations:

$$L_x(\omega) = \frac{U_c}{\alpha_x \omega} \tag{16}$$

$$L_y(\omega) = \frac{U_c}{\alpha_y \omega} \tag{17}$$

In Eqs.(16) and (17) αx e αy are constants with typical values for smooth walls of α_x=0.7 e α_y =0.1, Uc is the convection velocity given by Eq. (18):

$$U_c = \beta_c U_\infty \tag{18}$$

The constant of convection (βc) assume values among 0.6 and 0.75 and U_∞ is the free stream flow velocity.

In (SOUTO, 2014) was presented an estimation of reference values for the power spectral density of sound pressure level (SREF (ω)) generated by the action of a turbulent boundary layer in the center of VLM's equipment bay for an ambient condition where the Mach number and the altitude are 0.8527 and 3,9102km respectively. The curve is shown in Fig. (3). It is noted that the greatest increase in the sound pressure level occurred for the lowest natural frequencies, demonstrating that these have a great influence in this type of acoustic excitation.

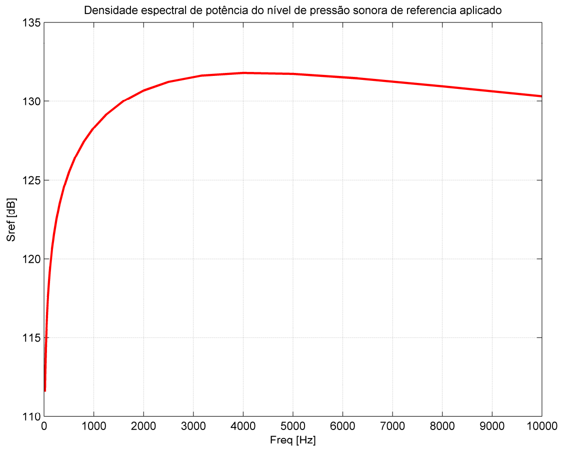

Figure 3. Power spectral density of sound pressure level due to the turbulent boundary layer in VLM's equipment bay for Mach 0.8527 and altitude of 3.9102km

7 –METHODOLOGY

A finite element model of the simplified structure of the equipment bay was drafted using ANSYS in order to calculate its modal parameters (natural frequencies and mode shapes), which were recorded in data files. Then a MATALB routine performs the following tasks:

a) Reading of the data files generated by ANSYS and the data reference power spectral density file provided by IAE from the flight conditions that VLM will be submitted;
b) Conversion of external pressures to a vector of nodal forces;
c) Calculation of the matrix of power spectral densities of the forces applied to the structure based on values from the vector of power spectral density of external pressures, which is obtained considering the items a and b;
d) Calculating the matrix of the frequency response function matrix and its conjugate for different frequency values from the known modal parameters;
e) Calculating the matrix of the power spectral densities of responses by matrix multiplication shown in Eq. (5);
f) Plotting of the results;

The cylinder analyzed in this work has radius of 0,730m, with height and thickness of 0,6780m 3,5x10-3m respectively. The adopted material is an isotropic composite material, with the Young's modulus equal to 0,689x1011N/m² and the assumed boundary conditions considered translational and rotational movement restriction of the three coordinate axes at both ends of the cylinder since the equipment bay is located between the 2nd stage and the hood of the vehicle, parts considered far more rigid.

The cylinder was modeled using ANSYS and discretized in a mesh with 1600 Shell 63 elements with 6 degrees of freedom each. The mesh has 1680 nodes. The cylinder type obtained can be seen in Figure 4 (15):

(a) (b)

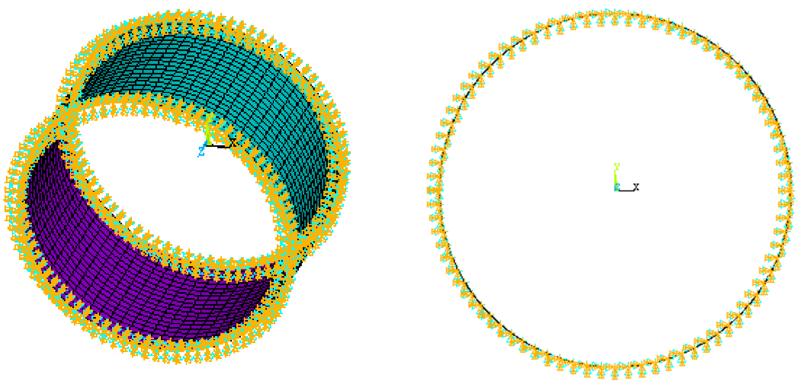

Figure 4. Isometric (a) and Frontal (b) views of the cylinder modeled by using ANSYS

8 RESULTS AND DISCUSSION

8.1 Cylinder natural frequencies and mode shapes

The natural frequencies of the studied cylinder were obtained through analysis with ANSYS software. Below the first 16 natural frequencies of the cylinder are shown in Table 1, where one can notice there is a repetition of the frequency of subsequent modes values. This is because the structure is symmetrical. In Fig. (5) some mode shapes of the cylinder are shown.

Table1. Cylinder first 16 natural frequencies

Modes	Natural Frequencies (Hz)	Modes	Natural Frequencies (Hz))
1	287,38	9	321,18
2	287,38	10	321,18
3	289,08	11	336,66
4	289,08	12	336,66
5	298,67	13	353,03
6	298,67	14	353,03
7	305,14	15	385,27
8	305,14	16	385,27

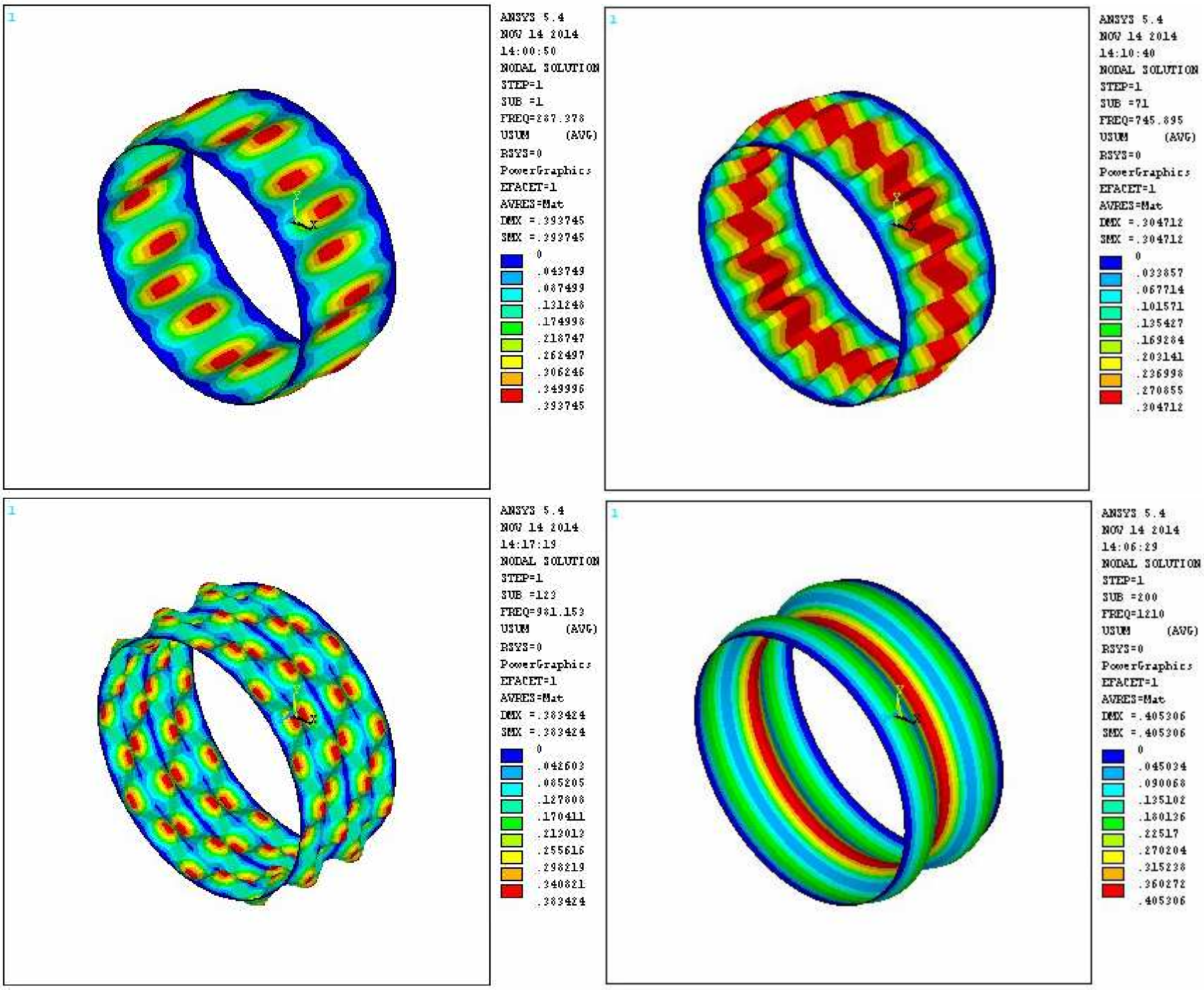

Figure 5. Cylinder mode shapes: mode 1, mode 71, mode 200 e mode 123 (clockwise direction)

8.2 Power spectral density of the cylinder structural response (Saa(ω))

Figure (6) displays the power spectral density of the structure acceleration response.

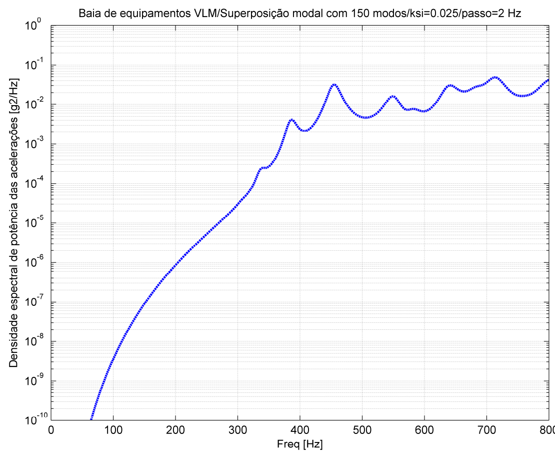

Figure 6. Power spectral density of the cylinder acceleration response

The response was calculated based on a damping coefficient (ksi) of 0.025 and using the first 150 cylinder vibration modes in the modal superposition. Although the analysis appear to have been made with a great number of modes, the mode 150 corresponds to a frequency 1082,60Hz, covering slightly more than 10% of the maximum value of the frequencies used in the calculation of Sref (ω) (10000Hz) . An analysis of the behavior at higher frequencies demands the inclusion of a much larger number of modes in the modal superposition, which was not executed due to computational limitations.

9 CONCLUSIONS

This work conducted a preliminary study of the dynamic response of VLM's equipment bay to acoustic excitation by turbulent boundary layer. By knowing the dynamic response of the structure one can determine whether the designed structure will withstand the stresses and strains generated and thus meet the safety factors required for a space vehicle.

The methodology demands reasonable computational resources, even using modal superposition, to perform analysis in a high frequency band. However at high frequencies the finite element method itself has some limitations due to the need of finer meshes and the sensitivity to geometric deviations even within the manufacturing tolerances. To overcome these difficulties a common practice is the use of methods like the statistical energy analysis.

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