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# ALGORITHM FOR COMPENSATORS DESIGN BY ROOT LOCUS METHOD

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***Abstract.*** *Compensators are used to modify the response of a system so that its output meets the desired results. Often the steps of control system design result of a trial and error procedure until the desired specifications are met. This work proposes to study plant linear systems of different types and degrees in order to develop algorithms that could help in the creation of compensators by the root locus method. For better accuracy, the algorithm created in MatLab® software was built to improve the transient response time (eg maximum overshoot and settling time) and/or the steady-state error. The user has the freedom to choose or not the system parameters (natural undamped frequency, the coefficient of damping, the position of the pole and zero of the compensator), and type the desired compensator: Phase Lead, Lag Phase or Lead-Lag Phase, if do not choose, the algorithm returns a transfer function that improves the transient response time of the process. Finally, the algorithm shows the compensator transfer function of the plant and the offset system.*

***Keywords:*** *Algorithm, Compensator, Lead-Lag Phase, Root Locus*

## INTRODUCTION

The Automatic Control has played a vital role on the progress of Engineering and Science, being an important tool in state of the art technology applications and became an important and integrant part of the industrial processes and of modern manufacture.

Controlling a magnitude or a physical variable means to alter its value, according to one intention. Not always this is perfectly achievable, or because enough energy is not available or because there are very important or very fast disturbs [1].

The root locus is a classic method for analysis and design of controllers to linear and time invariant plants, described through the transfer function [2]. This fact has motivated several research addressing the use of this method in the teaching of automatic control, considering, for example, the negative gains [3][4] and Lag [5] and Lead-Lag controllers [6].

Currently, it is possible to obtain poles of closed loop systems in a fast and exact way using computational methods. Nevertheless, the root locus method continues to be very useful in the design of control systems for allowing the designer to define accordingly the structure of the appropriate controller to each system.

Designing using the root locus method, means redraw the system's root locus by the addition of poles and zeros in the open loop transfer function of the system, forcing the new root locus to pass through the desired closed loop poles in the complex plan [7]. A typical control schematic of a process in closed loop with output feedback is presented in Fig. 1.

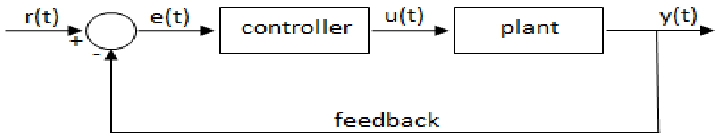


Figure 1 - Block Diagram of a control system in Closed Loop

In the design of a linear control system, it is verified that the root locus method becomes very useful, once it indicates the way in which the closed loop poles and zeros should be modified so that the response meets the performance specifications of the system.

The compensator is a physical device with transfer function designed to meet certain performance configurations. They can be electronic devices (Operational Amplifiers); RC nets – electrical, mechanical, pneumatics, hydraulics or the combination of those; amplifiers. The choice of which one to use, depends on the plant type. Used when the original system is unstable for all the gain values or is stable, but has undesired characteristics of transient response [8].

Overall, in a design of a control system, three important specifications are considered: System Stability, Steady-state Error and Transient Temporal Response.

## PURPOSE

The purpose of this paper is presenting a new algorithm to facilitate the design of compensators by the root locus method and to the application in laboratories for learning and verification of the classic control theory.

The authors believe this paper can be useful in the teaching of automatic control, because it addresses a classic compensating method (root locus), a poorly described in the specialized literature, and that can be used to improve different systems, as: control of level, temperature, pressure, industrial processes, Auto Guided Vehicles (AVGs), among others.

## METHODS

Using the steps for design of compensators by the root locus method described in [9] and [10] and the interactive software for assistance in mathematical calculus, MATLAB®.

The method used for the development of the algorithm is showed in the diagram of Fig. 2 and each stage is described as follows, as well as the obtained results in each one.

Initially the Algorithm asks the user to input the System's Transfer Function, giving three options to insert it, inputting the data or the values of the numerator and denominator polynomial's coefficients, or Poles, Zeros and Gain or the Laplacian function (in function of S). Then, it asked to choose which type of compensation is desired to be designed for this plant, depending on the parameters that are desired to insert adjustments.

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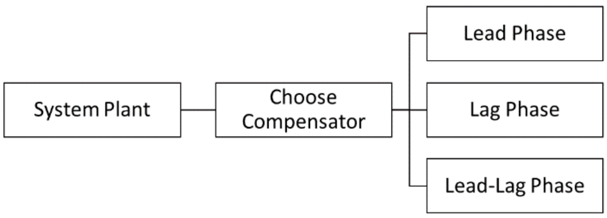



Figure 2 - Methodology for the development of the algorithm

### Lead Phase Compensation

Used when the original system is unstable for all the gain values or is stable, but has undesirable characteristics of Transient Response. It has the property of improving the system's temporal response reducing the overshoot and the settling time. When a lead phase compensator is included in the loop, the root locus is displaced to the left, causing the reduction of the overshoot and the transient response time.

The typical block of a lead phase compensator has the following transfer function (1), according to [11] and [12]:

$$Gc(s) = Kc\left(\frac{S+Zc}{S+Pc}\right), Pc > Zc \tag{1}$$

Where the $Z_C$ and $P_C$ are the values of the compensator's pole and zero, respectively.

The diagram on Figure 3 describes the procedure for Lead Phase Compensation:

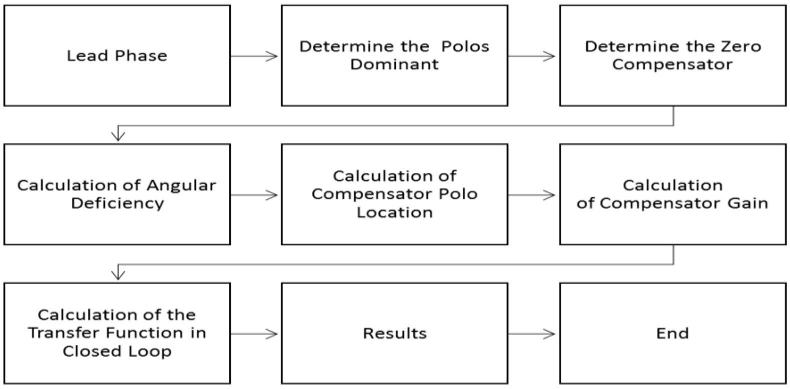



Figure 3 - Compensation method for Lead Phase

- **Dominant Poles**

The determination of the closed loop poles' positions that satisfies the desired performance specifications, or can be given initially by the user, or the user can define the desired Overshoot (MP) an the Settling Time (Ts, of 2% or 5%). To satisfies the Algorithm specifications, it is going to find the parameters Not Dumped Natural Frequency (ωn) and the Dumping Coefficient (ζ) to find the position of the S poles through the approximation of the characteristic polynomial of the Second Order Transfer Function as presented in (2):

$$S^2 + 2\xi\omega_n + \omega_n^2 \qquad (2)$$

- **Zero of the Compensator**

Once more the freedom to chose is given to the user, who can inform the desired Zero or the Algorithm defines it automatically as the Real part of the Dominant Pole defined previously [13].

- **Angular Deficiency**

To determine the Angular Deficiency, the phase condition is used as in Eq. (3).

$$DA = 180^{\circ} - \sum \theta polos + \sum \theta zeros \qquad (3)$$

- **Pole of the Compensator**

To determine the Pole of the Compensator is used as in Eq. (4).

$$\big(\angle(s - z_1) + \angle(s - z_2) + \cdots + \angle(s - z_m)\big) - \big(\angle(s - p_1) + \angle(s - p_2) + \cdots + \angle(s - p_m)\big) = \pm k\pi \qquad (4)$$

- **Gain of the Compensator**

To determine the gain of the compensator, the module condition is used as in Eq. (5).

$$\frac{k|s - z_1|.|s - z_2| \ldots |s - z_m|}{k|s - p_1|.|s - p_2| \ldots |s - p_m|} = 1 \qquad (5)$$

- **Result**

Show the Transfer Function of the Compensator and of the Compensated System and show all the Closed Loop Poles, the Transient Temporal Response of the Compensated System with the informed by user input, compared with the Not Compensated System.

**Lag Phase Compensation**

Used when the system's temporal response is satisfactory, but the steady-state error needs to be improved, but, in the opposite of the lead phase compensation, the lag phase can compromise the system's temporal response, so that there be no changes on the geometric root locus with the inclusion of the compensator in the loop, the contribution of the lead compensator's phase must be near to zero, this is only possible if the compensator's pole and the zero are near each other.

The typical block of a Lag phase compensator has the following transfer function (6), according to [14] and [15]:

$$Gc(s) = Kc\left(\frac{S+Zc}{S+Pc}\right), Zc > Pc \qquad (6)$$

The diagram of Figure 4 describes the procedure for the Lag Phase Compensation.

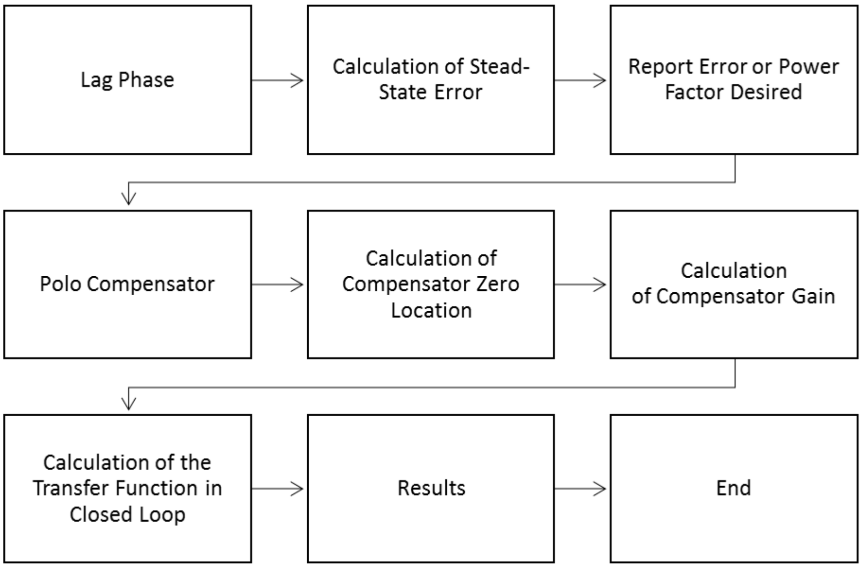



Figure 4 - Compensation method for Lag Phase

- **Steady-state Error**

Steady-state Error and the Coefficient of Static Speed Error.

- **Desired Error**

Inform the Desired Error or the Error's Attenuation Factor.

- **Pole of the Compensator**

Inform the desired Pole or the Algorithm is going to match with 0.01 or 0.005 according to [16].

- **Zero of the Compensator**

To determine the Zero of the Compensator the Eq. (7) was applied.

$$e_C(\infty) = \lim_{x \to 0} s\left(\frac{1}{1 + G_{(S)}}\right) R_{(s)} \tag{7}$$

Substituting the values by the new steady-state error and being G(s) the connection in series between the Compensator and the System's Plant.

- **Gain of the Compensator**

The MATLAB®'s function rlocfind() is used to return the gain of the dominant pole, which is used as parameter.

- **Result**

Show the Transfer Function of the Compensator and of the Compensated System and show all the Closed Loop Poles, the Transient Temporal Response of the Compensated System with the informed by user input, compared with the Not Compensated System.

**Lead-Lag Phase Compensation**

This Compensator is used when there is the desire of improving simultaneously the transient response and the steady-state error of the system, designing, first of all, a lead phase compensator to improve the transient response in function of time and after, the lag phase compensator is designed, depending on how much the steady-state error needs to be changed, to meet the specifications.

The typical block of a Lead-Lag phase compensator has the following transfer function in (8), according to [17] and [18]:

$$Gc(s) = Kc\left(\frac{S + Z_{av}}{S + P_{av}}\right)\left(\frac{S + Z_{at}}{S + P_{at}}\right) \tag{8}$$

Where Zav, Pav, Zat and Pat are the values of the zero and pole of the lead phase compensator and lag phase compensator, respectively.

The Algorithm for the Lead-Lag Phase compensation is nothing more than applying the Lead Phase compensator and next the Lag Phase compensator. The diagram in Fig. 5 describes the procedure for the Lead-Lag Phase compensation.

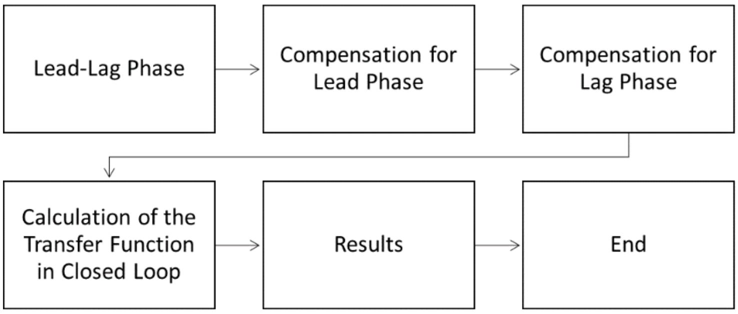



Figure 5 - Compensation method for Lead-Lag Phase

Different of the application of a Lead Phase or a Lag Phase compensator, this one increase not only in one, but in two degrees the transfer function of the system, because it is the addition of two poles and two zeros. And for the compensation dominant pole, it uses the dominant pole obtained in the lead phase compensation.

## RESULTS

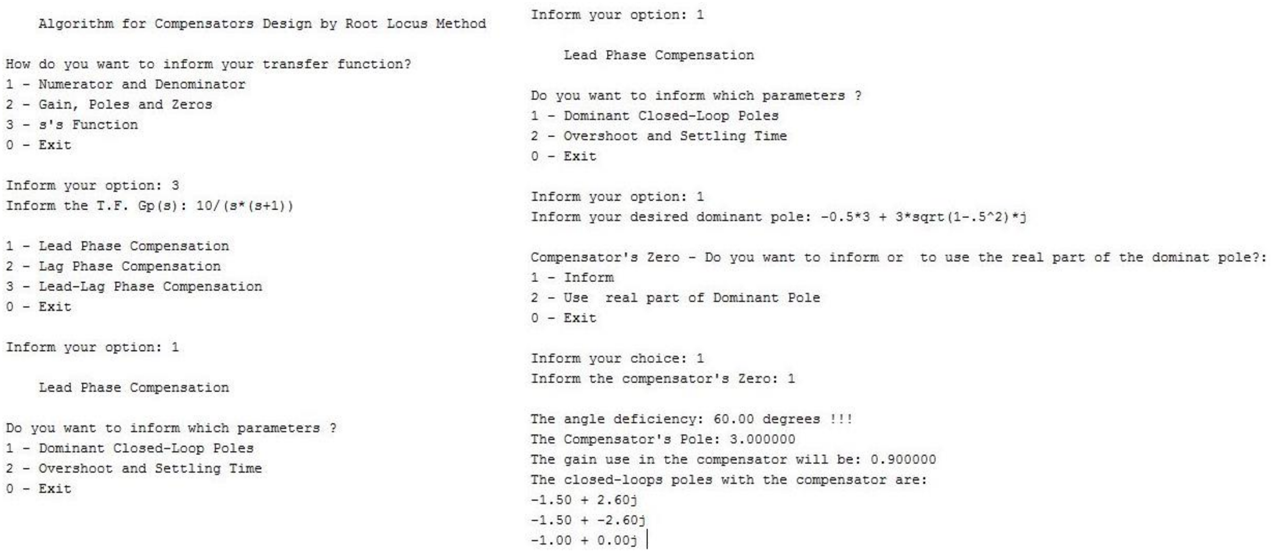
Two studies of transfer functions were conducted, taken from [19] and [20]. For a subsequent comparison of results, the first example was used. The example presented a position control system with a feedforward transfer function as in (9) and closed-loop poles as in (10).

$$G(s) = \frac{10}{s(s+1)} \tag{9}$$

$$s = -0.5 \pm j2.5981 \tag{10}$$

The desired damping ratio and undamped natural frequency of the closed-loop poles were 0.5 and 3 rad/sec, respectively.

Entering the information to the developed algorithm in the software MatLab®, this returns the following results:



```
    Algorithm for Compensators Design by Root Locus Method

How do you want to inform your transfer function?
1 - Numerator and Denominator
2 - Gain, Poles and Zeros
3 - s's Function
0 - Exit

Inform your option: 3
Inform the T.F. Gp(s): 10/(s*(s+1))

1 - Lead Phase Compensation
2 - Lag Phase Compensation
3 - Lead-Lag Phase Compensation
0 - Exit

Inform your option: 1

    Lead Phase Compensation

Do you want to inform which parameters ?
1 - Dominant Closed-Loop Poles
2 - Overshoot and Settling Time
0 - Exit
```

```
Inform your option: 1

    Lead Phase Compensation

Do you want to inform which parameters ?
1 - Dominant Closed-Loop Poles
2 - Overshoot and Settling Time
0 - Exit

Inform your option: 1
Inform your desired dominant pole: -0.5*3 + 3*sqrt(1-.5^2)*j

Compensator's Zero - Do you want to inform or  to use the real part of the dominat pole?:
1 - Inform
2 - Use  real part of Dominant Pole
0 - Exit

Inform your choice: 1
Inform the compensator's Zero: 1

The angle deficiency: 60.00 degrees !!!
The Compensator's Pole: 3.000000
The gain use in the compensator will be: 0.900000
The closed-loops poles with the compensator are:
-1.50 + 2.60j
-1.50 + -2.60j
-1.00 + 0.00j
```

Figure 6 – Algorithm for Compensator Design by Root Locus in MatLab ®

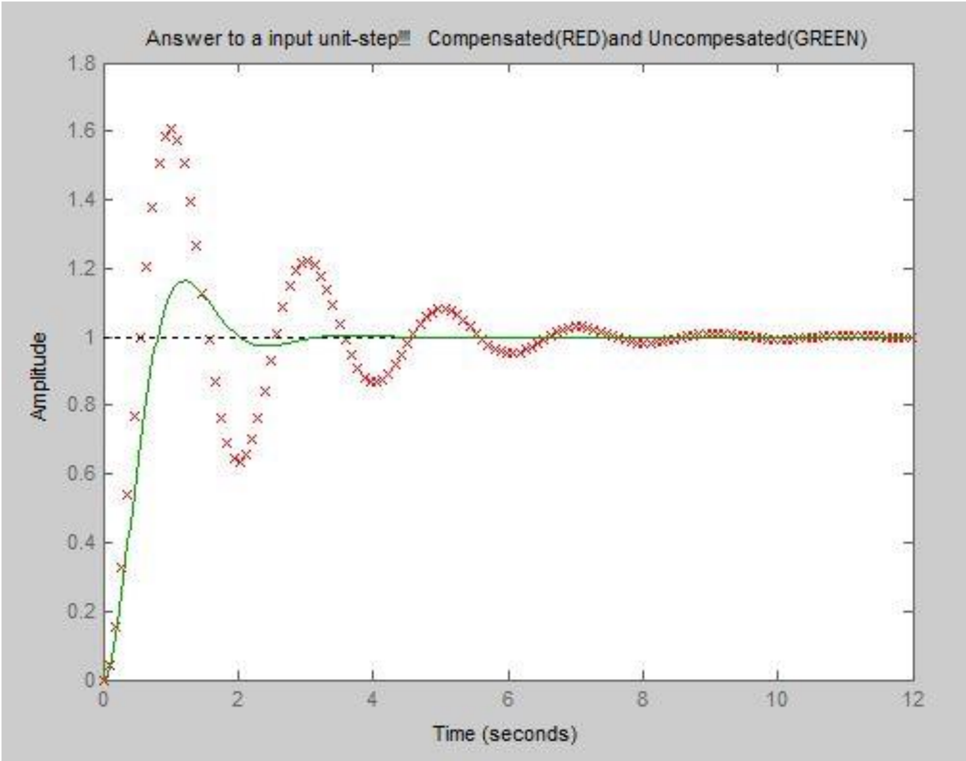



Figure 7 - Response Algorithm

In another case study used a sample taken from Castrucci et al (2011), this example give a transference's function Eq. (9) and is required to design a compensator for the overshoot (Mp(%)) is 16% and the Settling Time for 2% criterion is 2 seconds.

Using the algorithm, we get the following result:

```
    Algorithm for Compensators Design by Root Locus Method

How do you want to inform your transfer function?
1 - Numerator and Denominator
2 - Gain, Poles and Zeros
3 - s's Function
0 - Exit

Inform your option: 1
Inform the numerator vector of the plan Gp(s): [5]
Inform the denominator vector of the plan Gp(s): [1 1 0]

1 - Lead Phase Compensation
2 - Lag Phase Compensation
3 - Lead-Lag Phase Compensation
0 - Exit

Inform your option: 1

    Lead Phase Compensation

Do you want to inform which parameters ?
1 - Dominant Closed-Loop Poles
2 - Overshoot and Settling Time
0 - Exit

Inform your option: 2

Inform the Oveshoot(in %): 16

2 - Settling Time: 2% criterion
5 - Settling Time: 5% criterion

Inform which criterion: 2

Settling Time 2%% criterion (in seconds): 2

Compensator's Zero - Do you want to inform or  to use the real part of the dominat pole?:
1 - Inform
2 - Use  real part of Dominant Pole
0 - Exit

Inform your choice: 2

The compensator's Zero: 2.000000
The angle deficiency: 43.48 degrees !!!
The Compensator's Pole: 5.615051
The gain use in the compensator will be: 4.120088
The closed-loops poles with the compensator are:
-2.00 + 3.43j
-2.00 + -3.43j
-2.62 + 0.00j
```

Figure 8 - Algorithm for Compensator Design by Root Locus in MatLab ®

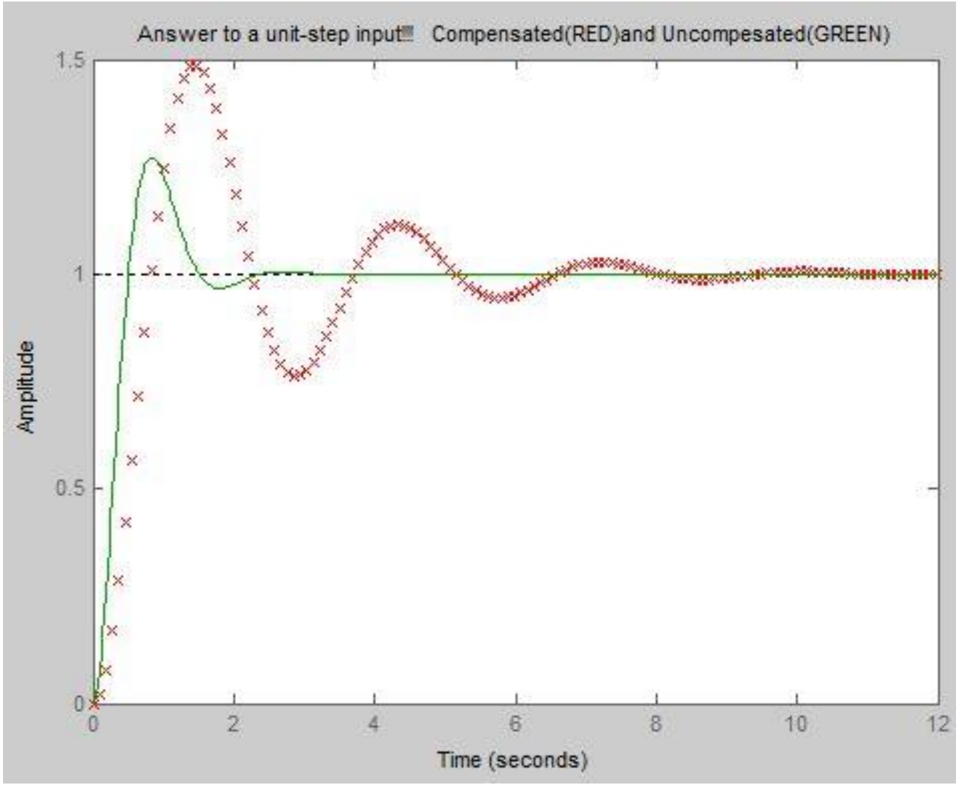



Figure 9 – Response Algorithm

## DISCUSSION

As can be seen in the results, the algorithm provides the main information of the compensator to be implemented to obtain user's desired system. This algorithm does not distinguish between Plants that can be analyzed, what makes it very versatile for the academic applications.

The type of compensator can be informed by the user or suggested by the algorithm, what allows it to be very didactic. The user also can choose the Compensator's Poles or use the one suggested by the algorithm, what gives more flexibility to the design of Compensators.

With the use of the software MATLAB® the results were obtained in a fast way and without the necessity of heavy handmade calculus.

## CONCLUSION

The algorithm developed proved to be reliable because of the fact of being compatible with any kind of Plants' Transfer Function analyzed by it.

By using a high performance and usability software, the algorithm brought the possibility of solving the problem of designing the most adequate compensator in a fast and organized way. In addition, it gives more flexibility to the user to define the parameters best suited to the application.

Based on the above, it can be said the algorithm developed is a powerful didactic tool to facilitate the study and design of Compensators by the root locus method, made by professors and students.

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