TRANSVERSE VIBRATION OF HORIZONTAL TUBES PARTIALLY FILLED WITH LIQUID

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Abstract. *This paper presents the results of an experimental and numerical investigation on the flexural vibration of horizontal tubes partially filled with liquid. Due to symmetry, tubes full of liquid have the same natural frequencies for transverse vibration in the vertical or horizontal direction. However, vibration tests performed with tubes partially filled with liquid show that the natural frequencies of the tube are not the same for vibrations occurring in the vertical and horizontal directions. CFD simulations of partially filled tubes oscillating harmonically indicate that the effective inertia of the pipe is different for oscillations in the vertical and horizontal directions. Through experimental tests and CFD analysis, added mass coefficients are proposed in order to calculate the natural frequencies of partially filled pipes vibrating in the horizontal plane.*

Keywords: *partially filled pipes, added mass coefficients, vibration*

1. INTRODUCTION

The knowledge of vibration characteristics of partially filled liquid systems is important for a great variety of engineering applications, such as storage tanks, flexible pipe systems and pipelines. The dynamic behavior of completely filled liquid pipes has been extensively investigated by many authors however; few studies have been reported concerning the vibration of partially filled horizontal systems, where the free surface of liquid is parallel to the horizontal axis of the container. An experimental investigation on the vibration characteristics of a partially filled horizontal cylindrical shell has been performed by Amabili and Dalpiaz (1995). Approximate analytical solutions to calculate the natural frequencies and mode shapes of a partially filled horizontal cylindrical shell are proposed by Amabili (1996). The dynamic characteristics of a partially filled and/or submerged, horizontal cylindrical shell were studied by Ergin and Temarel (2002). The paper of Chan and Zhang (1995) was the only study found about the vibration of a partially filled tube; nonetheless the authors have studied the case of free vibration of a vertical partially filled tube, where the tube can be separated in two uniform parts, one with liquid and the other empty. For the partially filled horizontal tube, the axial symmetry is lost, resulting in a rather complex problem.

The Presence of fluid may add mass, stiffness and damping, changing the structural response of liquid filled systems. Natural frequencies of flexural vibration of completely liquid filled tubes can be calculated adding the total mass of liquid to the tube, as the main consequence of the liquid is to increase the mass. The same is not true for the partially liquid filled horizontal tube. In the present work, the authors have investigated the flexural vibration of partially liquid-filled horizontal tubes. We shall restrict our attention to the case where the transversal amplitudes of tube oscillations are small and the sloshing phenomenon doesn't occur.

Firstly, based on vibration tests with an acrylic tube partially filled with water, the authors concluded that the natural frequencies of vibration of the partially filled horizontal tube, corresponding to the modes in the vertical plane, can be calculated adding the mass of liquid to the mass of tube. However, only part of the mass of liquid is effectively added to the tube when it oscillates in the horizontal plane. Two coupling mechanisms between the liquid and the pipe are considered to explain these experimental results: the liquid pressure exerted upon the surface of pipe and the friction coupling. The liquid pressure acting on the internal surface of the pipe is considered to be due to the inertial forces of liquid (Amabili, 1996). The friction couple is due to the shear stress acting at the interface between the liquid and pipe wall in resistance to the relative motion between liquid and pipe.

To calculate the natural frequencies of the flexural vibration modes in the horizontal plane, it's necessary to know the mass of liquid added to the tube. In this work, the authors propose the use of added mass coefficients to calculate the natural frequencies of flexural vibration modes of partially liquid filled horizontal tube. Experimental and numerical analyses have been performed to determine the coefficients of added mass for different filling levels of liquid. These coefficients are used to calculate the natural frequencies of vibration of the partially filled tube corresponding to the vibration modes in the horizontal plane. To the best of the authors' knowledge, the results reported in this work have not been reported elsewhere in the literature.

2. EXPERIMENTAL VERIFICATION

The experimental verification of the effect of liquid on the natural frequencies of partially filled horizontal tubes was performed using the experimental setup shown in Figure 1. An acrylic tube was employed in the experiments for two reasons: transparency and low density. The transparency of the acrylic allows visualization of the liquid motion during the tests and allows us to place the tube in the exact horizontal position, as the liquid level along the length of the tube can be observed and used to find the horizontal position of the tube. The lower density of acrylic, when compared to metals, is important because, when the liquid density is comparable with the tube density, the added mas becomes a significant portion of the total mass, therefore the effect of the liquid on the natural frequencies of the tube will be higher.

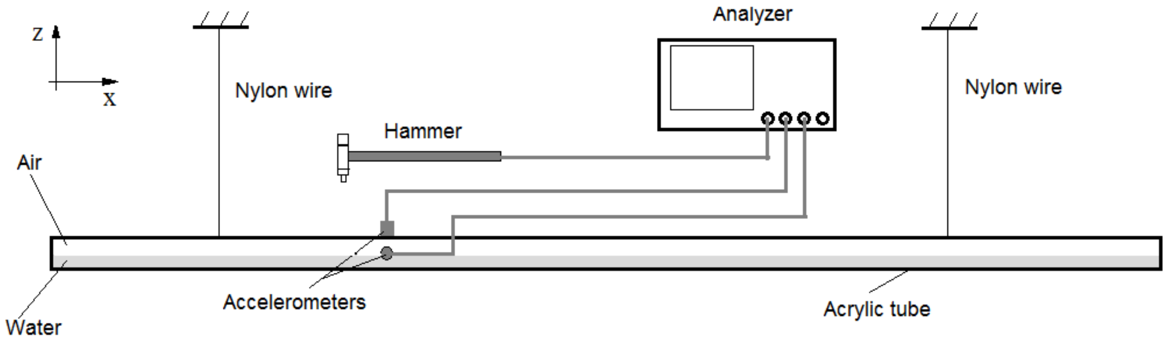

Figure 1. Experimental setup to measure the natural frequencies of the horizontal tube which is partially filled with liquid.

A 1804 mm long acrylic tube of 50.8 mm outside diameter and 44.8 inside diameter, closed at both ends with plastic caps and supported horizontally by two nylon wires, was excited with impacts applied by an instrumented hammer model 086C02 from PCB™. Lightweight accelerometers model 333B50 from PCB™ were used to measure the vibration response of the tube in the z and y direction. The signals from the instrumented impact hammer and accelerometers were sent to a Agilent™ 35670A dynamic signal analyzer (DSA) to obtain the frequency response functions (FRF). Vibration tests were performed with the tube empty, partially filled with water to different filling levels and completely filled. With the DSA on the average mode set to 10 samples, and using an exponential window and a force window, a series of slight impacts were applied to the tube with the impact hammer. It was observed that the impacts produced only very small waves on the surface of water. Axial movement of liquid was not observed.

Two FRF were obtained for each filling level of water, and for the tube empty and completely filled: a FRF corresponding to the relationship between the impact force and the acceleration measured in the z direction and a FRF corresponding to the relationship between the impact force and the acceleration measured in the y direction.

Due to the lack of axial symmetry of the partially filled pipe, and to facilitate the comprehension of the present work, the natural frequencies of vibration were separated in two groups: "the vertical natural frequencies of vibration" and the "horizontal natural frequencies of vibration". These natural frequencies correspond respectively to the vibration modes of the tube in the vertical plane, "the vertical modes", and the vibration modes in the horizontal plane, "the horizontal modes". Figure 2 shows the variation of the first natural frequency of vibration in function of the volume fraction of water. The lower line in the plot corresponds to the vertical natural frequencies and the upper line in the plot to the horizontal natural frequencies of vibration in function of the volume fraction of water.

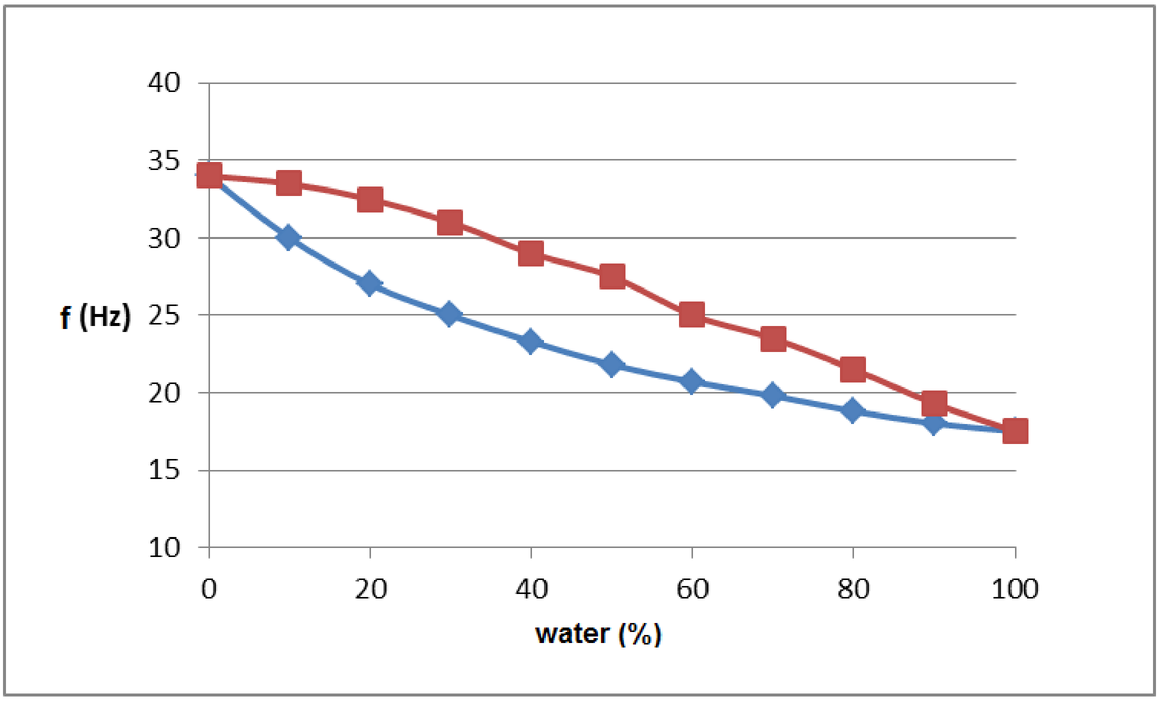

Figure 2. First natural frequency of vibration of the horizontal tube partially filled with liquid in function of the volume fraction of water. The lower line in the plot corresponds to the vertical natural frequencies and the upper one to the horizontal natural frequencies.

One can note that for the same volume fraction of water, the vertical natural frequency is lower than the horizontal natural frequency of vibration. This difference is because only part of the mass of liquid is effectively added to the tube when it oscillates in the horizontal plane. Note also that for the empty (0% of water) or completely filled tube (100 % of water) the horizontal and vertical natural frequencies are the same, as the axial symmetry exists. As will be demonstrated later, the vertical natural frequencies of the partially filled tube can be calculated adding the mass of water to the mass of tube.

A simple equation, based on the Euler-Bernoulli beam theory, is proposed to calculate the natural frequencies of vibration of the horizontal tube partially filled with liquid.

$$\omega = \beta^2 \sqrt{\frac{EI}{m_T + \alpha m_L}} \tag{1}$$

Where:

ω – Natural frequency of vibration
β – Factor that depends on the boundary condition and the vibration mode
E – Young's elastic modulus of the tube material
I – cross sectional area moment of inertia of tube
m_T – mass of tube per unit length
m_L – mass of liquid per unit length
α – coefficient of added mass of liquid

The coefficient of added mass of liquid α, $0 < \alpha \leq 1$, is a function of liquid level, among other factors, as the liquid viscosity. This coefficient is used to compute the part of the mass of liquid oscillating together with the tube, and is obtained dividing the mass of liquid displaced m_{Ld} by the mass of liquid m_L ($\alpha = m_{Ld} / m_L$) . If the tube is completely filled, then $\alpha = 1$. For the tube vibrating in the plane xz (vertical plane), it is assumed that all the mass of liquid per unit length m_L oscillates with the tube ($\alpha = 1$), for any liquid level. Experimental analyses using the experimental setup shown in Fig. 1 and CFD analyses presented in the next section were performed to determine the coefficient of added mass of liquid α when the tube oscillates in the horizontal plane.

3. CFD MODEL

The unsteady incompressible free-surface flow is modeled in three-dimensions, with mass and momentum being conserved in the fluid domain. To determine the velocity and pressure fields, a homogeneous multiphase Eulerian-Eulerian fluid approach is used, with a volume fraction ϕ being defined for water and air. Cartesian coordinates are defined with the *x-z* plane at the interface, such that water occupies the region $z \leq 0$ when at rest. Continuity and Navier-Stokes equations are given by,

$$\frac{\partial(\rho)}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \tag{2}$$

$$\frac{D(\rho \vec{u})}{Dt} = \nabla \cdot \left(\vec{\vec{T}}\right) + \rho \vec{f} \tag{3}$$

where,

$$\vec{\vec{T}} = -p\delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{4}$$

$$\rho = \sum_{l=1}^{2} \phi_l \rho_l \,, \qquad , \qquad \mu = \sum_{l=1}^{2} \phi_l \mu_l \tag{5}$$

with $l=1,2$ representing air or water; ρ and μ are the fluid's density and viscosity; $\boldsymbol{u}=(u,v,w)$ is the fluid velocity; $\boldsymbol{f} = (0,-g,0)$ is the acceleration due to gravity; $\boldsymbol{T}$ is the stress tensor for a Newtonian fluid, which includes the effects of pressure p and viscous forces. The Shear Stress Transport (SST) model of turbulence was used. Neumann and Dirichlet conditions are applied to the pipe's wall i.e. the pipe is rigid, impermeable and has no-slip boundary condition.

To complete our model we assume that initially all the fluid domain is at rest; suddenly a harmonic horizontal displacement is imposed to the reservoir at time $t = 0$, representing approximately the hammer impact in the wall.

$$y(t) = 0.0001\sin(100\pi t) \tag{6}$$

where the amplitude of displacement is 0,1 mm and the frequency is 50 Hz, representing approximately the average of tube’s movement in the impact testing.

The initial value problem is solved by the commercial CFD package ANSYS CFX release 15.0 (ANSYS 2015), which makes use of the finite volume method (Versteeg and Malalasekera 1995, Maliska 2004). The mesh was refined near the free surface and walls in order to better predict the nonlinear profile of the fluid motion. Figure 3 shows one of the computational grids used, which contains 20,800 hexahedral elements. A refined time grid is employed with time steps of 0,3 ms for 0,03 s of total time. All the computations were carried out on a 64 bit, 3.40 GHz Intel Core i7-2600 processor with 16 Gb of RAM. Each test has CPU time of $1.153\text{x}10^3$ s.

The pressure field is estimated and integrated across the pipe's area for each time step. The evolution of the horizontal and vertical forcing at the pipe's wall is shown in figure 4. A harmonic behavior was observed in the case of horizontal forcing. An added mass coefficient is calculated by,

$$\alpha = \frac{m_{Ld}}{m_L} \tag{7}$$

where the denominator is the total mass of water inside the pipe and the numerator is the ratio of the maximum horizontal force and the maximum acceleration due to the imposed movement.

If we derive Eq. (6) twice we can find the maximum acceleration in the movement

$$a_{max} = 9{,}87\ m/s^2 \tag{8}$$

The force peaks, in the harmonic movement seen in Fig. 4, are divided by this maximum acceleration to obtain the mass of fluid that follow the tube.

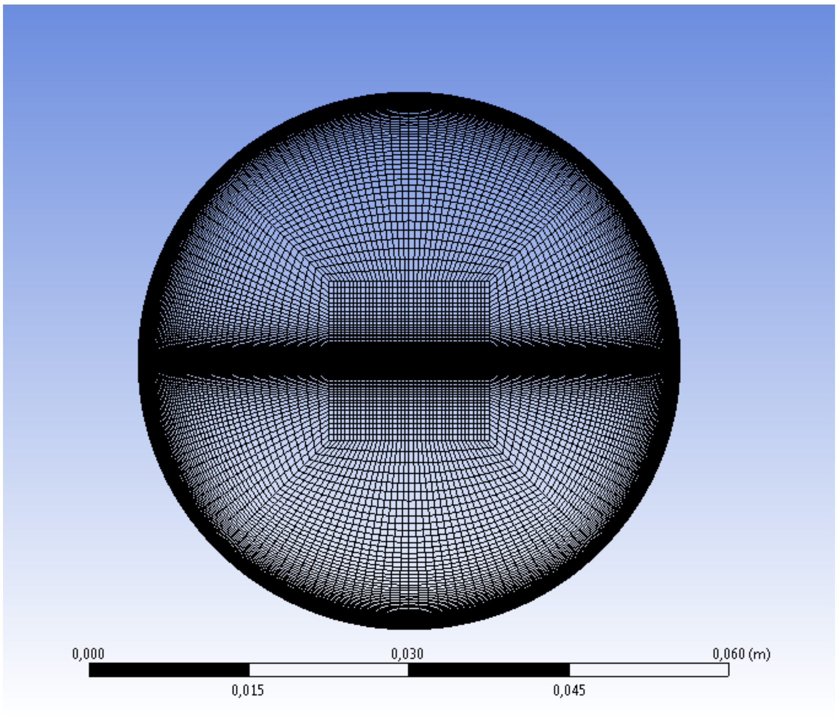

Figure 3. Computational Grid for a 50% water-filled pipe.

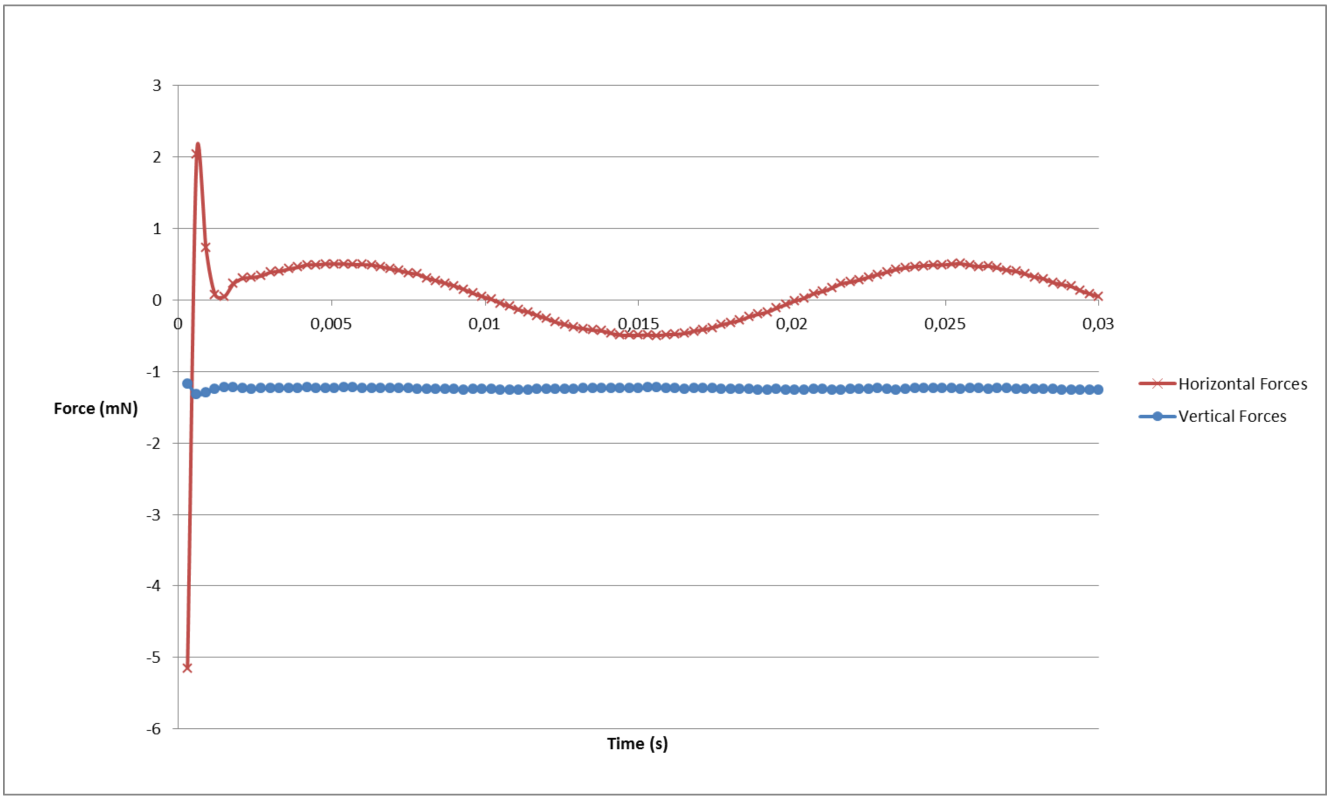

Figure 4. Horizontal and vertical forcing at a 50% water-filled pipe

4. RESULTS

Figure 5 shows a comparison between the coefficients of added mass of liquid α determined numerically and experimentally, as previously described. One can note that the numerical results are in good agreement with the experimental ones. The experimental values of α were obtained as follows: given the measured values of natural frequencies ω, one can calculate the corresponding values of α manipulating the eq.(1).

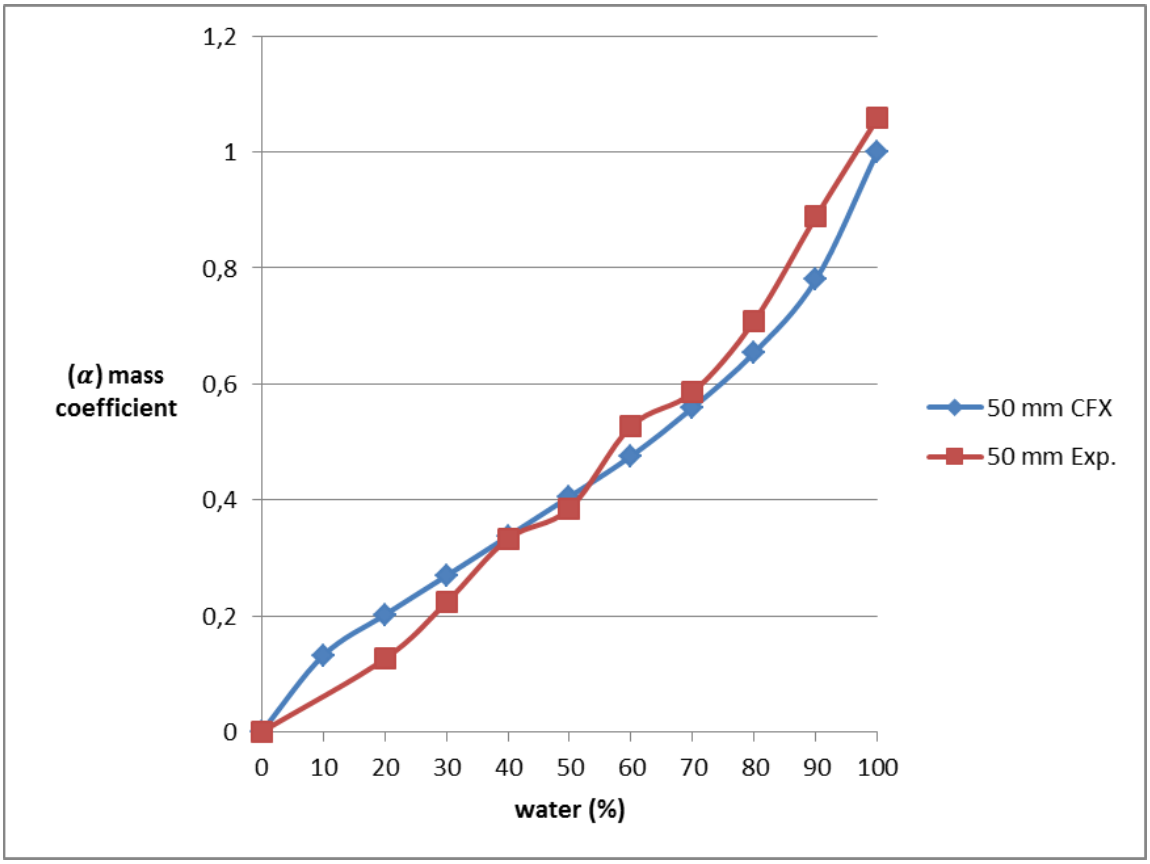

Figure 5. Variation of the coefficient of added mass of liquid α in function of the volume fraction of water

The graph in Fig. 5 shows that, for the tube with circular cross section, the coefficient α increases with the level of liquid inside the tube. The mass of liquid added to the tube is very low at low filling levels and, for the tube completely filled, all the mass of liquid oscillates together with the tube ($\alpha = 1$).

Figure 6 shows a comparison between the natural frequencies obtained experimentally and the natural frequencies calculated using eq. (1). The upper lines in the plot correspond to the first natural frequency of tube vibration in the vertical plane, in function of the volume fraction of water. The lower lines correspond to the first natural frequency of tube vibration in the horizontal plane. The calculated natural frequencies for vibration in the vertical plane were obtained using $\alpha = 1$ in eq. (1), while the natural frequencies for vibration in the horizontal plane were obtained using the results for α given in Fig. 5, in eq. (1).

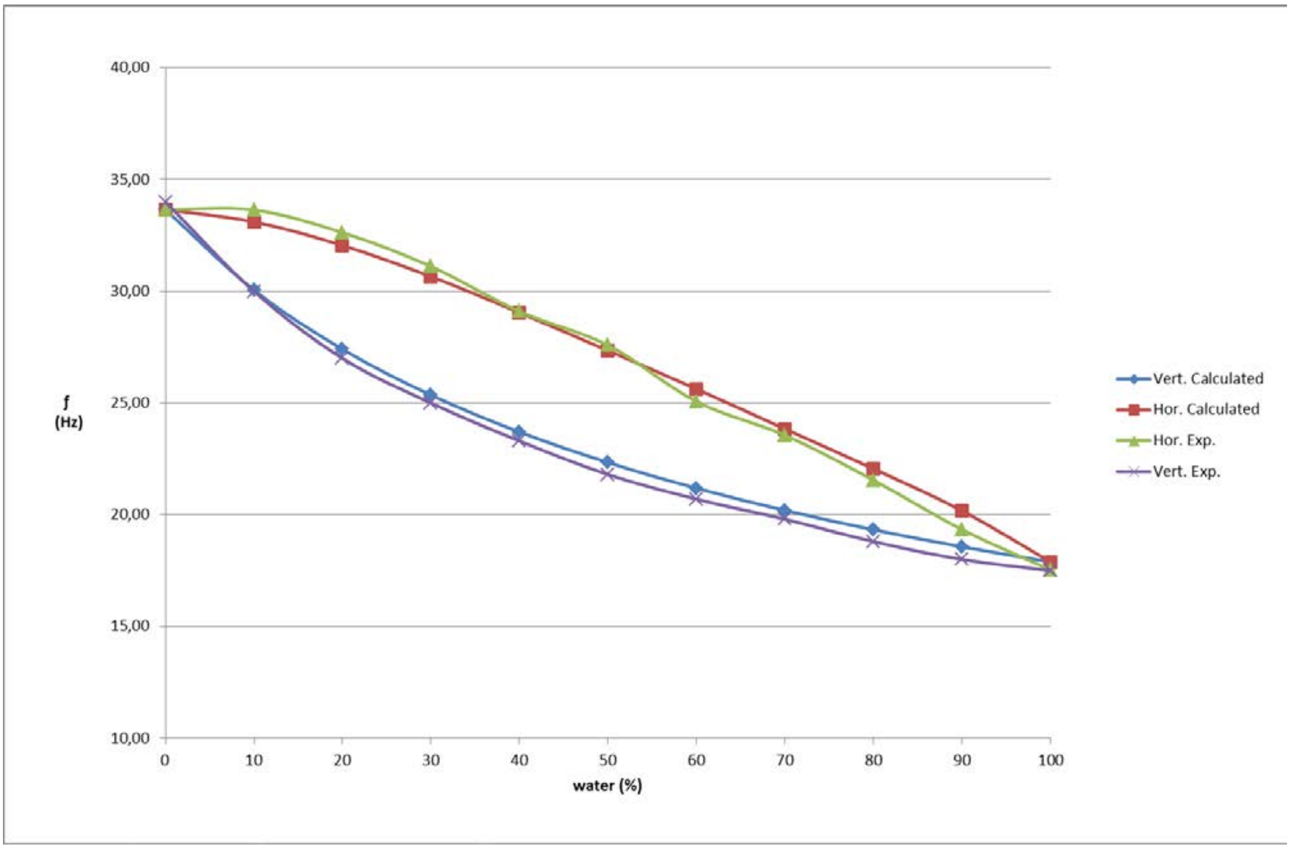

Figure 6. Comparison between the natural frequencies obtained experimentally and calculated using eq. (1).

One can note in Fig. 6 that the natural frequencies calculated using eq. (1) agree very well with the natural frequencies obtained experimentally.

5. CONCLUSIONS

This paper presented an experimental and numerical study on the transverse vibration of horizontal tubes partially filled with liquid. Preliminary results of vibration tests shown that, when the tube oscillates in the horizontal plane, only part of the liquid oscillates together with the tube. To determine the mass of liquid effectively added to the tube, a coefficient of added mass of liquid was determined using vibration tests and CFD analyses. A simple equation based on the Euler-Bernoulli beam model, where the coefficient of added mass of liquid is used to consider the mass of liquid, is proposed to calculate the natural frequencies of the horizontal tube partially filled with liquid. Further studies are being done by the authors in order to evaluate the effect of liquid viscosity and frequency of vibration on the coefficient of added mass of liquid.

6. ACKNOWLEDGEMENTS

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