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A COMPARISON STUDY OF UNCERTAINTY ANALYSIS APPROACHES FOR ROTOR DYNAMICS APPLICATIONS

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Abstract. *The models of mechanical systems are inherently uncertain due to design fluctuations or to lack of knowledge about the system. Thus, it is essential to have a methodology that takes into account these uncertainties so that a reliable model results. Considering the various existing methods for modeling uncertainties in mechanical systems, the stochastic approach and the fuzzy logic are found among the most widely used. Each approach has its pros and cons and the knowledge of these features aids the choice of which methodology would be more appropriate for a given application. Therefore, this paper aims at analyzing the characteristics and performance of these two approaches. For this aim, a flexible rotor with operational uncertainties was modeled by using both stochastic methodologies and fuzzy logic as based on the so-called α – optimization method. Through numerical simulations the methodologies have been compared in terms of rotor orbits and frequency response functions.*

Keywords: *stochastic analysis, fuzzy analysis, rotor dynamics*

1. INTRODUCTION

Uncertainties are inherent aspects of mechanical systems, whether by actual lack of information or due to design fluctuations such as variations in operational conditions or due to damage. Knowing how these uncertainties affect the system performance is essential in designing robust systems. Thus, the replacement of the deterministic approach for one that takes into account such uncertainties is a natural evolution of design techniques.

There are various approaches for modeling uncertainties in mechanical systems. Among them, the stochastic approach and fuzzy logic are the most used. The stochastic methodology is based on the probability theory, while the fuzzy logic, first presented by Zadeh (1965), is an intuitive approach based on fuzzy sets and in the possibility theory (Zadeh, 1968). Both methodologies can be potentially applied to several problems in engineering, whereas for mechanical systems the stochastic approach shows a longer history of applications, and consequently is more established than fuzzy logic. However, the latter is gaining more attention recently and the literature shows some relevant applications.

Stochastic approaches, based on the stochastic finite element method, have been used to perform uncertainty analysis in rotor dynamics. Didier *et al.* (2011) quantified the effects of the uncertainties on the variation of the response of rotating systems. For this purpose, the authors coupled a Polynomial Chaos expansion with the stochastic finite element method. Koroishi *et al.* (2012) applied the stochastic finite element method to a flexible rotor with uncertain parameters. The uncertainties were modeled as homogeneous Gaussian random fields and discretized by the Karhunen-Lòeve series expansion.

Within the scope of rotor dynamics, the study of the fuzzy logic approach is recent. Lara-Molina *et al.* (2014b) analyzed the dynamics of flexible rotors under uncertain parameters modeled as fuzzy variables and fuzzy random variables. Cavalini *et al.* (2014) evaluated the dynamic behavior of a flexible rotor supported by two fluid film bearings with uncertain oil viscosity and radial clearance in both bearings. Cavalini *et al.* (2015) characterized the uncertain parameters of fluid film bearings as fuzzy variables. The numerical responses determined by the Fuzzy Finite Element Model were compared with those from experimental results.

It is worth mentioning that both above-mentioned approaches have applications in rotor dynamics. However, it is not of our knowledge the existence of a systematic study to find out which methodology is more suitable in modeling uncertainties in rotor dynamic applications. Then, in this context, the present contribution intends to evaluate the pros and cons of each approach for modeling uncertain parameters. First a review of the main features of each methodology are presented; then, the methods are compared through numerical simulations of a flexible rotor with uncertain Young's Module supported by hydrodynamic bearings with uncertain oil nominal temperature.

2. STOCHASTIC APPROACH

In the stochastic approach, the purpose is to evaluate the randomness and variability of the model uncertainties; thereby the uncertainties are modeled as random variables or as random fields. Among the various stochastic techniques that aim at accomplishing this goal, the stochastic finite element and the Monte Carlo simulation are certainly the most widely used.

2.1 Monte Carlo Simulation

The Monte Carlo simulation is a class of methods that can be understood as an exhaustive search technique. The idea is to infer the statistical behavior of the model response without previous knowledge of the statistical behavior of the uncertain quantities.

To accomplish this task, the uncertain quantities are considered as random fields, and then a large number of samples, that are simply realizations of this field, are simulated. After, the uncertain quantity response is obtained by means of the deterministic model of the system. With this result the statistical behavior of the outputs (the model response) are estimated. Finally, from these estimates the variability of the model is inferred.

This approach requires the analysis of a large number of simulations, which may imply high computational cost, depending on the complexity of the system.

2.2 Stochastic Finite Element

The stochastic finite element method (SFEM) can be viewed as an extension of the finite element method. Then, it can be understood as a finite element method with uncertain quantities.

Consider a mechanical system with deterministic properties and ruled by partial differential equations. In the finite element method the geometry of the system is discretized on a set of points that form the nodal mesh. With this discretization the model response is approximated by the nodal displacements. If the properties of the system are uncertain, it can be modeled as random fields, and the system is now ruled by stochastic partial differential equations. A random variable is completely determined by its value in every event. Adopting the same spatial discretization as the one used in the finite element method results in selecting a finite set of events, so that the random field is represented incompletely.

To discretize the "random dimension" more efficiently, a series expansion technique is usually associated with SFEM. The most used one is the Karhunen-Lòeve series expansion, as presented by Ghanem and Spanos (1991). Another possibility is the use of other existing orthogonal series expansions, as showed by Zhang and Ellingwood (1994).

2.2.1 Karhunen-Loève Series Expansion

The Karhunen-Loève expansion of a random field is based on the spectral decomposition of its autocovariance function, and is a continuous representation of the random field expressed by the superposition of orthogonal random variables weighted by deterministic spatial functions (Ghanem and Spanos, 1991).

The autocovariance of a one-dimensional random field $H(\boldsymbol{x},\theta)$, where $\boldsymbol{x}$ denotes the spatial dependence of the field and θ represents a random process, can be defined as shown in Eq. (1).

$$C_{HH}(\boldsymbol{x},\boldsymbol{x}') = \sigma(\boldsymbol{x})\sigma(\boldsymbol{x}')\rho(\boldsymbol{x},\boldsymbol{x}') \tag{1}$$

The set of deterministic functions for which any event of the field is expanded with respect to a given geometry Ω is defined by the eigenvalue problem presented in Eq. (2).

$$\forall\ i = 1, \dots\ \int_{\Omega} C_{HH}(\boldsymbol{x},\boldsymbol{x}')\,\varphi_i(\boldsymbol{x}')d\Omega_{x'} = \lambda_i\varphi_i(\boldsymbol{x}) \tag{2}$$

Due to the mathematical properties of the deterministic function $\{\varphi_i\}$ (the eigenfunctions of the autocovariance $C_{HH}(\boldsymbol{x},\boldsymbol{x}')$) that are the solution of Eq. (2), any event of the random field can be expanded as show in

Eq. (3), where $\mu(x)$ is the mean value of the field, λ_i are the eigenvalues of the autocovariance function $C_{HH}(\boldsymbol{x}, \boldsymbol{x}')$, and $\xi_i(\theta)$ are random variables.

$$H(\boldsymbol{x}, \theta) = \mu(\boldsymbol{x}) + \sum_{i=1}^{\infty} \sqrt{\lambda_i}\, \xi_i(\theta) \varphi_i(x) \tag{3}$$

The intrinsic eigenvalue problem in the expansion can be analytically solved only for a few autocovariance functions and for some geometries. Detailed solutions for triangular and exponential covariance functions and one-dimensional homogeneous random fields are shown in Ghanem and Spanos (1991), for $\Omega = [-a, a]$. Except for these particular cases the eigenvalue problem has to be solved numerically, which may prove to be a complex task.

2.2.2 Ortoghonal Series Expansion

The Karhunen-Loève series expansion is an efficient form of representing a random field. However it demands solving an integer eigenvalue problem to determine the complete set of orthogonal functions that form the base of the expansion. However, the expansion can be performed with a previous definition of a complete set of orthogonal functions, as proposed by Zhang and Ellingwood (1994), thus avoiding the eigenvalue problem.

This set of orthogonal functions can form the basis of a vectorial space of real random variables with finite second moment (a *Hilbert* space). A random field $H(\boldsymbol{x}, \theta)$ of mean function $\mu(\boldsymbol{x})$ can be defined as a curve in this space. Any realization of this random field is a function in the *Hilbert* space, and it can be expanded through orthogonal functions. Thus the expansion in Eq. (4), where $\xi_i(\theta)$ are zero-mean random variables, is valid.

$$H(x, \theta) = \mu(x) + \sum_{i=1}^{\infty} \xi_i(\theta) h_i(x) \tag{4}$$

Zhang and Ellingwood (1994) showed that through spectral decomposition of the covariance matrix it is possible to transform the correlated random variables used in the orthogonal series expansion into uncorrelated random variables. With this transformation, it is possible to demonstrate that expanding a random field on an orthogonal base is equivalent to expanding the random field by using the Karhunen-Loève expansion, with the advantage of not solving an integer eigenvalue problem.

2.3 Fuzzy Logic

The fuzzy sets theory was first introduced by Zadeh (1965) with the purpose of formalizing the notion of graded membership, represented by the membership function. With this concept, the fuzzy sets are understood as the counterpart of the *Boolean* notion of regular sets.

The membership function of a fuzzy set can basically be seen from three different perspectives: a *degree of similarity*, in which the membership function quantifies how similar an element of the set is compared to a prototype element; a *degree of preference,* where the membership function quantifies the preference or the feasibility of an element of the set; or a *degree of uncertainty,* in which the membership function is the degree of possibility that a determined parameter u has the value of an specific element x of the set (Dubois and Prade, 1997). The *degree of uncertainty* notion is the base of the possibility theory (Zadeh, 1968) and the main tool for fuzzy uncertainty analysis.

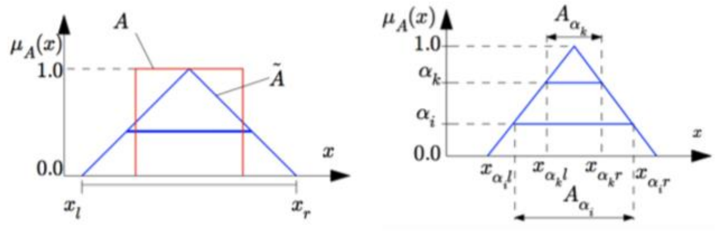
Through the fuzzy set theory, uncertainties in which the statistical process that describes the random variables is unknown are modeled. Computationally the uncertainties are modeled as fuzzy numbers where the actual value of the parameter is unknown but limited by an interval weighted by a membership function.

Following the degree of uncertainty interpretation, the membership function of a fuzzy set is the $[0; 1]$ continuous interval that weights the degree of pertinence of the element x with respect to the fuzzy set $\tilde{A}$. Thus the values of $\tilde{A}(x)$ close to 1 indicates a high compatibility of the element x with the set $\tilde{A}$.

For computational purposes, the fuzzy set $\tilde{A}$ can be represented by means of subsets that are denominated α-levels. The α-levels are *crisp* sets (continuos and real intervals) of all elements that has membership functions values equal or higher than α. Figure 1 highlight the concepts of fuzzy set and α-levels.

In the uncertainty analysis of a model via fuzzy logic, the uncertainties and innacuracies of the model are represented as fuzzy inputs and the model itself is a fuzzy function that maps these inputs. There are various forms of mapping a fuzzy vector. The most known method is the extension principle, a mathematical precept that insures a fuzzy extension for any mathematical function, i.e., the mapping of the model uncertainties (fuzzy inputs) is made by the deterministic evaluation (model evaluation) of the fuzzy vector.

Based on the generalization of a function $f: \boldsymbol{X} \rightarrow \boldsymbol{Y}$ mapping fuzzy subsets of $\boldsymbol{X}$ into fuzzy subsets of $\boldsymbol{Y}$, if $\boldsymbol{x}$ has a degree of membership μ_X, $y = f(x)$ is assigned a degree of membership μ_Y. Let the fuzzy sets $A_i, i = 1 \dots n$ defined on X_i, and $B = f(A_i)$; then, the membership function of B, by the extension principle, is given as $\mu_B(y) = max_{y=f(A_i)}\left(\min\left[\mu_{A_i}(x_i)\right]\right)$.



a) Fuzzy set.

b) The α-levels.

Figure 1.Fuzzy set and α-level representation.

Another method for mapping fuzzy input vectors using the deterministic model is the α-level optimization. The method can be summarized as follows: the fuzzy inputs are discretized by means of α-cuts creating then a crisp subspace that is the search space of the optimization problem; the optimization is then carried out with the purpose of finding the minimum and maximum values of the model output in each α-level, whereas the minimum value corresponds to the lower bound and the maximum value is the upper bound of the α-level interval of the output. Figure 2 ilustrates the α-level optimization method.

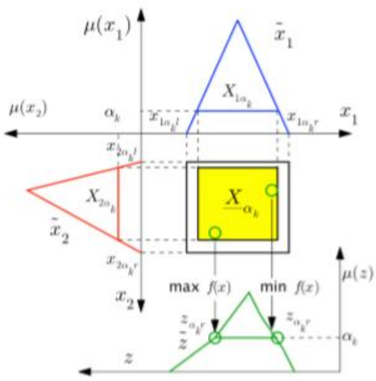


Figure 2.α-level optimization.

The α-level optimization method is an eficient methodology for mapping fuzzy vectors. Besides, it is a powerful tool for fuzzy uncentainty analysis. However, the effectiveness of the method is highlly related to the performance of the optimization technique.

3. NUMERICAL SIMULATIONS

Aiming at evaluating the performance of the techniques previously described, two scenarios regarding rotor dynamics were studied. The first scenario resumes to the frequency domain analysis of a flexible rotor with uncertain Young's Module, while in the second scenario the time domain analysis concerning the orbits of the same flexible rotor was evaluated. In the first scenario, the analysis and the parametrization of the finite element stiffness matrix was performed following the procedure presented by Koroishi *et al.* (2012).

In the simulations, the flexible rotor is composed of a horizontal flexible steel shaft discretized into 13 equally spaced Timoshenko's beam elements, three rigid steel discs, namely D_1 (node #3), D_2 (node #6), and D_3(node #11), and two cylindrical hydrodynamic bearings (B_1 and B_2, located at the nodes #1 and #13, respectively). The values of the physical and geometrical characteristics used to generate the FE model are given in Tab. 1 and a schematic representation of the FE model is shown in Fig. 3.

Three different methods relying in the stochastic approach were used considering Gaussian random fields with 5% deviation of the nominal value: the Monte Carlo simulation combined with Latin Hypercube, the SFEM associated with Karhunen-Loève series expansion with ten terms and exponential autocovariance function, and SFEM associated with Legendre Polynomials series expansion (an orthogonal series expansion) with four terms and the same autocovariance function as the Karhunen Loève expansion. Both series expansion were combined with Latin Hypercube considering 100 samples.

Table 1.Physical and geometrical characteristics of the rotor.

Elements	Properties	Values		
Shaft	Length (m)	1.3		
	Poisson's coefficient	0.3		
	Young's Module (Pa)	2.10^{11}		
	Density (kg/m^3)	7800		
Disks		D_1	D_2	D_3
	Thickness (m)	0.05	0.05	0.06
	Inner Radius (m)	0.05	0.05	0.05
	Outer Radius (m)	0.12	0.2	0.2
Bearings	Length (m)	0.025		
	Radial Clearance (μm)	0.058		
Proportional coefficients	α	1.10^{-3}		
	β	1.10^{-7}		

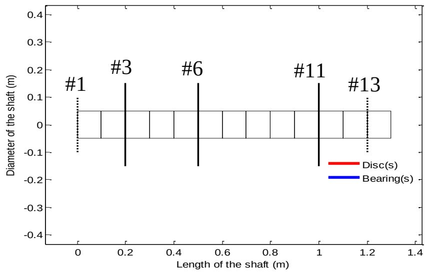


Figure 3.FE model.

Koroishi *et al.* (2012) showed that for a truncated Karhunen Loève expansion with exponential autocovariance function and correlation length equal to the finite element length, the mean square sense converge retaining ten terms. Zhang and Ellingwood (1994) demonstrated the convergence of a Legendre expansion of four terms with exponential covariance function. Also the recurrence characteristic of the Legendre series facilitates computation when the necessary integrations are performed by Gaussian quadrature as in this contribution.

In the Karhunen-Loève expansion, the eigenvalues and eigenfunctions of Eq. (3) are written as functions of the roots $\omega_i (i \geq 1)$ of two transcendental equations as showed by Koroishi *et al.* (2012) and summarized in Fig. 4.

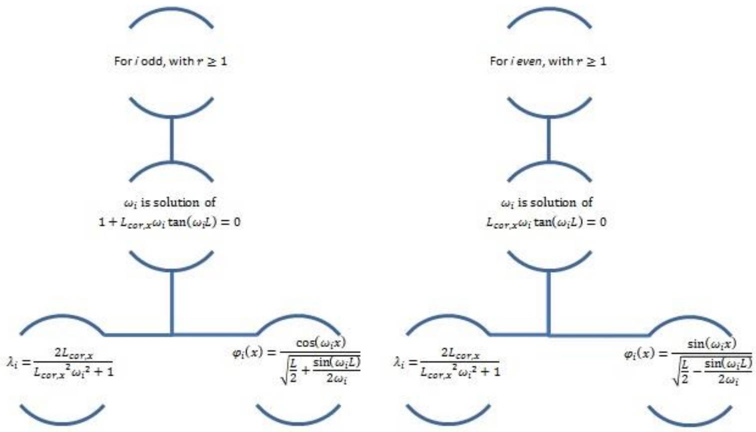


Figure 4.Eigenvalues and eigenfunctions of the Karhunen-Lòeve expansion.

The stiffness random matrix of the shaft is given by Eq. (5).

$$\boldsymbol{K}_s^{(e)}(\theta) = \boldsymbol{K}_s^{(e)} + \sum_{i=1}^{n} \overline{\boldsymbol{K}}_i^{(e)}\, \xi_i(\theta) \tag{5}$$

where $\boldsymbol{K}_s^{(e)}$ is the mean elementary stiffness matrix, and $\overline{\boldsymbol{K}}_i^{(e)}$ is given by Eq. (6).

$$\boldsymbol{K}_i^{(e)}(\theta) = \int_{x=0}^{L} \sqrt{\lambda_i}\, \varphi_i(x) \boldsymbol{B}^T(x) \overline{\boldsymbol{E}} \boldsymbol{B}(x) dx \tag{6}$$

wherein $\overline{\boldsymbol{E}}$ is the factored isentropic matrix containing the elastic material properties and $\boldsymbol{B}(x)$ is formed by differential operators appearing in the strain-displacement relations.

In the Legendre Polynomials expansion, the orthogonal base is obtained by normalizing and transforming the Legendre functions from the interval $[-1,1]$ to the problem domain. Using a covariance function as presented in Eq. (7) the random variables of the expansion are transformed into uncorrelated random variables through the spectral decomposition of the covariance matrix shown in Eq. (8).

$$C(x_1, x_2) = e^{-\frac{|x_1 - x_2|}{L_{cor,x}}} \tag{7}$$

$$C_{ij} = \int_\Omega \int_\Omega C(x_1, x_2) h_i(x_1) h_j(x_2) dx_1 dx_2 \tag{8}$$

With this transformation the stiffness random matrix in the expansion is obtained as shown in Eq. (6), with the difference that λ_i are the eigenvalues of the covariance matrix and the eigenfunctions are calculated through Eq. (9) in which $a_j^{(i)}$ is the ith column of the inverse of the transposed eigenvector matrix of C_{ij}.

$$\varphi_i(x) = \sum_j^n a_j^{(i)} h_j(x) \tag{9}$$

The fuzzy logic was implemented through fuzzy finite element wherein the stiffness matrix was factored following the same procedure as in the SFEM, and the uncertain Young's module was treated as fuzzy triangular number in which the bounds had the same values that the extreme values of the Gaussian fields considered in the stochastic analysis. The fuzzy variables were mapped through α-level optimization in which the optmization was performed by means of sequential quadratic programing (SQP) using the subroutine *fmincon* of MatLab®, given that the optimization problem for each α-level is an unconstrained linear problem (LP).

Figure 5 presents the results of the frequency analysis: it can be noticed that the Monte Carlo simulation and the fuzzy logic obtained equivalent uncertain envelopes, while the Legendre Polynomials expansion generates a "smaller" uncertain envelope, and the Karhunen-Loève expansion a "bigger" envelope. Regarding computational time, the three stochastic methods required more time to obtain the envelope than the fuzzy logic; however, none had prohibitive computational time.

Based on these results the second scenario was evaluated only for the fuzzy analysis and the Monte Carlo simulation, aiming at deeping tthe evaluation of the similarity among the methodologies responses. Uncertain nominal temperature was considered in this simulation, such as the viscosity varies around 10% its nominal value ($\mu = 0.4$ Pa.s).

As in the first analysis, the results of the dynamic analysis (the orbits) through fuzzy uncertainty analysis and Monte Carlo simulation presented identical values for both bearings and the three discs, thus confirming the equivalence of the responses generated by the methods. Figure 6 illustrates this point by presenting the responses obtained at the position of bearing #1 (close to the motor) and those at the position of disc # 3 (close to bearing # 2).

Again, the SQP was used in the α-level optimization regarding the linear influence of the viscosity both in the hydrodynamic coefficients and hydrodynamic forces of the bearings.

4. FINAL REMARKS

The goal of this paper was to determine the most suitable methodology for uncertainty analysis of rotating systems. Through numerical simulations, the most used stochastic techniques and the relatively new fuzzy analysis were compared evaluating the uncertain envelopes generated in the analysis of the dynamic behavior of a flexible rotor supported by hydrodynamic bearings. The uncertainties were introduced both in the Young's module of the shaft and in the nominal temperature. Through the uncertain envelopes, it can be inferred that the Monte Carlo simulation and the fuzzy logic techniques presented similar responses, thus indicating that these techniques lead to equivalent results, while the series expansion techniques associated with the stochastic finite element under or overestimate the uncertain envelopes indicating that these techniques may be inaccurately representing the variance of the uncertain parameter (Zhang and Ellingwood, 1994). However further investigations must be made to confirm this result.

Due to the relative mathematical complexity of the series expansion methods, and the similarity observed in the responses obtained by the Monte Carlo simulation and fuzzy analysis, these methodologies seems more suitable to perform uncertainty analysis of rotating systems. Nevertheless, the SFEM associated to series expansion are important methodologies for stochastic analysis due to their practically guaranteed convergence (Loève, 1977).

A form of understanding the similarity between Monte Carlo simulation and fuzzy logic is that both methodologies can be related to optimization problems. The Monte Carlo simulation can be viewed as a population optimization problem, where the uncertain envelope is obtained through the evaluation of a large number of events (realizations of

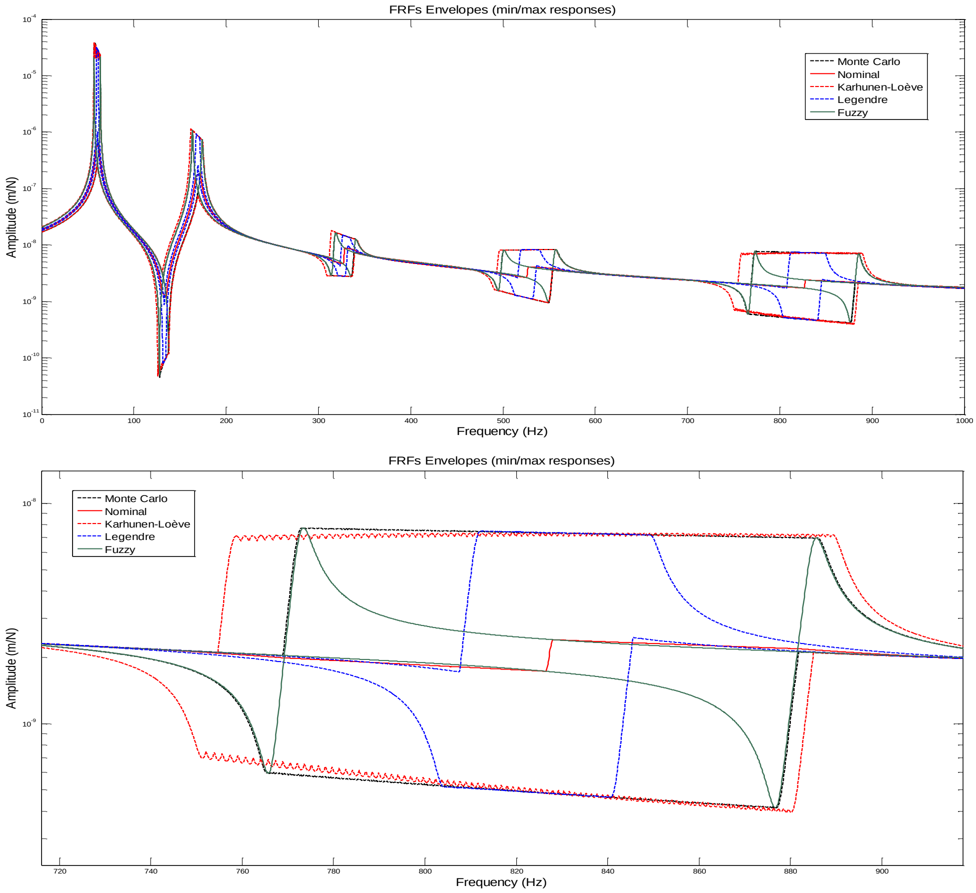


Figure 5.FRFs envelopes.

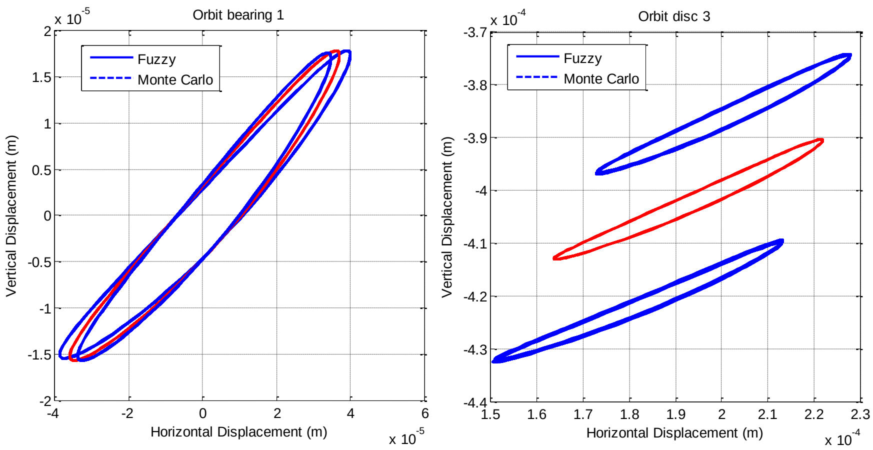


Figure 6.Orbits envelopes (min/max responses) obtained in the dynamic analysis.

the random field) in which is sought the individuals that generate the maximum and minimum responses. The Fuzzy logic based in the α-level optimization leads to unconstrained linear or nonlinear optimization problems formulated for each α-level. In each of the α-levels the maximum and minimum values of the model response are obtained, thereby creating the uncertain envelope. If the population optimization problem of the Monte Carlo Simulation and the unconstrained optimization problems of the fuzzy analisys are carried out properly, it is expected that the responses (Monte Carlo and Fuzzy) to be identical.

Another important feature of the Monte Carlo simulation and the fuzzy logic is its mathematical simplicity; however, this mathematical simplicity is counterweighted by the computational cost of the correponding algorithms. Depending on the complexity of the problem analyzed, both approaches can impose unaffordable computational cost.

Based in these points it may be concluded that the choice of which methodology to be used for uncertainty analysis in mechanical systems is problem dependent. However, the Monte Carlo simuation and the fuzzy logic appears to be the most indicated methodologies to perform uncertainty analysis of rotating systems, if the computational time is acceptable.

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