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AN ANALYTICAL STUDY THROUGH THE JACOBI INTEGRAL TO OBTAIN MULTIPLE REVOLUTION TRAJECTORIES TO THE MOON

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Abstract. *A study about Kepler's energy of the space vehicle on the arrival trajectory at the insertion point in a low Moon orbit for Earth-Moon bi-impulsive internal trajectories is performed in this paper. The classic planar circular restricted three-body problem (PCR3BP) is utilized as the dynamical model to describe the motion of the space vehicle. In order to determine the internal trajectories, a new two-point boundary value problem (TPBVP) is proposed in such way that the Jacobi Integral is prescribed. In this new problem, analytical expressions for the velocities increments are derived from the Jacobi Integral expression and a relation between the Jacobi Integral and the arrival Kepler's energy is obtained. In this way, a critical value for the Jacobi Integral is obtained in which the Kepler's energy becomes negative and a ballistic capture by the Moon can be accomplished by the space vehicle. The results also prove that an increasingly value of the time of flight is not followed by the increase of the Jacobi Integral value, which means that the determination of new trajectories becomes more difficult with large Jacobi Integral values.*

Keywords: *Earth-Moon trajectories, Jacobi Integral, ballistic capture, three-body problem.*

1. INTRODUCTION

In the last fifty years, the mankind has sent almost one hundred missions to the moon (Moskovitz, 2011). The Grail mission is one of the latest mission and its objective is to study the lunar gravitational field (Zuber *et al.*, 2013) in order to find some information about the origin of the Moon, and, consequently, the origin of Earth. In 1990, the Hiten spacecraft was launched and, in 1991, this spacecraft was placed into a lunar orbit performing a ballistic capture. This maneuver was done thanks to the work of Belbruno and Miller (1993) who have developed a new method to determine low-energy transfers based on the notion of weak stability boundary (WSB). Low-energy transfers were also obtained by connecting invariant manifolds (Koon *et al.*, 2001). These new methods explores the dynamic of the space vehicle in complex models (Yagasaki, 2004) such as the planar circular restricted three-body problem (PCR3BP) and the planar bi-circular restricted four-body problem (PBR4BP). According to the literature, there are four classes of dynamic models (Prado and Rios-Neto, 1993) which rule the motion of the space vehicle: the two body model, the perturbed two-body model, the three-body model and the N-body model. The patched-conic approximation and the so-called Hohmann transfer (Szebehely and Mark, 1998) are based on the two-body model and could be used as a preliminary analysis or to provide initial conditions for more complex models (da Silva Fernandes and Marinho, 2011). Besides the notion of weak stability boundary, there is another way to determine trajectories in which the fuel consumption must be minimum. This way of study is characterized by using optimization algorithms, especially those based on gradient methods, where the total velocity increment, which represents the fuel consumption, is minimized. This work is inserted in this second way of study, and it proposes a two-point boundary value problem (TPBVP) in order to obtain bi-impulsive Earth-Moon trajectories where the Jacobi Integral is prescribed. In this way, an analysis of the arrival Kepler's energy is allowed. Differently from the trajectory proposed by Belbruno and Miller (1993), where external trajectories with the Sun's influence are determined, this work also investigates the possibility of ballistic capture considering internal trajectories in the PCR3BP. Moreover, the proposed method determines both velocity increments through analytical expressions originated by developing the Jacobi Integral, which provides, initially, an idea of the total fuel consumption. Another advantage of prescribing the Jacobi Integral is that the intuition of the problem is facilitated. Thus, the search space to determine the initial condition can be reduced. However, a search for the initial condition usually must be performed. For instance, the work of Topputo (2013) has accomplished a search in a four dimensional space to generate the initial conditions in his algorithm to obtain optimal Earth-Moon trajectories with different times of flight.

2. OBJECTIVES

The main purpose of this work is to determine Earth-Moon interior bi-impulsive trajectories by solving a TPBVP with the classic PCR3BP as dynamical model and to study the possibility of ballistic capture by the Moon. The boundary

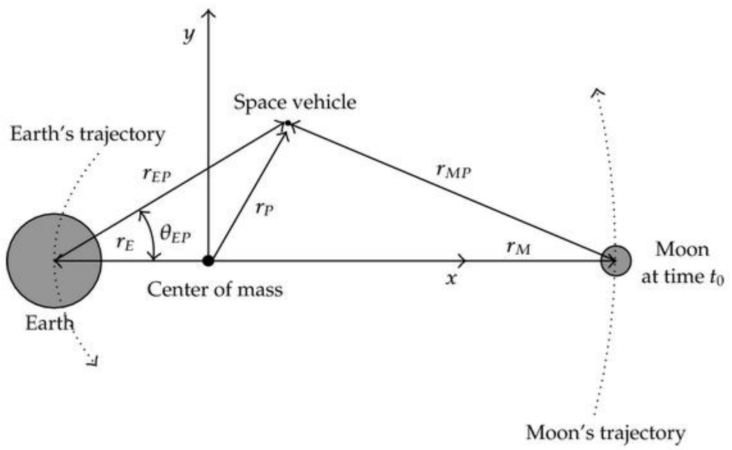

Figure 1: Inertial reference frame for the classic PCR3BP. Font: (da Silva Fernandes and Marinho, 2011)

problem is formulated in such way that the Jacobi Integral can be prescribed. In order to estimate the fuel consumption before solving the TPBVP, analytical expressions for the velocities increments are determined by developing the Jacobi Integral expression.

3. MATHEMATICAL FORMULATION

The purpose of this formulation is motivated by the orbital transfer problem, in which is desirable to transfer a space vehicle with infinitesimal mass from a low Earth orbit (LEO) to a low Moon orbit (LMO) by the application of two velocities increments. These increments are assumed to be impulsive and tangential to the terminal orbits. The first velocity increment is applied in the LEO and it accelerates the space vehicle putting it in a transfer trajectory. The second velocity increment is applied in the arrival near the Moon and it slowdowns the space vehicle circularizing its movement in the LMO. The terminal orbits are assumed to be circular. In this work, the dynamical model utilized to describe the motion of the vehicle is the classic PCR3BP. With this dynamical model in mind, the mathematical formulation described below is the foundation to construct a TPBVP in order to determine Earth-Moon trajectories.

3.1 The classic planar circular restricted three body problem (PCR3BP)

In this work, the two primary bodies of the PCR3BP are the Earth and the Moon, and, the third body is the space vehicle which is assumed to have infinitesimal mass. The mathematical formulation of this problem is taken from the paper by da Silva Fernandes and Marinho (2011). The classic PCR3BP considers that the Earth and the Moon rotate around the barycenter CM (center of mass) of the Earth-Moon system. The following hypotheses are considered:

1. Earth and Moon rotate around the barycenter CM of the Earth-Moon system;
2. The eccentricities of the Moon's orbit and the Earth's orbit around the barycenter CM are neglected;
3. The flight of the space vehicle takes places in the plane that contains the Moon's orbit and the Earth's orbit around CM;
4. The space vehicle is influenced only by the gravitational fields of the Moon and the Earth;
5. The gravitational fields of the Earth and the Moon are central and obey the inverse square distance law;
6. Only bi-impulsive trajectories are considered with the velocities increments tangential to the terminal orbits.

Consider an inertial reference frame G_{xyz}, Fig. 1, which contains the orbital plane of the Moon and with origin G at the CM. The x axis points toward the Moon at the initial time $t = t_0$, and, the y axis is perpendicular to the x axis and points toward to the movement of the Moon.

3.1.1 Differential equations

Assuming that the space vehicle has coordinates (x_P, y_P) in the inertial reference frame G_{xyz}, where the subscript P defines quantities related to the space vehicle, its the motion is described by the following differential equations:

$$\ddot{x}_p = -\frac{\mu_E}{r_{EP}^3}(x_p - x_E) - \frac{\mu_M}{r_{MP}^3}(x_p - x_M) \quad (1)$$

$$\ddot{y}_p = -\frac{\mu_E}{r_{EP}^3}(y_p - y_E) - \frac{\mu_M}{r_{MP}^3}(y_p - y_M), \quad (2)$$

where the subscripts E and M define quantities related, respectively, to the Earth and to the Moon. Thus, μ_E is the gravitational parameter of the Earth, and, μ_M is the gravitational parameter of the Moon. The operator $\cdot$ above the variables represents time derivatives quantities. r_{EP} e r_{MP} are the radial distances of the vehicle to the Earth and to the Moon respectively, and, are given by:

$$r_{EP} = \sqrt{(x_P - x_E)^2 + (y_P - y_E)^2} \quad (3)$$

$$r_{MP} = \sqrt{(x_P - x_M)^2 + (y_P - y_M)^2}. \quad (4)$$

The position vector $\mathbf{r_E} = (x_E, y_E)$ of the Earth and the position vector $\mathbf{r_M} = (x_M, y_M)$ of the Moon are calculated at each time as:

$$x_E(t) = -\mu x_M(t) \quad (5)$$

$$y_E(t) = -\mu y_M(t) \quad (6)$$

$$x_M(t) = \frac{D}{1+\mu} cos(\omega t) \quad (7)$$

$$y_M(t) = \frac{D}{1+\mu} sin(\omega t), \quad (8)$$

where $\mu = \mu_M/\mu_E$, $\omega = \sqrt{\frac{\mu_M + \mu_E}{D^3}}$ is the angular velocity of the Moon around CM, and D is the mean distance from the Earth to the Moon.

3.1.2 Initial Conditions

The initial conditions for the differential equations system, Eq. (1) and Eq. (2), are written as:

$$x_P(0) = x_{EP}(0) + x_E(0) = r_{EP}(0) cos\theta_{EP}(0) + x_E(0) \quad (9)$$

$$y_P(0) = y_{EP}(0) + y_E(0) = r_{EP}(0) sin\theta_{EP}(0) + y_E(0) \quad (10)$$

$$\dot{x}_P(0) = \dot{x}_{EP}(0) + \dot{x}_E(0) = -\left[\sqrt{\frac{\mu_E}{r_{EP}(0)}} + \Delta v_1\right] sin\theta_{EP}(0) + \dot{x}_E(0) \quad (11)$$

$$\dot{y}_P(0) = \dot{y}_{EP}(0) + \dot{y}_E(0) = \left[\sqrt{\frac{\mu_E}{r_{EP}(0)}} + \Delta v_1\right] cos\theta_{EP}(0) + \dot{y}_E(0), \quad (12)$$

where $r_{EP}(0) = r_{EP_0}$ is the distance from the center of the Earth to the space vehicle, which is calculated with the Earth radius R_E and the altitude h_{LEO} of the LEO, *i.e*, $r_{EP_0} = R_E + h_{LEO}$. Δv_1 is the velocity increment applied in the

LEO; θ_{EP} is the angle between the position vector $\mathbf{r_{EP}}$ of the space vehicle with respect to the Earth and the x axis of the inertial reference frame. The initial time $t_0 = 0$ corresponds to the moment immediately after the application of the first velocity increment. Since this velocity increment is applied tangentially to the LEO, the position vector $\mathbf{r_{EP}}\left(0\right)$ and the velocity vector $\mathbf{v_{EP}}\left(0\right)$ of the space vehicle with respect to the Earth are orthogonal. From Eq. (5) and Eq. (6), the initial position and velocity of the Earth at $t = t_0$ are:

$$x_E\left(0\right) = -\frac{\mu D}{1+\mu} \tag{13}$$

$$y_E\left(0\right) = 0 \tag{14}$$

$$\dot{x}_E\left(0\right) = 0 \tag{15}$$

$$\dot{y}_E\left(0\right) = -\frac{\mu D\omega}{1+\mu}. \tag{16}$$

3.1.3 Final Conditions

The final conditions establish relations at the moment $t_f = T$ immediately before the application of the second velocity increment Δv_2 at LMO. There are three final conditions: the distance condition; the velocity condition and the orthogonality condition. The distance condition imposes that, at the time $t_f = T$, the space vehicle be in the desirable altitude of the LMO. The velocity condition imposes that the velocity of the vehicle after the application of the second velocity increment be equal to the circular velocity at the LMO. And, the orthogonality condition imposes that the second velocity increment is applied tangentially to the final orbit (LMO). Therefore, these three conditions are, respectively, expressed as:

$$\left(x_P\left(T\right) - x_M\left(T\right)\right)^2 + \left(y_P\left(T\right) - y_M\left(T\right)\right)^2 = \left(r_{MP}\left(T\right)\right)^2 \tag{17}$$

$$\left(\dot{x}_P\left(T\right) - \dot{x}_M\left(T\right)\right)^2 + \left(\dot{y}_P\left(T\right) - \dot{y}_M\left(T\right)\right)^2 = \left[\sqrt{\frac{\mu_M}{r_{MP}\left(T\right)}} + \Delta v_2\right]^2 \tag{18}$$

$$\begin{aligned}\left(x_P\left(T\right) - x_M\left(T\right)\right)\left(\dot{y}_P\left(T\right) - \dot{y}_M\left(T\right)\right) - \left(\dot{x}_P\left(T\right) - \dot{x}_M\left(T\right)\right)\left(y_P\left(T\right) - y_M\left(T\right)\right) = \\ \mp r_{MP}\left(T\right)\left[\sqrt{\frac{\mu_M}{r_{MP}\left(T\right)}} + v_2\right],\end{aligned} \tag{19}$$

where the upper and the lower sign refers to a clockwise and a counterclockwise arrival in the LMO respectively. $r_{MP}\left(T\right) = r_{MP_f}$ is the distance from the center of the Moon to the space vehicle at the final time, which is calculated with the Moon radius R_M and the altitude h_{LMO} of the LMO, *i.e*, $r_{MP_f} = R_M + h_{LMO}$. From Eq. (7) and Eq. (8), the position and the velocity of the Moon at $t = t_f = T$ are:

$$x_M\left(T\right) = \frac{D}{1+\mu}cos\left(\omega T\right) \tag{20}$$

$$y_M\left(T\right) = \frac{D}{1+\mu}sin\left(\omega T\right) \tag{21}$$

$$\dot{x}_M\left(T\right) = -\frac{D\omega}{1+\mu}sin\left(\omega T\right) \tag{22}$$

$$\dot{y}_M\left(T\right) = \frac{D\omega}{1+\mu}cos\left(\omega T\right). \tag{23}$$

3.1.4 Analytical Expressions for the Velocities Increments

Consider the expression of the Jacobi Integral as below:

$$J\left(\xi_P,\eta_P,\dot{\xi}_P,\dot{\eta}_P\right)=2\left(\frac{1}{2}\omega^2\left({\xi_P}^2+{\eta_P}^2\right)+\frac{\mu_E}{r_{EP}}+\frac{\mu_M}{r_{MP}}\right)-\left(\dot{\xi}_P^2+\dot{\eta}_P^2\right), \quad (24)$$

where $J\left(\xi_P,\eta_P,\dot{\xi}_P,\dot{\eta}_P\right)$ is the Jacobi Integral, (ξ_P,η_P) and $(\dot{\xi}_P,\dot{\eta}_P)$ are the position and velocity coordinates of the space vehicle in a rotating reference frame $G_{\xi\eta}$, which contains the orbital plane of the Moon and with its origin at the CM point. The ξ axis points toward the Moon at each time instant t, and, the η axis is perpendicular to the ξ axis.

From the Jacobi Integral, one finds that the first velocity increment Δv_1, applied in the LEO, is given by

$$\begin{aligned}\Delta v_1 &= \omega r_{EP}(0)-\sqrt{\frac{\mu_E}{r_{EP}(0)}}\\ &\pm\{2\left[\frac{1}{2}\omega^2\left((r_{EP}(0))^2+\left(\frac{\mu D}{1+\mu}\right)^2-2\left(\frac{\mu D}{1+\mu}\right)(r_{EP}(0)\,cos\,(\theta_{EP}(0)))\right)+\frac{\mu_E}{r_{EP}(0)}+\frac{\mu_M}{r_{MP}(0)}\right]\\ &\qquad -J\left(\xi_P,\eta_P,\dot{\xi}_P,\dot{\eta}_P\right)\}^{\frac{1}{2}}, \quad (25)\end{aligned}$$

where $r_{MP}(0)=\sqrt{D^2+r_{EP}^2(0)-2Dr_{EP}(0)\,cos\theta_{EP}(0)}$.

By an analog development from the Jacobi Integral, one finds that the second velocity increment Δv_2, applied in the LMO, is given by:

$$\begin{aligned}\Delta v_2 &= -\sqrt{\frac{\mu_M}{r_{MP}(T)}}\mp\{(\omega r_{MP}(T))\\ &\pm\{\left[\omega^2\left((r_{MP}(T))^2+\left(\frac{D}{1+\mu}\right)^2+2\,(r_{MP}(T))\left(\frac{D}{1+\mu}\right)(cos\,(\theta_{MP}(T)-\omega T))\right)+\frac{2\mu_E}{r_{EP}(T)}+\frac{2\mu_M}{r_{MP}(T)}\right]\\ &\qquad -J\left(\xi_P,\eta_P,\dot{\xi}_P,\dot{\eta}_P\right)\}^{\frac{1}{2}}, \quad (26)\end{aligned}$$

where $r_{EP}(T)=\sqrt{D^2+r_{MP}^2(T)+2Dr_{MP}(T)\,cos\,(\theta_{MP}(T)-\omega T)}$ and θ_{MP} is the angle between the position vector of the space vehicle with respect to the Moon and the x axis of the inertial reference frame.

The analytical expressions given by Eqs. (25) and (26) provide a way to estimate the velocities increments Δv_1 and Δv_2, respectively, for a given value of the Jacobi Integral. An analysis of magnitude order shows that the terms which depend on $\theta_{EP}(0)$ and $\theta_{MP}(T)$ can be neglected, since their contributions to the velocities increments are insignificant. Hence, the following approximations are suggested in order to determine the velocities increments: $cos\,(\theta_{EP})=1$ in Eq. (25); and, $cos\,(\theta_{MP}(T)-\omega T)=1$ in Eq. (26).

3.1.5 The two-point boundary value problem

Based on the analytical expressions for the velocities increments determined in the previous topics, a TPBVP is formulated for the PCR3BP in order to obtain Earth-Moon trajectories with a prescribed value of the Jacobi Integral. Therefore, the TPBVP is enunciated as:

"Given a value of the Jacobi Integral $C=J\left(\xi_p,\eta_p,\dot{\xi}_p,\dot{\eta}_p\right)$, determine the set of variables $(\theta_{EP}(0),\theta_{MP}(T),T)$ that satisfies the final constraints given by Eqs. (17)-(19) with the initial conditions given by Eqs. (9)-(12).

Note that this problem has three variables to be iterated. However, the search space of the variable T is reduced when the total time of flight of the space vehicle is relatively known. Moreover, this formulation allows setting the Jacobi Integral previously which gives three advantages: increases the problem intuition since the Jacobi Integral already provides, initially, an idea of the total fuel consumption; the determination of the desired trajectories is facilitated, especially for those trajectories with large time of flight; and, a detailed study of the Kepler's energy of the space vehicle on the arrival trajectory at the insertion point in a low Moon orbit (LMO) can be developed.

4. RESULTS

Table (1) shows the data used in the numerical simulations. The LEO altitude is 167 km, and, the LMO altitude is 100 km. The space vehicle can arrive to the LMO in the clockwise or counterclockwise sense. In this paper, only clockwise arrivals are considered. Table (2) illustrates some results of trajectories for several values of the Jacobi Integral. The trajectories are determined utilizing the TPBVP described in the previous section and only interior trajectories are considered. Table (2) also includes the Kepler's energy of the space vehicle on the arrival trajectory at the insertion point in the LMO. Figure (2) shows this relation between the Kepler's energy and the Jacobi Integral. The method 1 in the legend of the Fig. (2) and Fig. (3) illustrates the trajectories obtained by the problem formulated in this paper. In this figure, results of others trajectories, which are calculated utilizing the method described by da Silva Fernandes and Marinho (2011), denoted as method 2, are also shown. The continuous blue line in Fig. (2) represents the analytical relation between the Jacobi Integral and the arrival Kepler's energy, as given by the development of Eq. (26). The vertical lines represent the critical values of the Jacobi Integral corresponding to the Lagrangian points L_i, $i = 1, 2, 3, 4, 5$. Figure (3) shows the times of flight of all these trajectories against the Jacobi Integral. As the trajectories are not optimized, there are many trajectories for the same Jacobi Integral value. To illustrate this fact, Fig. (4) shows a different trajectory with $C = 2.90\ km^2/s^2$ from that one illustrated in Tab. (2).

Table 1: Parameters specification

ω_M	$2.649 \times 10^{-6}\ rad/s$
v_M	$1.018\ km/s$
R_E	$6378.2\ km$
R_M	$1738\ km$
μ_E	$398600\ km^3/s^2$
μ_M	$4902.83\ km^3/s^2$
D	$384400\ km$
R_S	$66300\ km$

Table 2: Trajectories for several Jacobi Integral values

Δv_1 $[km/s]$	Δv_2 $[km/s]$	Δv_{TOTAL} $[km/s]$	*Time of Flight [days]*	$\theta_{EP}(0)$ $[degrees]$	*Kepler's energy* $[km^2/s^2]$	*Jacobi Integral* $[km^2/s^2]$	*Feasibility*
3.123491	0.735282	3.858772	82.535	141.878	0.137473	2.80	Yes
3.123032	0.733173	3.856206	82.197	-102.004	0.132481	2.81	Yes
3.122574	0.731064	3.853638	85.336	152.038	0.127492	2.82	Yes
3.122116	0.728953	3.851068	92.605	-110.724	0.122502	2.83	Yes
3.121657	0.726839	3.848497	190.374	-126.842	0.117512	2.84	Yes
3.121199	0.724735	3.845934	162.724	-29.334	0.112549	2.85	No
3.120740	0.722608	3.843348	163.298	-3.085	0.107534	2.86	Yes
3.120282	0.720489	3.840771	175.193	-7.603	0.102544	2.87	Yes
3.119824	0.718368	3.838191	202.472	58.981	0.097555	2.88	Yes
3.119365	0.716245	3.835611	202.261	-81.910	0.092566	2.89	Yes
3.118907	0.714121	3.833027	202.166	-69.582	0.087576	2.90	Yes
3.118448	0.711994	3.830442	194.517	7.269	0.082586	2.91	Yes
3.117990	0.709866	3.827856	201.720	-95.487	0.077599	2.92	Yes
3.117531	0.707737	3.825268	201.345	-9.749	0.072611	2.93	Yes

5. DISCUSSION

Table (2) shows that interior trajectories with large times of flight are obtained with large values of Jacobi Integral. This means that trajectories of this kind must have a little value for the first velocity increment so that the space vehicle accomplish several swing-by maneuvers with the Moon. However, if a large Jacobi Integral value is prescribed, it does not mean that only trajectories with large time of flight are obtained, because there are many kinds of solutions for the same value of the Jacobi Integral. For instance, Figs. (4) and (5) illustrate a trajectory with a time of flight 82 days longer than the trajectory shown in Tab. (2) for $C = 2.90\ km^2/s^2$.

In the Figs. (2) and (3) the results obtained through method 2 are optimized. Trajectories with smaller Kepler's energy are obtained by method 1, as seen in Fig. (2), and they could be used as initial condition for a posteriorly optimization method. In this same figure, there is a critical value of the Jacobi Integral ($C = 3.0756\ km^2/s^2$) where the Kepler's Energy becomes negative, which means that the arrival in the LMO can be considered ballistic. But, the computation of trajectories with small values for the Kepler's energy becomes more difficult as the Jacobi Integral increases. This almost

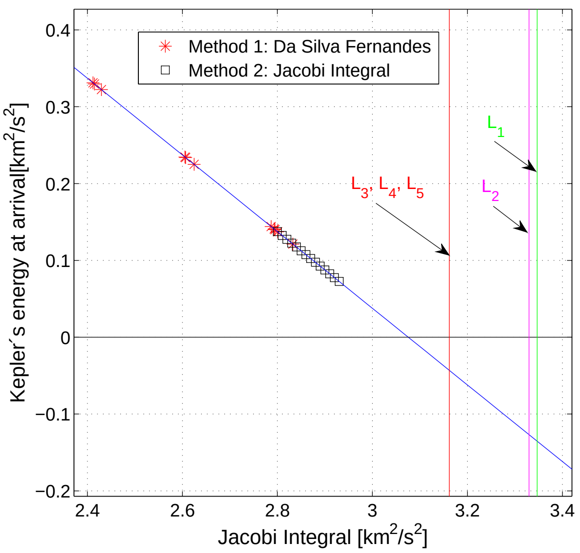

Figure 2: Kepler's energy × Jacobi Integral.

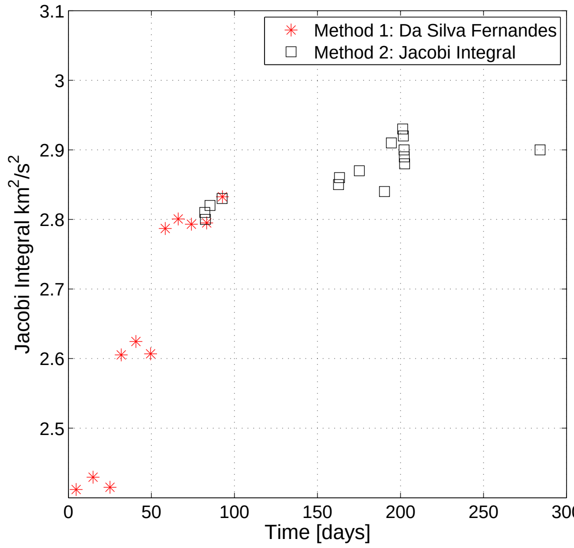

Figure 3: Time of flight × Jacobi Integral.

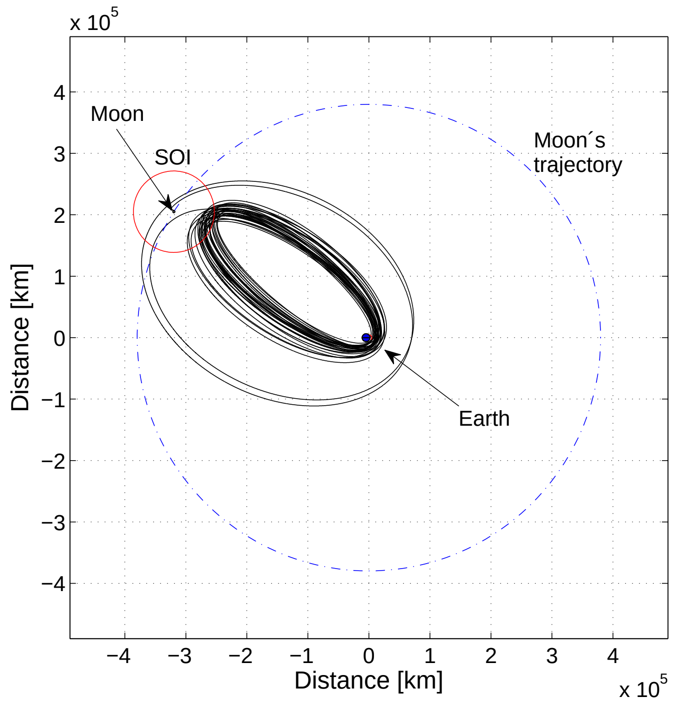

Figure 4: $C = 2.90\ km^2/s^2$, $T = 284.019\ days$, $\Delta v_{TOTAL} = 3.833027 km/s$. Inertial reference frame.

asymptotic behaviour can be seen in Fig. (3), where the time of flight increases considerably with a little change in the Jacobi Integral.

6. CONCLUSION

This work proposes a new method to determine Earth-Moon trajectories based on the solution of a TPBVP with a prescribed value of the Jacobi Integral. Several internal trajectories are determined and a study for the arrival Kepler's energy is performed. The results show that there is a critical value for the Jacobi Integral ($C = 3.0756\ km^2/s^2$) where the Kepler's energy becomes negative and a ballistic capture can be accomplish by the Moon. This method facilitates the determination of trajectories with small positive values of the Kepler's energy and the determination of trajectories with large times of flight.

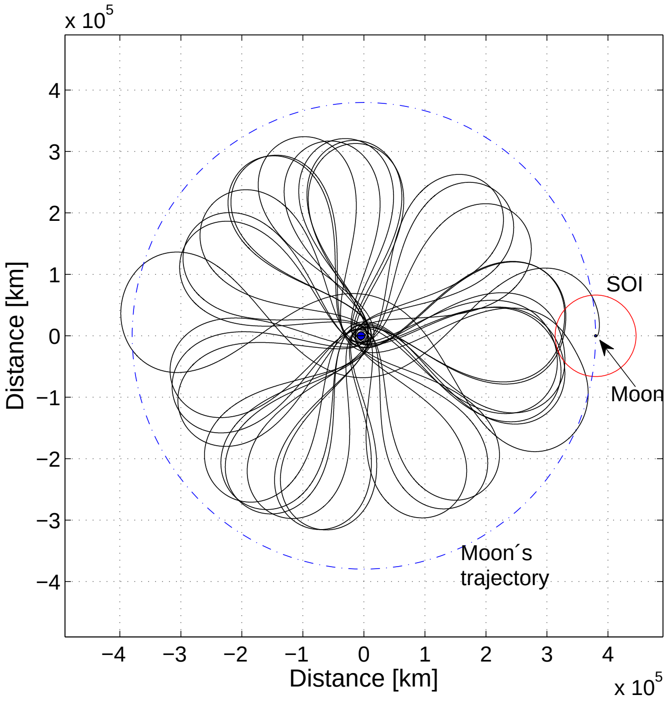

Figure 5: $C = 2.90\ km^2/s^2$, $T = 284.019\ days$, $\Delta v_{TOTAL} = 3.833027 km/s$. Rotating reference frame.

7. ACKNOWLEDGEMENTS

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