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A STUDY OF OPTIMAL ROUND TRIP TRAJECTORIES TO THE MOON AND THE INFLUENCE OF THE SUN

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Abstract. *In the last two decades, new kinds of trajectories have been proposed in order to transfer a space vehicle from a low Earth orbit (LEO) to a low Moon orbit (LMO). This work studies and compares several models to determine optimal bi-impulsive trajectories in a round trip journey to the Moon which minimize the fuel consumption, represented by the total velocity increment applied to the space vehicle. The velocity increments are assumed to be impulsive and are applied tangentially to the terminal orbits. The models considered in this work are four: the lunar patched-conic approximation, the simplified and the classic planar circular restricted three-body problem (PCR3BP), and, the planar bi-circular restricted four-body problem (PBR4BP). The Sequential Gradient-Restoration algorithm with constraints, developed by Miele and collaborators, is used to determine the optimal trajectories.*

Keywords: *patched-conic approximation, three-body problem, four-body problem, optimal Earth-Moon trajectories.*

1. INTRODUCTION

In the last decades, new types of Earth-to-Moon transfers have been designed and used in lunar missions which could not be possible using traditional approaches. New methods make use of the nonlinear dynamics of the circular restricted three-body problem and of the bi-circular restricted four-body problem to design low energy transfers. Low energy Earth-to-Moon transfers can be classified into exterior or interior, according to the geometry (Topputo 2013). In the exterior transfers the spacecraft is injected into an orbit with large apogee which crosses the Moon orbit. The apogee distance is approximately four times the Earth-Moon distance. This kind of trajectories exploits the Sun's gravitational attraction (Yamakawa et al. 1992). In the interior transfers most part of the trajectory occurs within the Moon orbit. The majority of these new works considers outgoing lunar missions with moderate or large flight time.

In this work, a study and a comparison between several models to determine optimal bi-impulsive trajectories in a round trip journey to the Moon which minimize the fuel consumption, represented by the total velocity increment applied to the space vehicle, is presented. The velocity increments are assumed to be impulsive and are applied tangentially to the terminal orbits. The models utilized in this work are four: the lunar patched-conic approximation, the simplified and the classic planar circular restricted three-body problem (PCR3BP), and, the planar bi-circular restricted four-body problem (PBR4BP). The patched-conic approximation is an important model since it provides the first solutions which are used as initial condition to the other models. Before the optimization process, a two-point boundary value problem (TPBVP) has to be formulated for each model. For the optimization process, the Sequential Gradient-Restoration algorithm with constraints, developed by Miele et al. (1969), is used. In the PBR4BP, the influence of the Sun is analyzed. Initially, an optimization of one degree of freedom is conducted considering a specified position of the initial Sun's phase angle. In this way, several solutions are obtained for different initial Sun's phase angles, and, a curve of fuel consumption is constructed with these optimal solutions. After this procedure, an optimization of two degrees of freedom is employed in order to optimize the initial Sun's phase angle to give the minimum fuel consumption. For all models, the trajectories studied are direct ascent maneuvers, and, both the outgoing and return trips are considered. Three values of LEO altitude are utilized: 167 km, 320 km and 463 km. For each LEO altitude, five values of LMO are used between 100km and 500km. The models are compared with each other and with some results found in the literature. The optimal results in the PBR4BP show that a small reduction of the fuel consumption can be achieve if the initial Sun's phase angle is chosen properly.

2. FORMULATION OF THE OPTIMZATION PROBLEM

This section describes briefly the dynamical models used in this study. Details about these models can be found in the cited references along the text.

2.1 Patched-conic approximation

In the next paragraphs an explanation about the round trip lunar patched-conic approximation and the corresponding optimization problem is presented. To achieve this purpose the outgoing trip is explained first. It consists of transferring a space vehicle from a circular low Earth orbit (LEO) to a circular low Moon orbit (LMO) minimizing the fuel consumption, represented by the total velocity increment. Next, the return flight is formulated similarly to the outgoing trip, but transferring the space vehicle from a LMO to a LEO.

2.1.1 Outgoing trip

The patched-conic approximation is based on the two-body dynamics, in which the motion of the space vehicle is described by a conic, and it uses the concept of sphere of influence. It is a simple strategy of patching two conics to describe the entire trajectory. In the lunar patched-conic approximation an ellipse and a hyperbole are used. The ellipse describes the motion of the space vehicle when the Earth gravitational field is considered (geocentric phase), and, the hyperbole describes the motion when the Moon gravitational field is considered (selenocentric phase). To obtain the complete trajectory of the relative motion of the vehicle, the two conics are connected together at the edge of the sphere of influence of the Moon. Similar to the Hohmann transfer, the patched-conic approximation involves two impulses with the total increment of velocity representing the fuel consumption. A mathematical formulation of the patched-conic approximation for the outgoing trip can be found in Da Silva Fernandes and Marinho (2012). The following hypotheses are assumed:

1) The Earth is fixed in space;
2) The Moon orbit around the Earth is circular;
3) The flight of the space vehicle takes place in the orbital plane of the Moon;
4) The gravitational field of the Earth and the Moon are central and obey the inverse square law;
5) The transfer trajectory has two distinct phases: the geocentric phase, which starts immediately after the first velocity increment; and, the selenocentric phase, which begins when the vehicle reaches the sphere of influence of the Moon;
6) The class of two-impulse trajectories is considered. Each velocity increment is applied tangentially to the initial orbit (LEO) and to the final (LMO) orbit.

The patched-conic approximation for the outgoing trip is completely determined by four variables: the radial distance r_0, the velocity v_0, the flight path angle φ_0 of the space vehicle at the point of insertion in the geocentric trajectory; and, the phase angle λ_1 of the vehicle with the Earth relatively to the Moon at the moment when the space vehicle reaches the sphere of influence of the Moon. From the hypothesis 6), $\varphi_0 = 0$.

2.1.2 Return trip

The mathematical formulation of the return trip is basically the same as the outgoing mission, but with the selenocentric trajectory as the starting path. The hypotheses remain the same; however, the fifth hypothesis can be better stated as it follows:

5) The transfer trajectory has two distinct phases: the selenocentric phase, which starts immediately after the first velocity increment; and, the geocentric phase, which begins when the vehicle reaches the edge of the sphere of influence of the Moon.

The return trip based on the patched-conic approximation is completely solved when the set of variables $(r_0, v_0, \varphi_0, \lambda_1)$ is known. Unlike the outgoing trip, these variables are related with the beginning of the return mission so that the radial distance r_0, the velocity v_0 and the flight path angle φ_0 are related at the point of insertion in the selenocentric trajectory. The angle λ_1 is the same phase angle previously defined, but with a distinct configuration. Again, according to the hypothesis 6), $\varphi_0 = 0$.

2.1.3 Optimization problem based on patched-conic approximation

The patched-conic approximations for the outgoing and return trips are determined through the solution of a boundary value problem, in which the iteration variable is the velocity v_0. The other variables, r_0, φ_0 and λ_1, are prescribed. The initial guess for v_0 can be obtained from the Hohmann transfer, as described in Bate et al. (1971). If the set (r_0, φ_0) is prescribed, then there is just one solution for each value of λ_1 if the solution exists. In this way, an optimization problem can be enunciated in order to determine some value of λ_1 which minimizes the total fuel consumption, represented by the total velocity increment. The complete optimization problem is stated as below:

"Determine the set of variables (λ_1, v_0) which minimizes the function

$$F(\lambda_1, v_0): \Delta v_{Total} = \Delta v_{LEO} + \Delta v_{LMO},$$

subject to the constraint $g(\lambda_1, v_0): r_f - r_p = 0$, where Δv_{LEO} is the velocity increment at the circular low Earth orbit (LEO) and Δv_{LMO} is the velocity increment at the circular low Moon orbit (LMO). Note that $r_p = r_{pM}$ for the outgoing trip, and, $r_p = r_{pE}$ for the return trip, where r_{pM} and r_{pE} are, respectively, the distance at the periapsis of the selenocentric trajectory in the outgoing trip, and, the distance at the periapsis of the geocentric trajectory in the return trip. In both cases r_f is the prescribed radius"

The Sequential Gradient Restoration Algorithm (SGRA), developed by Miele and collaborators (1969), is used to solve this optimization problem with two degrees of freedom. To initialize this algorithm, an initial guess that satisfies the constraint must be provided. Therefore, the boundary value problem must be solved in order to obtain this initial guess and to ensure the algorithm's convergence. The boundary value problem consists of determining v_0 to a specified set (r_0, φ_0) such that it satisfies the final condition $r_p = r_f$ to a certain λ_1.

2.2 Bi-circular restricted four-body problem

The optimization problem based on the planar bi-circular restricted four-body problem – PBR4BP – is formulated as described below. The following assumptions are employed:

1) Earth and Moon move in circular orbits around the center of mass of the Earth-Moon system;
2) The center of mass of Earth-Moon system moves in circular orbit around the center of mass of the Sun-Earth-Moon system;
3) The flight of the space vehicle takes place in the Moon orbital plane;
4) The space vehicle is subject only to the gravitational fields of Earth, Moon and Sun;
5) The gravitational fields of Earth, Moon and Sun are central and obey the inverse square law;
6) The class of two-impulse trajectories is considered. The impulses are applied tangentially to the space vehicle velocity relative to Earth (first impulse) and to Moon (second impulse).

Consider a moving reference frame *Bxy* contained in the Moon orbital plane: its origin is the center of mass of Earth-Moon system; the *x*-axis points towards the Moon position at the initial time $t_0 = 0$ and the *y*-axis is perpendicular to the *x*-axis. See Fig. 1.

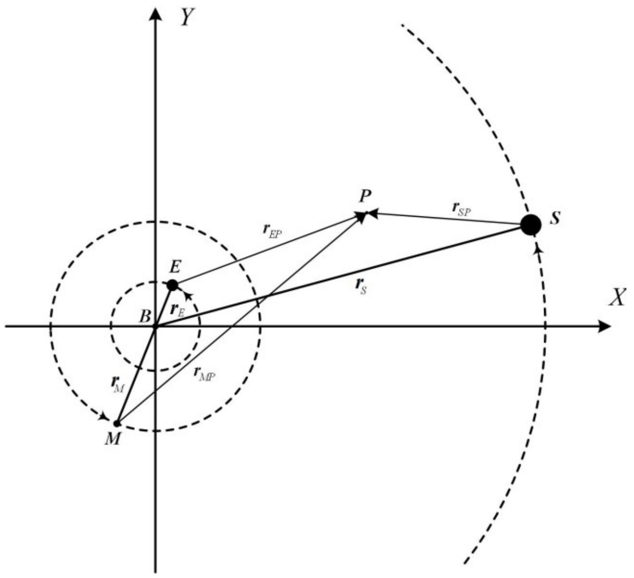


Figure 1. The reference frame *Bxy*

In Bxy reference frame, the motion of the space vehicle is described by the following set of differential equations:

$$\frac{dx_P}{dt} = u_P \qquad \frac{du_P}{dt} = -\frac{\mu_S}{r_{SP}^3}(x_P - x_S) - \frac{\mu_E}{r_{EP}^3}(x_P - x_E) - \frac{\mu_M}{r_{MP}^3}(x_P - x_M) - \frac{\mu_S}{r_S^2}\cos(\omega_S t + \theta_{S0})$$

$$\frac{dy_P}{dt} = v_P \qquad \frac{dv_P}{dt} = -\frac{\mu_S}{r_{SP}^3}(y_P - y_S) - \frac{\mu_E}{r_{EP}^3}(y_P - y_E) - \frac{\mu_M}{r_{MP}^3}(y_P - y_M) - \frac{\mu_S}{r_S^2}\sin(\omega_S t + \theta_{S0}), \tag{1}$$

where μ_E, μ_M and μ_S are the gravitational parameters of Earth, Moon and Sun, respectively; r_{EP}, r_{MP} and r_{SP} are the distances of the space vehicle from Earth (E), from Moon (M) and from Sun (S), respectively; that is, $r_{EP}^2 = (x_P - x_E)^2 + (y_P - y_E)^2$, $r_{MP}^2 = (x_P - x_M)^2 + (y_P - y_M)^2$ and $r_{SP}^2 = (x_P - x_S)^2 + (y_P - y_S)^2$. The distance from B to Earth, Moon and Sun are denoted by r_E, r_M and r_S, respectively. So, the position vectors of Earth, Moon and Sun are defined in the reference frame Bxy by the equations

$$x_E(t) = -r_E\cos\theta_M(t) \qquad x_M(t) = r_M\cos\theta_M(t) \qquad x_S(t) = r_S\cos\theta_S(t)$$

$$y_E(t) = -r_E\sin\theta_M(t) \qquad y_M(t) = r_M\sin\theta_M(t) \qquad y_S(t) = r_S\sin\theta_M(t), \tag{2}$$

with $\theta_M(t) = \theta_0 + \sqrt{(\mu_E + \mu_M)/D^3}\, t$, $\theta_S(t) = \theta_{S0} + \omega_S t$, $\omega_S = \sqrt{(\mu_S + \mu_E + \mu_M)/r_S^3}$, $\theta_0 = 0$ is the initial phase of the Moon, θ_{S0} is the initial phase of the Sun, $r_E = \mu D/(1+\mu)$ and $r_M = D/(1+\mu)$, $\mu = \mu_M/\mu_E$ and D is the distance from the Earth to the Moon.

The initial conditions of the system of differential equations (1) correspond to the position and velocity vectors of the space vehicle after the application of the first impulse. The initial conditions $(t_0 = 0)$ can be written as follows

$$x_P(0) = x_E(0) + r_{EP}(0)\cos\theta_{EP}(0) \qquad u_P(0) = -\left[\sqrt{\frac{\mu_E}{r_{EP}(0)}} + \Delta v_{LEO}\right]\sin\theta_{EP}(0) + \dot{x}_E(0)$$

$$y_P(0) = y_E(0) + r_{EP}(0)\sin\theta_{EP}(0) \qquad v_P(0) = \left[\sqrt{\frac{\mu_E}{r_{EP}(0)}} + \Delta v_{LEO}\right]\cos\theta_{EP}(0) + \dot{y}_E(0), \tag{3}$$

where Δv_{LEO} is the velocity change at the first impulse, $r_{EP}(0) = h_0 + a_E$ and $\theta_{EP}(t)$ is the angle which the position vector $\mathbf{r}_{EP}$ forms with x-axis. h_0 is the altitude of LEO and a_E is the Earth radius. It should be noted that $\mathbf{r}_{EP}(0)$ and $\mathbf{v}_{EP}(0)$ are orthogonal (impulse is applied tangentially to LEO). From Eq. (2),

$$x_E(0) = -\mu\frac{D}{1+\mu} \qquad y_E(0) = 0 \qquad \dot{x}_E(0) = 0 \qquad \dot{y}_E(0) = -\mu\frac{D}{1+\mu}\dot{\theta}_M\,.$$

The final conditions of the system of differential equations (1) correspond to the position and velocity vectors of the space vehicle before the application of the second impulse. The final conditions $(t_f = T)$ can be put in the following form:

$$(x_P(T) - x_M(T))^2 + (y_P(T) - y_M(T))^2 = (r_{MP}(T))^2, \tag{4}$$

$$(u_P(T) - \dot{x}_M(T))^2 + (v_P(T) - \dot{y}_M(T))^2 = \left[\sqrt{\frac{\mu_M}{r_{MP}(T)}} + \Delta v_{LMO}\right]^2, \tag{5}$$

$$\begin{aligned}&(x_P(T) - x_M(T))(v_P(T) - \dot{y}_M(T)) - (y_P(T) - y_M(T))(u_P(T) - \dot{x}_M(T)) \\ &= \mp r_{MP}(T)\left[\sqrt{\frac{\mu_M}{r_{MP}(T)}} + \Delta v_{LMO}\right].\end{aligned} \tag{6}$$

where Δv_{LMO} is the velocity change at the second impulse, $r_{MP}(T) = a_M + h_f$, h_f is the altitude of LMO and a_M is the Moon radius. The upper sign refers to clockwise arrival to LMO and the lower sign refers to counterclockwise arrival to LMO. From Eqns. (4) and (6), one finds

$$x_M(T) = \frac{D}{1+\mu}\cos\theta_M(T) \qquad y_M(T) = \frac{D}{1+\mu}\sin\theta_M(T)$$

$$\dot{x}_M(T) = -\frac{D\dot{\theta}_M}{1+\mu}\sin\theta_M(T) \qquad \dot{y}_M(T) = \frac{D\dot{\theta}_M}{1+\mu}\cos\theta_M(T). \tag{7}$$

The problem defined by Eqns. (1)–(7) involves five unknowns Δv_{LEO}, Δv_{LMO}, T, $\theta_{EP}(0)$ and θ_{S0} that must be determined in order to satisfy the three final conditions – Eqns. (4), (5) and (6). This problem has two degrees of freedom, so an optimization problem can be formulated as follows: Determine Δv_{LEO}, Δv_{LMO}, T, $\theta_{EP}(0)$ and θ_{S0} which satisfy the final constraints, Eqns. (4)–(6), and, minimize the total characteristic velocity $\Delta v_{Total} = \Delta v_{LEO} + \Delta v_{LMO}$. This problem has been solved by using the SGRA algorithm.

2.3 Classic planar circular restricted three-body problem

In this section, the optimization problem based on the planar circular restricted three-body problem – PCR3BP – is formulated as described below. The following assumptions are employed:

1) Earth and Moon move around the center of mass of the Earth-Moon system;
2) The eccentricity of the Moon orbit around Earth is neglected;
3) The flight of the space vehicle takes place in the Moon orbital plane;
4) The space vehicle is subject only to the gravitational fields of Earth and Moon;
5) The gravitational fields of Earth and Moon are central and obey the inverse square law;
6) The class of two impulse trajectories is considered. The impulses are applied tangentially to the space vehicle velocity relative to Earth (first impulse) and to Moon (second impulse).

These assumptions are very similar to those ones stated in the preceding section. Now, the reference frame *Bxy* is inertial. By taking out the terms related do Sun in Eq. (1), one finds that the motion of the space vehicle is described by the following set of differential equations:

$$\frac{dx_P}{dt} = u_P \qquad \frac{du_P}{dt} = -\frac{\mu_E}{r_{EP}^3}(x_P - x_E) - \frac{\mu_M}{r_{MP}^3}(x_P - x_M)$$

$$\frac{dy_P}{dt} = v_P \qquad \frac{dv_P}{dt} = -\frac{\mu_E}{r_{EP}^3}(y_P - y_E) - \frac{\mu_M}{r_{MP}^3}(y_P - y_M). \tag{8}$$

The initial conditions of the system of differential equations (8) are given by Eq. (3) and the final conditions are given by Eqns. (4) – (6).

The boundary value problem is now defined by Eq. (8) in the place of Eq. (1) and it involves four unknowns Δv_{LEO}, Δv_{LMO}, T and $\theta_{EP}(0)$ that must be determined in order to satisfy the three final conditions – Eqns. (4), (5) and (6). Since this problem has one degree of freedom, an optimization problem can be formulated as follows: Determine Δv_{LEO}, Δv_{LMO}, T and $\theta_{EP}(0)$ which satisfy the final constraints (4) – (6) and minimize the total characteristic velocity $\Delta v_{Total} = \Delta v_{LEO} + \Delta v_{LMO}$. This optimization problem has been solved using the same algorithm described in the preceding sections.

2.4 Simplified planar circular restricted three-body problem

The simplified planar circular restricted three-body problem was used by Miele and Mancuso (2001). In this formulation, the Earth is fixed in space and the origin of the inertial reference frame is the center of the Earth. Accordingly, the mathematical formulation of the optimization problem based on the simplified model is quite similar to the previous one. The unique difference is that the terms related to the position vector and to the velocity vector of Earth are null.

The formulations of the return trip in the classic PCR3BP, in the simplified PCR3BP and in the PBR4BP are similar to the outgoing trip, however the initial conditions must be related to the Moon and the final conditions must be written with respect to the Earth.

3. RESULTS

This section shows the main results obtained to the problem of minimizing the fuel consumption of a space vehicle in a round trip mission from Earth to Moon. Three values of altitudes are specified for the LEO: 167 km, 320 km and 463 km. For each one of these values, several LMO altitudes are considered within the interval between 100 km and 500 km. Table 1 illustrates a list with the values of others parameters necessary to specify the entire problem.

Table 1: Specification of the parameters

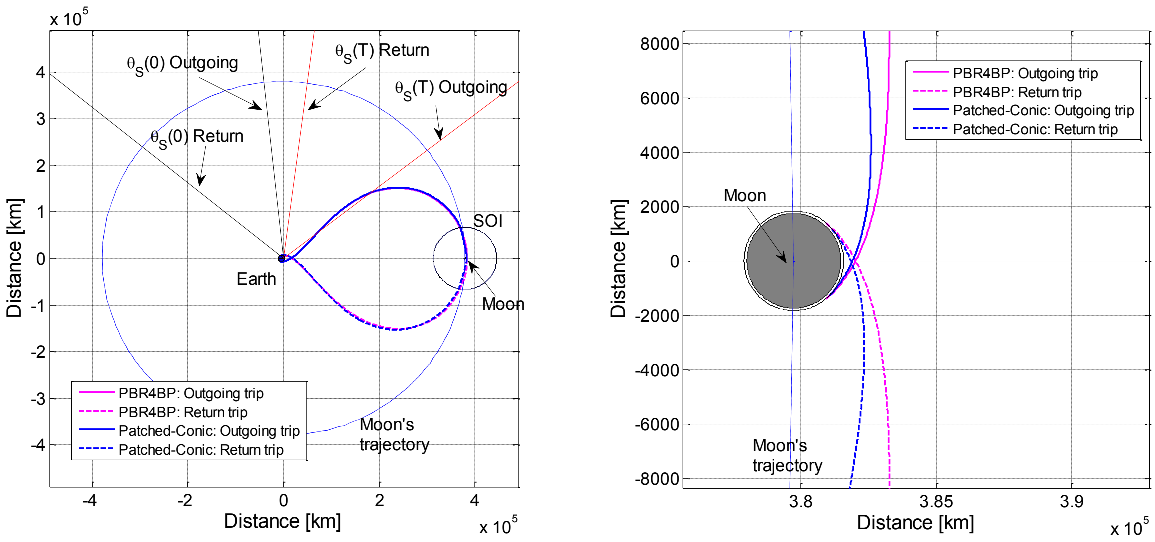
ω [(1)](angular velocity of the Moon)	2.66524×10^{-6} rad/s
a_E (Earth's radius)	6378.2 km
a_M (Moon's radius)	1738 km
μ_E (Earth's gravitational parameter)	398600 km^3/s^2
μ_M (Moon's gravitational parameter)	4902.83 km^3/s^2
μ_S (Sun's gravitational parameter)	1.327×10^{11} km^3/s^2
D (Earth-Moon distance)	384400 km
R_S (Radius of the Moon's sphere of influence	66300 km
r_S (distance from the center of mass of the Earth-Moon system to the Sun)	1.496×10^{8} km

[(1)]For the patched-conic approximation and for the simplified PCR3BP, the angular velocity of the Moon used is ω =2.649×10^{-6} rad/s in order to be compatible with the parameters of Bate et al. (1971) and Miele and Mancuso (2001).

Table 2 compares some results considering the different models. The trajectories of the most complex model (PBR4BP) and the simplest model (patched-conic approximation) shown in Tab. 2 are depicted in Fig. 3. The Figs. 3 and 4 illustrate the influence of the Sun in optimal trajectories considering a specified position of the Sun's initial phase angle θ_{S0} .

Table 2: Results from different models. *LEO = 167 km*, *LMO = 100 km.* Clockwise arrival.

Model	**Mission**	Δv_{LEO} **[km/s]**	Δv_{LMO} **[km/s]**	Δv_{TOTAL} **[km/s]**	***Time of Flight*** **[days]**	**Initial phase angle [degrees]**	λ_1 **[degrees]**	θ_{S0} **[degrees]**
Patched-Conic approximation	*Outgoing trip*	3.140943	0.785913	3.926855	4.944	-113.628	82.32	-
	Return trip	3.140943	0.785913	3.926855	4.945	44.197	82.33	-
simplified PCR3BP	*Outgoing trip*	3.140364	0.814824	3.955188	4.756	-114.194	-	-
	Return trip	3.140364	0.814824	3.955188	4.756	48.983	-	-
classic PCR3BP	*Outgoing trip*	3.141269	0.815714	3.956983	4.769	-113.776	-	-
	Return trip	3.141269	0.815714	3.956983	4.769	49.020	-	-
PBR4BP	*Outgoing trip*	3.141058	0.813658	3.954716	4.792	-113.687	-	96.166
	Return trip	3.141076	0.813650	3.954727	4.794	49.812	-	141.029



(a)

(b) Zoom at LMO

Figure 2: Patched Conic x PBR4BP. *LEO = 167 km*, *LMO = 100 km.* Clockwise arrival.

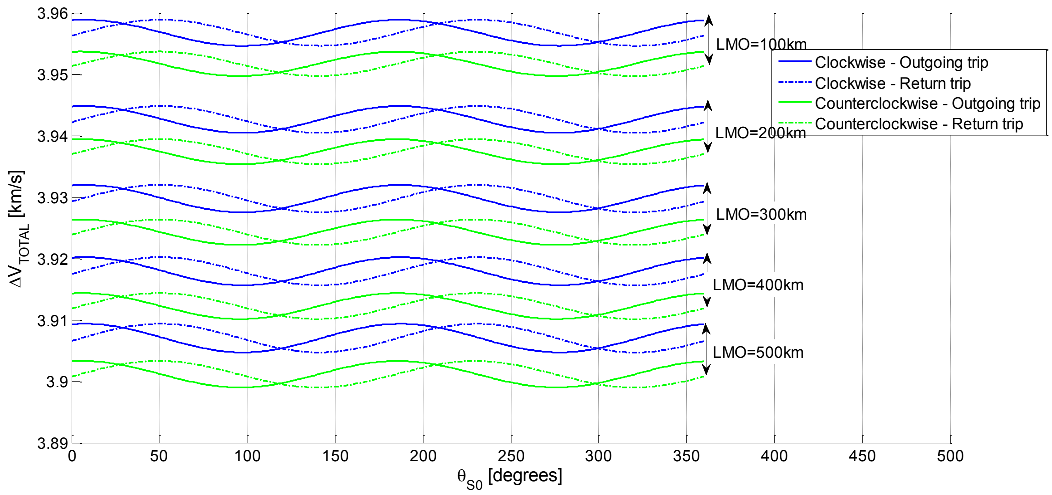


Figure 3: Sun's influence on optimal trajectories. *LEO = 167 km*.

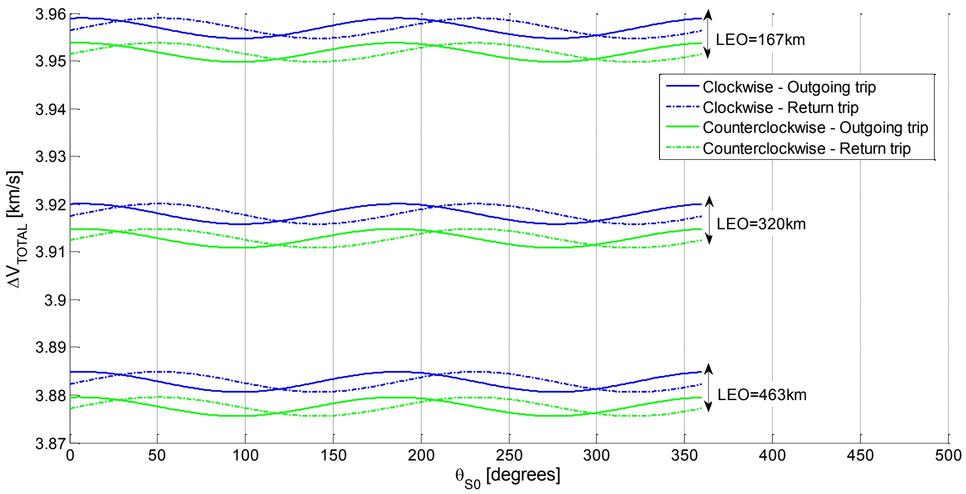


Figure 4: Sun's influence on optimal trajectories. *LMO = 100 km*.

4. DISCUSSION

Table 2 and Fig. 2 show that the results obtained by different models are very close. Therefore, the patched-conic approximation could be used as a preliminary analysis or could be used as initial guess to solve more complex models. Moreover, a symmetry between the outgoing trip and the return trip is observed since the velocities increments are practically equals for the same model. This symmetry is seen in Fig 2, where the symmetry axis lies on the Earth-Moon axis in the rotating reference frame. The analysis of the Sun's influence in Figs. 3 and 4 provide the following discussions:

1) For each curve in Figs. 3 and 4 there are two values of θ_{S0} where the Δv_{TOTAL} reaches the minimum.
2) For direct ascent maneuvers, the trajectories with clockwise arrival spend more fuel consumption than the trajectories with counterclockwise arrival.
3) The trajectories in the return trip have basically the same fuel consumption than the trajectories in the outgoing trip. However, the curves Δv_{TOTAL} vs θ_{S0} of the return trip are lightly lagged from the curves Δv_{TOTAL} vs θ_{S0} of the outgoing trip. Therefore, the two points of minimum values for Δv_{TOTAL} occur for values θ_{S0} lightly different. On the other hand, the outgoing trip trajectories are practically in phase, independently of the altitudes LEO and LMO, and of the direction in the arrival at the LMO. Consequently, the points of minimum values for θ_{S0} are close to each other in this case. The same behaviour is observed in the return trip trajectories. Therefore, a strong symmetry is also observed in the Sun's position, as seen in Fig. 2 as well.

5. CONCLUSION

This work presents a study on direct ascent trajectories of Earth-Moon round trip missions. The trajectories are optimized utilizing the SGRA in order to minimize the total fuel consumption. Several models are used to describe the motion of the space vehicle. The results show that the optimal trajectories obtained from different model are very close. The patched-conic approximation, for instance, is a good approximation even for the most complex model (PBR4BP) presented in this paper, which considers the gravitational influence of the Sun. An analysis of the Sun's influence in optimal trajectories is also performed and the results prove that there is a strong symmetry in the optimal Sun's position independently of the LEO and LMO altitudes, and of the direction in arrival at the LMO.

6. ACKNOWLEDGEMENTS

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