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ANALYSIS OF HEAT TRANSFER IN TRANSIENT REGIME IN ANISOTROPIC MATERIALS APPLYING THE BEM

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Abstract. *This paper presents a study on effective thermal conductivity of porous media in transient heat transfer. The microporosity of the material is represented by the insertion of holes in the domain of the geometry. The Boundary Element Method (BEM) is employed to solve the differential equations governing the potential problems in the transient regime. The analysis was performed using Representative Volume Elements (RVE's), which is a method that applies the Midfields theory to find the effective properties (macroscopic) of this micro-porous material. Both the diameters of the holes as well as the volume fraction of the material are pre-determined. The thermal conductivity is evaluated locally by the ratio between the average volumetric heat flux and volumetric average temperature gradient of the field. The technique used for isotropic materials is extended to the study of the behavior of heat conduction in anisotropic media using the method known as domain mapping or linear transformation. Some numerical examples are evaluated by checking the behavior of the effective thermal conductivity in isotropic and anisotropic materials.*

Keywords: *Heat transfer, effective thermal conductivity, Boundary Element Method, Isotropy, Anisotropy.*

1. INTRODUCTION

When talking about engineered materials, almost everything in nature can be considered heterogeneous, and its properties are intimately dependent on its microstructure. The determination of macroscopic characteristics of heterogeneous materials is a major problem in many engineering and science applications. Find the physical properties of materials is currently a very promising and important branch in various sectors (industrial, electrical, scientific, quality control etc.) all these areas depend on the availability of materials with physical and chemical properties and have well-defined , accuracy and reliability.

Studies of thermal conductive properties and microscopic physical phenomena of porous materials have been attracting the attention of the scientific community, for example, an integral method was developed to estimate the thermal conductivity varying with temperature without using internal temperature measurements with a one-dimensional model in which one of the faces is heated and the other is isolated. The distribution of temperature to the integral method was assumed as a third order polynomial equation where the coefficients were determined using two determined heat flows and two temperatures measured in the contour (Kim et al., 2003) . Furthermore, a study was done to define the effective thermal conductivity of the main materials used in construction panels for "vacuum" thermal insulation, like fibers and foams. It was built a microstructure for each material in analysis using simplifications and idealizations for the modeling of solid conduction phenomenon for each case. The authors concluded that the effective thermal conductivity is a function of porosity, of pure solid conductivity and mechanical properties of the material (Kwon et al., 2003).

The BEM has been increasingly used in solving problems, its main feature is the model size reduction, due to the way the mesh of the system is discretized. The contour method has emerged as a powerful alternative to finite element method, in particular cases where better accuracy is required due to problems such as stress concentration or where the domain extends to infinity (Lee and Mal, 1990). There are many other studies on effective properties of heterogeneous materials where BEM is used as numerical tool. As such, elasticity problems and to micromechanical where BEM is shown to be a suitable tool (Brebbia, et al., 1984). This work used a computational approach scheme together with Midfields theory, also called Theory of Effective Properties or RVE Analysis Method (Buroni, 2006; Zohdi, 2002). Through a representative sample material this ratio is calculated, that is, a material sample that incorporates a sufficient amount of micro-inhomogeneities. This sample is referred as representative volume element (RVE) in the literature.

The technique used for isotropic materials in this work is extended to the heat conduction behavior studies in anisotropic media. In recent years the heat conduction study of behavior in anisotropic media grew considerably due to wide application of these materials in engineering (Nemat-Nasser and Hori, 1999). However, there are few results on heat transfer in anisotropic media in the literature, ie, relatively few textbooks that devote significant content heat conduction problems in anisotropic bodies (Yao, et al., 2004). Among the contributions to anisotropic media can be cited a study that showed a formulation for the BEM related to the anisotropic media analysis plan, where have been discretized integral equations in the complex plane, which has become different from other commonly used formulations (Dondero, 2011).

Thus, this work aims to study the influence of solids behavior microstructure level when subjected to transient heat transfer using the MEC. The formulation is applied to isotropic and anisotropic materials.

2. METHOD OF BOUNDARY ELEMENTS FOR THE EQUATION OF DIFFUSION

This section presents the Boundary Element Method for the solution of the transient diffusion equation with time-dependent fundamental solution. To obtain an approximate solution for the unknowns in contour, the actual geometry is replaced by a set of elements in which the original arc length is substituted by straight lines defined by two geometric nodes for each element (Fig. 1).

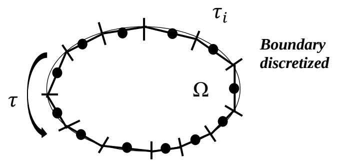


Figure 1. Approximation of the geometry

Transient heat conduction equation for one-dimensional domain isotropic and homogeneous materials without internal heat generation, from a time t is given by (Effren, 1997):

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha}\frac{\partial u}{\partial t} \tag{1}$$

Where u is the temperature and $\alpha = k/\rho c_p$ is the thermal diffusivity of the material and representes how fast the heat propagates through it. ρ is the specific mass and c_p is the specific heat of the material.

The differential equation that governs the heat flow problem given by equation (2) can be rewritten (Lee, et. Al, 1990):

$$\nabla^2 u(x,t) - \frac{1}{\alpha}\frac{\partial u(x,t)}{\partial t} = 0 \quad x \in \Omega \tag{2}$$

The Eq. (2) admits different interpretations according to the physical problem in question, assuming that is constant in time and space with the essential boundary conditions or Dirichlet and the natural boundary conditions or Neumann as Eq . (3) and (4) respectively.

$$u(x,t) = \bar{u}(x,t) \quad x \in \tau_u, t > 0, \tag{3}$$

$$q(x,t) = \frac{\partial u}{\partial n} = \bar{q}(x,t), x \in \tau_q, t > 0, \tag{4}$$

Since the problem is time-dependent should be prescribed initial conditions in time eq. (5).

$$u(x,t) = u_0(x,t), \text{ for t=0, in the } \omega \text{ domain} \tag{5}$$

Applying Weighted Residual method, the Eq. (2) can be transformed into an integral equation combined with the conditions given in Eqs. (3), (4) and (5). And considering his time dependence the fundamental solution u^* and q^* is obtained for the two-dimensional domain displayed in Eqs.(6) e (7).

$$u^{*}\left(\varepsilon,x,t_{F,}t\right)=\frac{1}{4\pi\alpha\tau}exp\left[\frac{-r^{2}\left(\varepsilon,x\right)}{4\alpha\tau}\right] \tag{6}$$

$$q^{*}\left(\varepsilon,x,t_{F,}t\right)=\frac{-r\dfrac{\partial r}{\partial n}}{8\pi\alpha^{2}\tau^{2}}exp\left[\frac{-r^{2}\left(\varepsilon,x\right)}{4\alpha\tau}\right] \tag{7}$$

Where: $\tau = t_F - t$, t_F is the observation time, t é the source application time, r is the distance between the point source ε and point field x. And α is the thermal diffusivity of the material.

Transient heat conduction equation for anisotropic materials will be considered in Section (4.2).

3. REPRESENTATIVE VOLUME ELEMENTS (RVE'S)

In this work, the numerical implementation generates holes in the field of a Representative Volume Element (RVE). This method applies to Midfields theory to find the effective properties (macroscopic) of a micro-porous material.

Thus, based on statistical analysis of a series of RVE's subjected to a thermal potential it is possible to determine the effective thermal conductivity k_{eff}.

Therefore, an RVE is a sufficiently small sample to be considered as a material point of the reporting area, but large enough to contain a statistically representative sample microstructure.

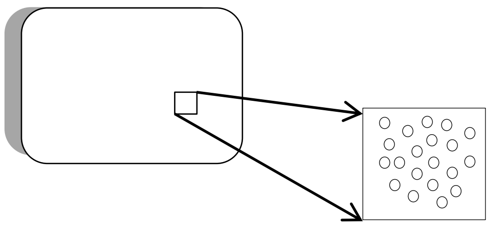


Figure 2. Scheme illustrating a RVE.

In heat conduction problems, the thermal conductivity is a second order tensor. Locally it is given by the ratio between the average volumetric heat flux and the volumetric average temperature gradient in a RVE. Thus, a simple way to estimate the effective thermal conductivity in the $\bar{q}$ direction can be presented (Pestana, 2014):

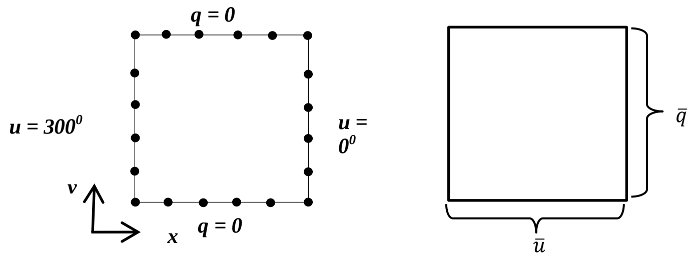
$$\mathrm{K}_{eff}=\frac{qL}{\left(U_{H}-U_{L}\right)} \tag{8}$$

Being $\bar{q}$ the heat flow, the differential potential $(U_H - U_L)$, L is the dimension that representes the side of the plate.

4. NUMERICAL RESULTS

To evaluate the influence of the number of holes in the time to reach steady state in heat transfer were evaluated six arrays, being the first solid, second, third, fourth, fifth and sixth, with 1, 4, 9, 16 and 25 holes, respectively. The outer contour of the plate was discretized with 24 constant elements, whereas the holes had boundary conditions prescribed isolated flows and were always discretized with the 16 constant elements (Fig. 3a).

The analysis time for heat transfer in the transient regime was 35 seconds and an interval of time of 200. The numerical integration was performed using the Gaussian quadrature with six points. The objective is to determine the effective thermal conductivity, the average flux at the right edge, and the average temperature in the lower edge of the plate (Fig. 3b).



(a) (b)

Figure 3. (a) Discretization and boundary conditions of RVE (b) Average flow $\bar{q}$ and average temperature $\bar{u}$

Another variable of interest is the time variation in relation to the number of holes until it reaches steady state. This study will point the influence of the amount of holes in relation to their thermal properties for micro-porous materials. In this paper, the term "microporous" is defined as circular shapes, insulated from each other and constant radius within the limited domain. The voids of the matrix are considered isolated due to the average thermal conductivity of air is very low (approximately 0.025 W / m.K) compared to other materials. Thus, each portion of the array can be considered an insulating (Shiah and Tan, 1997). The matrix conductivity tensor constants considered in this paper are the followings:

Isotropic model $k_{xx} = k_{yy}$
$$k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (9)$$

Anisotropic model $k_{xx} < k_{yy} > k_{xy}$
$$k = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 3 \end{bmatrix} \quad (10)$$

The conductivity tensor for isotropic materials only shows values of a unit in the x and y directions. However, for anisotropic materials values are adopted to emphasize the preferred direction of the heat flow in the material, which is the value three.

The effective thermal conductivity of this work is calculated as the integral of flow on the right side of the RVE.

$$k_{eff} = \frac{\sum_{i=1}^{\mathrm{ne}} \left(\frac{q_i}{\mathrm{ne}} \right) .L}{\left(U_H - U_L \right)} \quad (11)$$

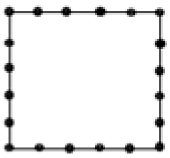
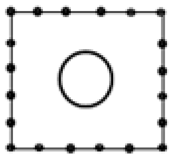
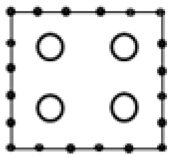
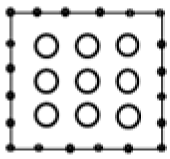
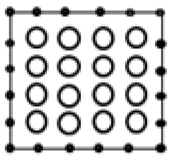
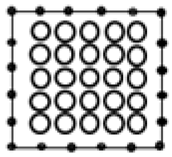
Where, ne is the number of elements used to discretize the edge in question and q_i is the heat flow in the "i-th" element of the edge calculated by the BEM. $(U_H - U_L)$ is the differential potential, L is the dimension that represents the right side of the plate.

4.1. Isotropic Potential Problems

A material is isotropic if its mechanical and thermal properties are the same in all directions. These materials may have homogeneous or non-homogeneous microscopic structures. For example, steel demonstrates isotropic behavior, although its microscopic structure is not homogeneous (Johann, 2005).

This section presents the results for isotropic material. In this regard, Table 1 shows the sum of flows performed in the right edge of the plate and K_{eff} for each of the cases. It can be seen that the solid plate had a flow of 50.5322 W / m^2 and an effective thermal conductivity of 1.0108 W / m ° C. However, in cases where holes are inserted in the flow field plate and the effective thermal conductivity undergone a minor change. Importantly, the representative RVE was always maintained at a constant volume ratio, which in this case is defined as the ratio between the area of holes and area of the solid plate. In the same table is also possible to conclude that the value of K_{eff} undergoes small oscillations which shows its dependence on the number and position of holes (Fig. 4).

Table 1. Solid Matrix and 1, 4, 9, 16 and 25 holes.

Solid Matrix	**1 hole**	**4 holes**
$q = \int q dL = 50.5376$ W/m² $k_{eff} = 1.0108$ W/m°C	$q = \int q dL = 42.5153$ W/m² $k_{eff} = 0.8503$ W/m°C	$q = \int q dL = 42.5078$ W/m² $k_{eff} = 0.8502$ W/m°C
		
9 holes	**16 holes**	**25 holes**
$q = \int q dL = 42.5052$ W/m² $k_{eff} = 0.8501$ W/m°C	$q = \int q dL = 42.5039$ W/m² $k_{eff} = 0.8501$ W/m°C	$q = \int q dL = 42.5048$ W/m² $k_{eff} = 0.8501$ W/m°C
		

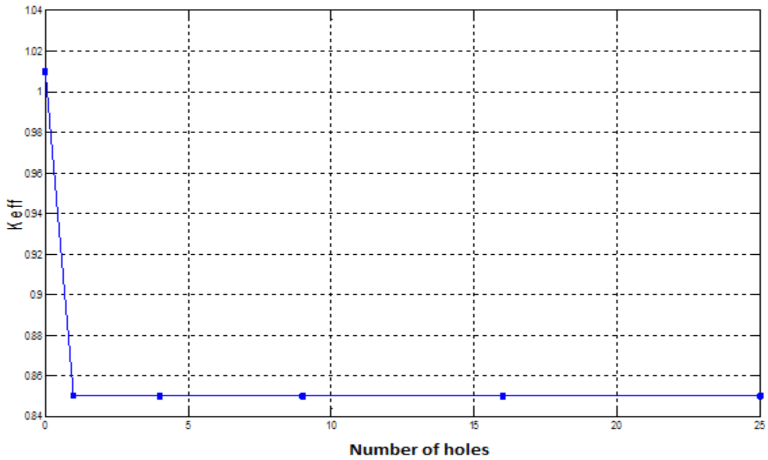


Figure 4. The thermal conductivity value compared to the number of holes.

4.2 Anisotropic Potential Problems

To study the behavior of anisotropic materials in this study it was used the known domain mapping method (or linear coordinate transformation). The method of linear transformation mapping coordinates consists of initially map an anisotropic domain to a new domain of isotropic behavior, however, mathematically equivalent to the original domain. It is important to emphasize that the mapping is done either in the geometry of the problem as well as the Neumann boundary conditions. And to find the final solution and the original flow field, applies the inverse of the geometric and flow mapping (Anflor, 2007; Pestana, 2014).

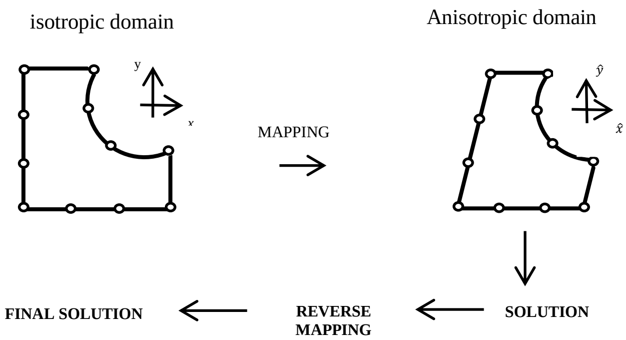


Figure 5. Illustrative geometric mapping scheme used.

The effective coordinates are used in matrix form to transform the anisotropic field in an equivalent isotropic:

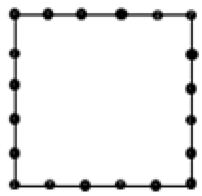
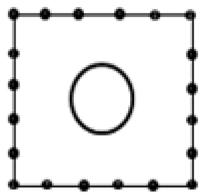
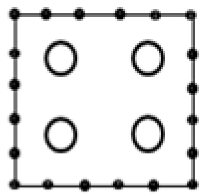
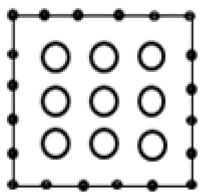
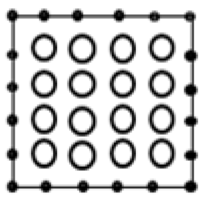
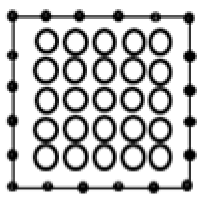
$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} 1 & \alpha \\ 0 & \beta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{12}$$

Where variables α, β e k are, respectively:

$$\alpha = \frac{-k_{xy}}{k_{yy}} \therefore \beta = \frac{k}{k_{yy}} \therefore k = \sqrt{k_{xx}k_{yy} - k_{xy}^2} \tag{13}$$

Thus, Table 2 shows the summation of flows performed in the right edge of the plate and k_eff for each of the cases examined in the study of anisotropic problems. It is noted that the solid plate had a flow rate of 70.56 W / m^2 and an effective thermal conductivity of 1.4113 $W / m\ °C$. However, for cases in which holes were inserted in the field of plate the flow and the effective thermal conductivity suffered a small oscillation that shows that such properties also depend on the position of the holes (Fig. 6). For the study of anisotropic materials RVE was also always maintained at a constant volume ratio.

Table 2. Solid Matrix 1, 4, 9, 16 and 25 holes.

Solid Matrix	**1 hole**	**4 holes**
$q = \int qdL = 70.56\ W/m^2$ $k_{eff} = 1.4113\ W/m°C$	$q = \int qdL = 64.18\ W/m^2$ $k_{eff} = 1.2837\ W/m°C$	$q = \int qdL = 64.15\ W/m^2$ $k_{eff} = 1.2834\ W/m°C$
		
9 holes	**16 holes**	**25 holes**
$q = \int qdL = 64.13\ W/m^2$ $k_{eff} = 1.2826\ W/m°C$	$q = \int qdL = 64.11\ W/m^2$ $k_{eff} = 1.2823\ W/m°C$	$q = \int qdL = 64.10\ W/m^2$ $k_{eff} = 1.2820\ W/m°C$
		

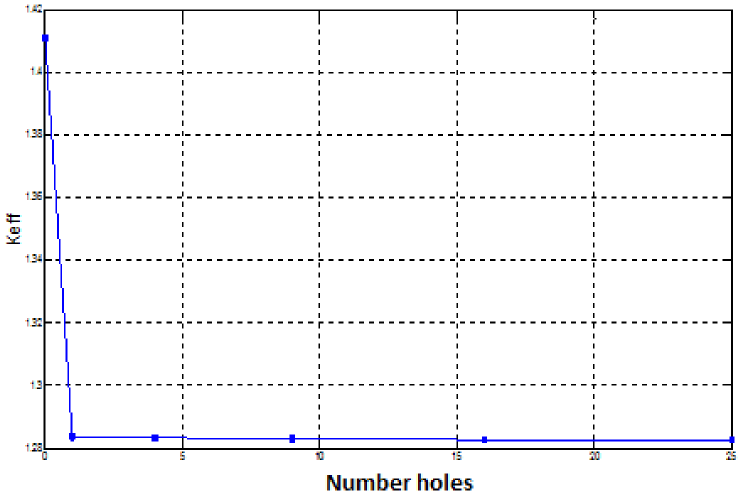


Figure 6. The thermal conductivity value compared to the number of holes.

Analyzing the temperature history of materials studied, it is noted that for the anisotropic case, the transience was longer than for isotropic (Fig. 7). This is because the effective thermal conductivity of a material depends not only on its structure, but also changes in that structure that will cause effective anisotropy in thermal conductivity. Thus, the anisotropy causes a heat dissipation at different rates in different directions and under these conditions, the time to reach steady state will be higher than for isotropic materials, having the same physical properties regardless of the direction considered (Pestana, 2014).

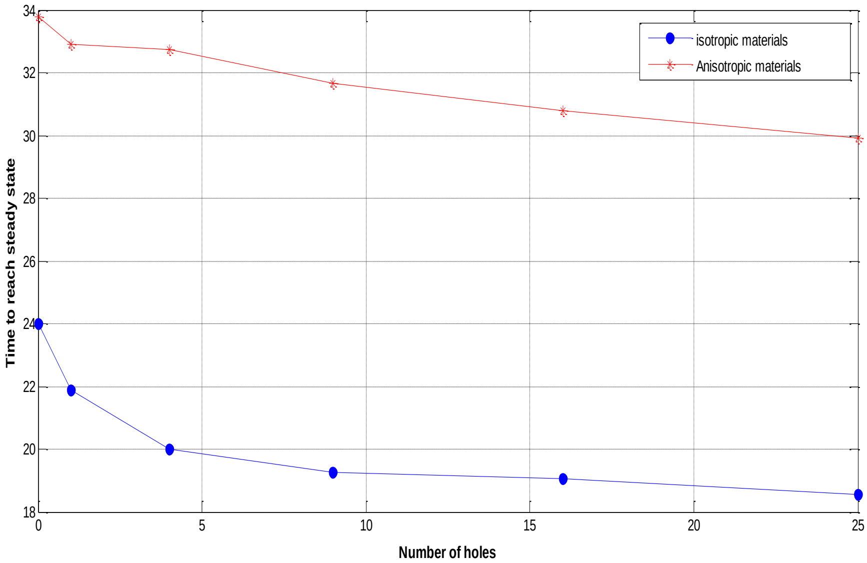


Figure 7. Time to reach steady state in arrays: solid, with 1, 4, 9, 16 and 25 holes (isotropic and anisotropic materials).

5. CONCLUSION

In this paper a study was made to estimate the thermal properties of a micro-porous material represented by a RVE with a constant volume ratio. The BEM was used for the solution of differential equations of heat conduction problem using time dependent fundamental solutions. The results showed that the tendency to achieve a constant volume ratio EVR for both isotropic materials as well as for the anisotropic is that the thermal properties don’t suffer more changes, however, the same is not true for the transient transition time for permanent. A larger number of holes in the solid matrix decreases the transience of time on both materials, although the volume ratio be maintained always constant.

However, it is noted that for the anisotropic materials transience time is longer than for isotropic materials, because the physical properties of that one are different in different directions, which justifies a greater time to reach steady state. Further investigation in this area are still being evaluated as a natural way to go in this work is to extend it for parameter optimization problems where it will be possible to analyze the influence of position, number of holes and stationary time.

6. ACKNOWLEDGEMENTS

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