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PID CONTROL AND ANALYSIS OF THE ROBUSTNESS IN A QUADROTOR

Gabriela Vieira Lima
Rafael Monteiro Jorge Alves de Souza
Faculdade de Engenharia Elétrica (FEELT) – Universidade Federal de Uberlândia (UFU)
Av. João Naves de Ávila, 2160 – Bloco 3N – Campus Santa Mônica – CEP: 38400-902. Uberlândia, MG, Brasil.
gabriela@eel.ufu.br; rafael.mjas@gmail.com

Fábio Vincenzi Romualdo da Silva
Faculdade de Engenharia Elétrica (FEELT) – Universidade Federal de Uberlândia (UFU)
Av. João Naves de Ávila, 2160 – Bloco 3N – Campus Santa Mônica – CEP: 38400-902. Uberlândia, MG, Brasil.
fabiovince@gmail.com

Aniel Silva de Morais
Faculdade de Engenharia Elétrica (FEELT) – Universidade Federal de Uberlândia (UFU)
Av. João Naves de Ávila, 2160 – Bloco 3N – Campus Santa Mônica – CEP: 38400-902. Uberlândia, MG, Brasil.
aniel@eletrica.ufu.br

Guênia Mara Vieira Ladeira
Faculdade de Engenharia Mecânica (FEMEC) – Universidade Federal de Uberlândia (UFU)
Av. João Naves de Ávila, 2160 – Bloco 1M – Campus Santa Mônica – CEP: 38400-902. Uberlândia, MG, Brasil.
guenia@mecanica.ufu.br

Josué Silva de Morais
Faculdade de Engenharia Elétrica (FEELT) – Universidade Federal de Uberlândia (UFU)
Av. João Naves de Ávila, 2160 – Bloco 3N – Campus Santa Mônica – CEP: 38400-902. Uberlândia, MG, Brasil.
josue@eletrica.ufu.br

Abstract. *Technological advances in microelectromechanical (MEMS) sensors and actuators, in energy storage and in data processing have enabled the development of unmanned aerial vehicles (UAVs). The quadrotor has emerged as one of these research platforms due to its mechanical simplicity and high maneuverability, as well as the ability to hover and perform vertical take-off and landing. This study demonstrates the implementation of a control strategy for providing a solution to problems of positioning and path tracking of the quadrotor. The dynamic model was obtained through the Euler-Lagrange formalism. As this is an underactuated mechanical system, its control system has to be of a hierarchical scheme, composed by two subsystems: the translational and rotational. The translational subsystem enables the positioning and path tracking, while the rotational subsystem stabilizes the vehicle orientation angles. For reaching such objectives, proportional integral derivative (PID) classical controls were used. The robust performance of the controller was verified through of simulations in the presence of aerodynamic disturbances and parametric uncertainties.*

Keywords: *PID Controller, Dynamic Model, Quadrotor, Path Tracking, Underactuated Mechanical System.*

1. INTRODUCTION

The progress performed in areas of sensors, energy storage and data processing allowed the development of unmanned aerial vehicles (UAVs). These vehicles can be used in civil and military applications, in tasks such as search and rescue, inspections over large areas, security, intervention in hostile environment, as well as in the film and entertainment industry.

The quadrotor belongs to a class of unmanned aerial vehicles with rotating wings. The system is similar to a helicopter, however, with four sets of rotors/propellers as shown in Fig. 1.

This configuration holds several advantages when compared to the conventional helicopter. As the quadrotor is lifted and propelled by four rotors, it is possible to reduce the individual rotor size and maintain or increase the total vehicle load capacity. In addition, this system does not require angular control on the propellers. Factors such as these allow for a significant reduction in project costs and vehicle maintenance (Bouabdallah *et. al*., 2004).

Other advantages associated with the quadrotor are: high maneuverability and the ability of performing vertical take-off and landing, as well as stabilize at a particular altitude (hover), characteristics that allow for its use in environments hostile to human beings (Lima *et. al.*, 2014).

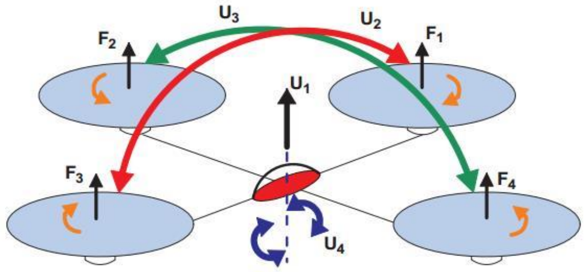

Figure 1. Schematic diagram of a quadrotor (Raffo, 2007).

The quadrotor is an underactuated mechanical system, once it possesses six degrees of freedom and only four actuators. Besides this, it is a nonlinear system, subject to parametric uncertainties and unmodeled dynamic. Such factors make the application of control strategies over the vehicle difficult (Raffo, 2007), (Souza *et. al*., 2014).

Several studies have been published describing the dynamic modeling and control strategies of the quadrotor. In the area of control, the first challenge was stabilizing the orientation angles of the vehicle. A number of strategies have had success in this task and are still used today. Following on, the proposal was to design control strategies for vehicle navigation in a three-dimensional space (3D).

Classic controllers of the PID type were developed for stabilizing the orientation angles of UAVs. According to Bouabdallah *et. al.*, (2004) and Azfar and Hazry (2011), the technique shows itself to be efficient, even in disturbance rejection.

In Castillo *et. al.* (2006), proportional derivative controllers (PD) were designed, for stabilizing the orientation and altitude of the aerial vehicle, which allows for the performing of hover maneuver.

In Salih *et. al.* (2010); Erginer and Altug (2007); Li e Li (2011), PID controllers were constructed for navigating the quadrotor in a three-dimensional space. Continuing the same line of research, in Zareb *et. al.*, (2013), a hybrid control technique was developed, the PID-fuzzy, for flights in outdoor or indoor environments.

Nevertheless, research associated with quadrotor navigation in three-dimensional space, does not present PID control results concerning situations involving aerodynamic disturbances and parametric uncertainties.

Thereby, this study has as its objective the analysis of the behavior of aerial vehicles in tasks, such as path tracking, observing the robustness of the PID controller in rejecting aerodynamic disturbances and parametric uncertainties of aerial vehicles.

2. DYNAMIC MODELING

The dynamic modeling of the system is obtained under the hypothesis that the vehicle is a body rigid in space, and subject to the main force (thrust) and three torques.

Without loss of generality, one can establish the following assumptions (Bouabdallah *et. al*., 2004):

- The quadrotor structure is supposedly rigid and symmetric.
- The propellers are supposedly rigid.
- The center of mass of the vehicle coincides with the origin of the rigid body's fixed coordinate system.
- The thrust and drag are proportional to the squared velocity of the propellers.

Consider a system of coordinates fixed to a rigid body $B = \{\vec{x}, \vec{y}, \vec{z}\}$ able to rotate and translate relative to another system of coordinates, the inertial frame $E = \{\vec{X}, \vec{Y}, \vec{Z}\}$ considered fixed to the earth, as in Fig. 2.

The position and orientation of the quadrotor are given through Eq. (1):

$$\xi = [x\ y\ z\ \phi\ \theta\ \psi]^T \tag{1}$$

where, x, y, z represents the translation on directions X, Y, Z of the coordinate frame E. With respect to orientation one has: ϕ that represents the roll angle, θ is the pitch angle and ψ is the yaw angle of the quadrotor.

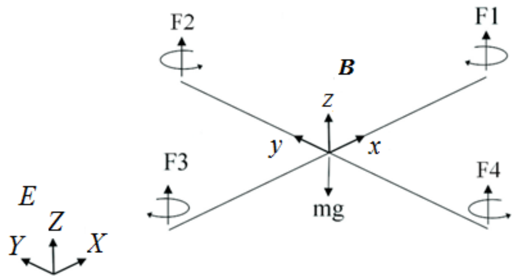

Figure 2. Coordinate system of Quadrotor (Bouabdallah *et. al*., 2004).

2.1 Modeling of the translation movement

Through the parametrization of the Euler angles XYZ, we can obtain the orientation of the aerial vehicle in three dimensions, using the rotation matrix R, Eq. (2):

$$R = \begin{bmatrix} C\psi C\theta & C\psi S\theta S\phi - S\psi C\phi & C\psi S\theta C\phi + S\psi S\phi \\ S\psi C\theta & S\psi S\theta S\phi + C\psi C\phi & S\psi S\theta C\phi - C\psi S\phi \\ -S\theta & C\theta S\phi & C\theta C\phi \end{bmatrix} \tag{2}$$

where, $C(\cdot) = \cos(\cdot)$ and $S(\cdot) = \text{sen}(\cdot)$.

The resultant force on the translation system is the vertical thrust (U_1) produced by the actuation of the four propellers. This force has the direction *z* on the coordinate system fixed to the rigid body and is proportional to the squared velocities of the propellers, as in Eq. (3).

$$U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \tag{3}$$

Where Ω_i refers to the angular speed of each motor, and b is the thrust coefficient of the propellers.

Thus obtains the movement equations referring to the translation system, using Newton's second law, represented in Eq. (4):

$$\begin{cases} \ddot{x} = \left(S\psi S\phi + C\psi S\theta C\phi\right)\dfrac{U_1}{m} \\ \ddot{y} = \left(-C\psi S\phi + S\psi S\theta C\phi\right)\dfrac{U_1}{m} \\ \ddot{z} = g - \left(C\theta C\phi\right)\dfrac{U_1}{m} \end{cases} \tag{4}$$

where *m* is the total mass of the quadrotor and *g* represents the gravitational acceleration.

2.2 Modeling of the rotation movement

The equations for the rotation movement can be expressed through the formalism of Euler-Lagrange, based on the concepts of kinetic and potential energy, as in Eq. (5).

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \gamma_i \tag{5}$$

Where L represents the Lagrangian of the system. The generalized coordinates are represented by q_i and the non-conservatives forces and torques are given by γ_i.

Assuming that the inertia matrix is diagonal (symmetrical structure), one arrives at the following expression for the kinetic energy of the quadrotor, demonstrated in Eq. (6):

$$T = \frac{1}{2} I_{xx}(\dot{\phi} - \dot{\psi} S\theta)^2 + \frac{1}{2} I_{yy}(\dot{\theta} C\phi + \dot{\psi} S\phi C\theta)^2 + \frac{1}{2} I_{zz}(\dot{\theta} S\phi - \dot{\psi} C\phi C\theta)^2 \tag{6}$$

where I_{xx}, I_{yy}, I_{zz} are the inertia moments on the axis *x,y,z.*

Considering that the acceleration due to gravity is the same at every point of the quadrotor, we can express the total potential energy of the vehicle as, Eq. (7):

$$V = \int x\ dm(x).(-gS\theta) + \int y\ dm(y).(gS\phi C\theta) + \int z\ dm(z).(gC\phi C\theta) \quad (7)$$

So, replacing (6) and (7) on the Lagrangian equation (5), we obtain the following movement equations, Eq. (8):

$$\ddot{\phi} = \frac{\dot{\theta}\dot{\psi}(I_{yy} - I_{zz})}{I_{xx}}$$
$$\ddot{\theta} = \frac{\dot{\phi}\dot{\psi}(I_{zz} - I_{xx})}{I_{yy}}$$
$$\ddot{\psi} = \frac{\dot{\theta}\dot{\phi}(I_{xx} - I_{yy})}{I_{zz}} \quad (8)$$

The non-conservative torques that act on the aerial vehicle are described on Eq. (9).

$$U_2 = bl\left(\Omega_4^2 - \Omega_2^2\right)$$
$$U_3 = bl\left(\Omega_3^2 - \Omega_1^2\right)$$
$$U_4 = d(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \quad (9)$$

Where l is the half span of the quadrotor and d is the drag coefficient of the propellers

In addition, the rotation system is also submitted to gyroscope effect, resulting from the propellers movement, Eq. (10):

$$\tau'_x = J\omega_y\left(\Omega_1 + \Omega_3 - \Omega_2 - \Omega_4\right)$$
$$\tau'_y = J\omega_x\left(\Omega_2 + \Omega_4 - \Omega_1 - \Omega_3\right) \quad (10)$$

where, J represents the inertia moment of the motor. The variables ω_x and ω_y represents the angular speed on the x and y axis, respectively.

One concludes here that the dynamic model of the quadrotor that describes the roll, pitch and yaw movements contains the gyroscope effect resulting from the rotation of the rigid body, the gyroscope effect resulting from the propeller rotation and finally the action of the actuator Eq. (11):

$$\begin{cases} \ddot{\phi} = \dfrac{\dot{\theta}\dot{\psi}(I_{yy} - I_{zz})}{I_{xx}} + \dfrac{J\dot{\theta}\Omega}{I_{xx}} + \dfrac{U_2}{I_{xx}} \\ \ddot{\theta} = \dfrac{\dot{\phi}\dot{\psi}(I_{zz} - I_{xx})}{I_{yy}} - \dfrac{J\dot{\phi}\Omega}{I_{yy}} + \dfrac{U_3}{I_{yy}} \\ \ddot{\psi} = \dfrac{\dot{\theta}\dot{\phi}(I_{xx} - I_{yy})}{I_{zz}} + \dfrac{U_4}{I_{zz}} \end{cases} \quad (11)$$

3. CONTROL

The control strategy used in this article is presented Fig. 3, known as the cascade structure.

The path to follow in the Cartesian space $\Re^3$ is supplied by the path generator block. Such a block provides the three position coordinates (X_d, Y_d, Z_d) and the yaw angle (ψ_d). The generated path does not consider the presence of aerodynamic disturbances and instability on the vehicle orientation along the track (Raffo, 2007).

The *Translational Controller* enables navigation and stabilizes the quadrotor in three-dimensional space. This type of control is executed in two stages. Initially, we control the altitude of quadrotor by manipulating the total thrust (U_1). In sequence, the references for the roll and pitch angles (ϕ_d, θ_d) are generated through two virtual inputs, calculated in a way that the quadrotor can execute the movement on the *xy* plane.

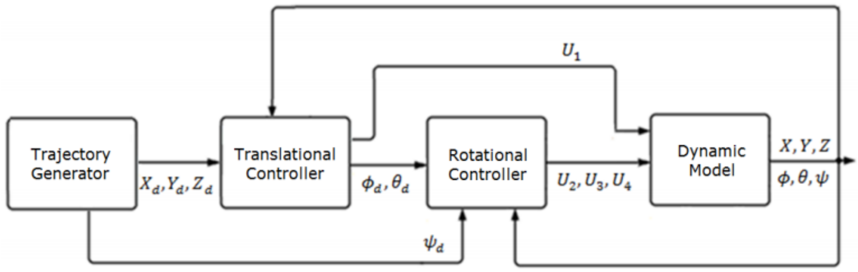

Figure 3. Control strategy of the quadrotor.

The *Rotational Controller* allows for the stabilization of the three orientation angles of the aerial vehicle: roll, pitch and yaw. The manipulated variables correspond to the torques applied to the three axis.

Then, we apply the main force (thrust) and the three torques on the quadrotor in order to provide the movement desired.

3.1 Translational Control

In this section the laws of control that allow the solution to path tracking are presented.

3.1.1 Altitude

The control of the vertical position can be obtained by using the following control input, Eq. (12):

$$U_1 = (r_1 - mg).\frac{1}{\cos\theta\cos\phi} \tag{12}$$

where r_1 represents a PID controller, such that:

$$e_z = z_d - z$$
$$r_1 = k_{pz}.e_z + k_{iz}.\int_0^t e_z dt + k_{dz}.\dot{e}_z \tag{13}$$

with: k_{pz}, k_{iz} and k_{dz} being the gains of the altitude controller.

3.1.2 Displacement on the xy plane

Through the dynamic system model (14) one observes that the displacement on the *xy* plane depends on the input of control U_1.

In fact, U_1 corresponds to the magnitude of thrust necessary to obtain the desired linear movement, while u_x and u_y can be considered as directions of U_1 that generate the movement on axis *x* and *y* respectively.

$$\ddot{x} = (u_x)\frac{U_1}{m}$$
$$\ddot{y} = (u_y)\frac{U_1}{m} \tag{14}$$

where:

$$u_x = \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi$$
$$u_y = -\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi \tag{15}$$

The virtual input calculations u_x and u_y are performed through equation 14, when PID control actions have already been applied to the x and y axis. Through these virtual inputs, the necessary values for the roll, and pitch (ϕ_d, θ_d) for guiding the quadrotor on the *xy* plane are calculated as shown in Eq. (15).

The PID controllers were tuned and the acquired gains can be seen in Table 1.

3.2 Rotational Control

Rotational control provides the stabilization of the three orientation angles of the quadrotor: roll, pitch and yaw. To this three independent PID controllers were constructed, where the torques applied to the axis correspond to the manipulated variables. The laws of control are represented in Eq. (16).

$$U_2 = k_{p\phi}.e_\phi + k_{i\phi}.\int_0^t e_\phi dt + k_{d\phi}.\dot{e}_\phi$$
$$U_3 = k_{p\theta}.e_\theta + k_{i\theta}.\int_0^t e_\theta dt + k_{d\theta}.\dot{e}_\theta$$
$$U_4 = k_{p\psi}.e_\psi + k_{i\psi}.\int_0^t e_\psi dt + k_{d\psi}.\dot{e}_\psi \quad (16)$$

The PID controllers were tuned and the acquired gains can be seen in Tab. 1.

Table 1. PID controllers gains.

Control	k_p	k_i	k_d
X	0.10	0.0005	1.30
Y	0.10	0.0005	1.30
Z	17.00	5.00	7.00
ϕ	0.12	0.0005	0.08
θ	0.12	0.0005	0.08
ψ	0.05	0.0005	0.02

4. RESULTS

The control strategy proposed in this study was verified through computational simulations. The behavior of the aerial vehicle was analyzed by means of tasks based on positioning and path tracking, coupled with the presence of parametric uncertainties and aerodynamic disturbances.

The nominal parameters of the quadrotor used in the simulation, are presented in Table 2.

The authors used the helicoid trajectory as their reference, this trajectory is defined in the following, Eq. (17).

$$x_d = \frac{1}{2}\cos(t/2);\, y_d = \frac{1}{2}\operatorname{sen}(t/2);\, z_d = 1 + \frac{t}{10} \quad (17)$$

The initial and end coordinates of the aerial vehicle were:

$[x, y, z] = [0.5, 0, 1]$ m $[\phi, \theta, \psi] = [0, 0, 0.5]$ $rad.$
$[x, y, z] = [0.5, 0, 6]$ m $[\phi, \theta, \psi] = [0, 0, 0]$ $rad.$

Table 2. Quadrotor parameters.

Parameter	**Units**	**Value**
m	kg	0.53
l	m	0.23
I_{xx}	$kg.m^2$	$6.22e^{-3}$
I_{yy}	$kg.m^2$	$6.22e^{-3}$
I_{zz}	$kg.m^2$	$1.12e^{-2}$

During the simulation uncertainties of $\pm 20\%$ were considered on inertial parameters, in this manner it was possible to evaluate the robustness of the controller.

In addition, external disturbances were applied to the translational system: $A_x = 1N$ in $t = 20s$; $A_y = 1N$ in $t = 30s$; $A_z = -1N$ in $t = 40s$.

The results from the task of path tracking can be seen in Figures 4 and 5.

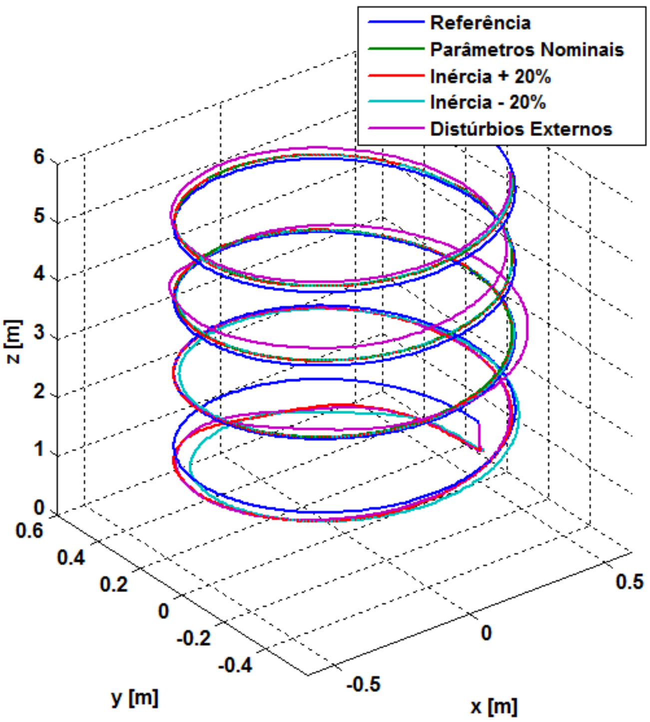

Figure 4. Helicoidal path tracking.

One notes, that the system presented a robust behaviour when submitted to uncertainty parameters concerning inertia, since the aerial vehicle follows practically the same path when compared to nominal parameters, besides not presenting instabilities.

In addition, it was possible to check that the system rejected small aerodynamic disturbances not sustained on the translational system. At the moment disturbances were applied, the quadrotor presented a small overshoot, leaving the desired path but was very quickly able to return to its ideal setting and complete the task. It was also verified that disturbances on the rotational system are not rejected, thus causing destabilization of the quadrotor.

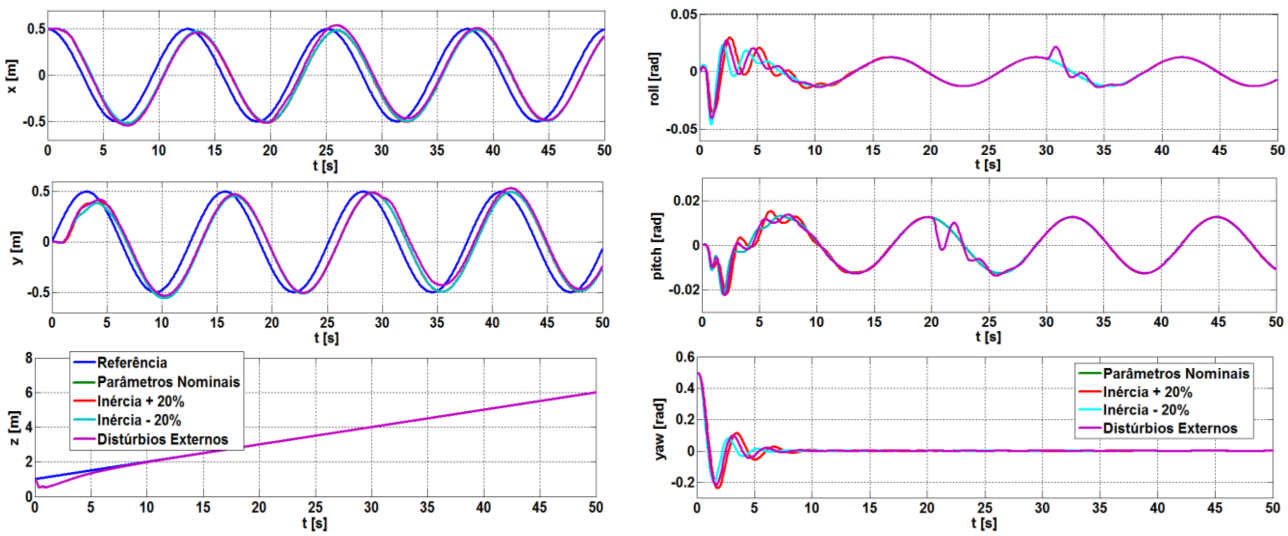

Figure 5. Position (x, y, z) and orientation (ϕ, θ, ψ) in the presence of external disturbances and parametric uncertainties.

Through Fig. 5 one understands that the controls present a short delay in the path tracking task, however, this it has not caused issues on the execution of the proposal goal. Besides, the system exhibited a stable behaviour without overshoot.

One also observes that the roll and pitch angles presented a small oscillation of around zero. However, the UAV executed the flight mission in a stable manner. The yaw angle stabilized quickly and presented a null stationary error.

5. CONCLUSIONS

In this article the authors presented the dynamic modeling of an unmanned aerial vehicle of the quadrotor type, obtained through the Euler-Lagrange formalism. Based on this model a cascade control strategy was developed, using PID controllers with the objective of guiding the quadrotor in its positioning tasks as well as in path tracking.

The efficiency of the PID controller was checked through closed loop simulation, where characteristics of the following type were observed: stability, response time, stationary error and overshoot.

Another noted factor was the robustness of the PID controller. Simulations were carried out taking into consideration uncertainties on the inertial parameters of the dynamic model. In addition, the behaviour of the aerial vehicle in the presence of aerodynamic disturbances as wind gust situations.

It should be highlighted that the control system presented herein was considered efficient, since it executed the positioning and path tracking tasks proposed, besides allowing for the analysis of robustness in the face of parametric uncertainty and aerodynamic disturbances.

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7. RESPONSIBILITY NOTICE

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