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AN IMPROVEMENT OF ENERGY HARVESTING USING A NON-ENERGY SINK (NES) DEVICE AS A PASSIVE CONTROL ON A 2:1 INTERNAL RESONANCE STRUCTURE

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Abstract. *Energy harvesting based on a simple portal frame structure with a non-energy sink (NES) device coupled to the symmetric mode is presented. Nonlinearities presented in piezoelectric material are considered in the piezoelectric coupling mathematical model. The system presents a 2:1 internal resonance (saturation phenomenon) also external resonance with a harmonic force excitation is considered. In order to find out correlation between values of the passive control device that optimizes power or average values that stabilizes a possible chaotic behaviour into the periodic orbit, we used some analysis of the average power, bifurcation diagrams and parametrical variations. The proposed passive control technique uses a simple mass-damping-springer device in order to tune the vibration of the structure to improve the energy harvesting. The results show that with the implementation of the control strategy it is possible to the captured energy to a desired operating frequency.*

Keywords: *Energy Harvesting, Passive Control, Non-ideal System, Saturation Phenomenon*

1. INTRODUCTION

In recent past years the research about vibration energy harvesting has increased substantially. Many of those vibration sources are found in structures that are excited by wind, sea waves, vehicle traffics, i.e., external excitations.

The research about piezoelectric material as a means of energy transduction has been widely studied as we see in the work of, for example, (Erturk and Inman, 2009; Litak, et al., 2012; Stanton, et al., 2010; Erturk and Inman, 2009; Stephen, 2006). Some nonlinearities of the piezoelectric material was experimentally found by (Crawley and Anderson, 1990), and an analytical approximated was proposed by (Tripplet and Quinn, 2009). Recently, the nonlinearities of vibratory energy harvesting was widely exploited by (Daqaq, et al., 2014).

Some kind of structures may present internal resonance between conditions, such as 2:1, so that the system transfer part of the energy available at a certain coordinate to the another one. This is the saturation phenomenon described by many authors, for example, among others, (Nayfeh and Mook, 2008; Mook, et al., 1985; Nayfeh, 2000). The implementation of saturation as a control method was proposed and studied by many authors, for example, (Pai, et al., 1998; Felix, et al. 2005; Tusset, et al., 2015).

In recent work, a model of an energy harvested based on a simple portal frame structure was presented (Iliuk, et al., 2013). The system was found to be a bi-stable Duffing oscillator presenting chaotic behaviour. The structure was controlled using a NES (Nonlinear Energy Sink) (Vakakis, 2008), and after addition of piezoelectric elements, energy harvesting was considered. The results showed that the energy harvesting with a NES passive controller was improved, since the system was forced to a periodic orbit. The elimination of chaotic behaviour by a passive control was investigated in (Tusset et al. 2012).

Hence, this work will investigate the influence of the passive control on energy harvesting of a simple portal frame of two-degrees-of-freedom structure with a piezoelectric material coupled to a column. Numerical results will be computed.

2. PHYSICAL AND MATHEMATICAL PROBLEM

The physical model studied in this paper, illustrated in Fig. 1, consists in a portal frame of two-degrees-of-freedom with a piezoelectric material coupled to a column and a nonlinear energy-sink (NES) device coupled to the mid span of the beam, i.e., the NES device will move to the symmetric mode direction.

The portal frame consists of two columns clamped in their bases with height h and a horizontal beam pinned to the columns at both ends with length L. Both column and beam have flexural stiffness EI. The mass at mid span of the beam is M. The masses of the columns are m. The structure is modelled as a lumped mass system with two-degrees-of-freedom. The coordinate q_1 is related to the horizontal displacement in the sway mode, with natural frequency ω_1, and q_2 to the mid-span vertical displacement of the beam in the symmetric mode, with natural frequency ω_2. The linear stiffness of the columns and the beam can be evaluated by a Rayleigh-Ritz procedure using cubic trial functions. Geometric nonlinearity is introduced by considering the shortening due to bending of the columns and of the beam.

The nonlinear energy-sink (NES) device consists of a spring-mass-damping system coupled to the mass at the mid span of the beam. The damping of the NES is c_n. We consider a nonlinear stiffness of a cubic term k_n. The mass of the device is m_4. The coordinate q_4 is related to the movement of the NES device. It will depend on the movement of the first symmetric mode.

The nonlinear piezoelectric material is coupled to the column as an electric circuit, which is excited by an internal voltage (back-emf) proportional to the mechanical velocity, in order to harvest energy from the vibration of the column. This circuit consists of a resistor R, a produced charge Q and a capacitance C_p of the capacitor. The dimensionless relation of nonlinearity of the piezoceramic is given by $d(q_1) = \theta(1+\Theta|q_1|)$ defined by (Triplett and Quinn, 2009), where θ is the linear piezoelectric coefficient and Θ is the nonlinear piezoelectric coefficient.

The mechanical system is based-excited by a harmonic force which has amplitude F_0 and external frequency ω_n. This external force frequency is set near resonance with the symmetric mode. Frequency ω_2 is also set twice the frequency of the sway mode as $2\omega_1 = \omega_2$. These conditions of resonance are necessary to have modal coupling in the nonlinear adopted model. In these conditions, we have the saturation phenomenon.

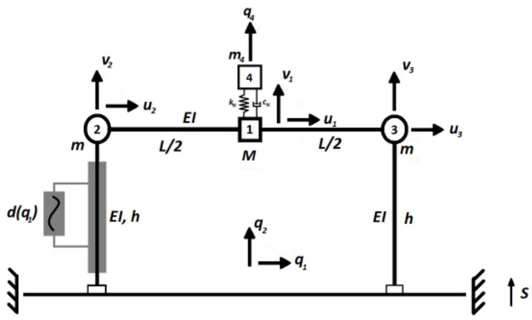

Figure 1 - Physical model of the system.

2.1 Modelling of the physical model

The modelling of the physical model will be used the energy Lagrange's method which uses the Lagrangian function and Euler-Lagrange equation.

Nodal displacements, shown in Fig. 1, are

$$\begin{aligned} &u_1 = q_1 \qquad && u_2 = u_1 + \frac{B}{4}v_1^2 \qquad && u_3 = u_1 - \frac{B}{4}v_1^2 \\ &v_1 = q_2 \qquad && v_2 = -\frac{A}{2}u_1^2 \qquad && v_3 = -\frac{A}{2}u_1^2 \end{aligned} \tag{1}$$

where A = 6/5h and B = 24/5L. The stiffness of the beam and column calculated by the Rayleigh-Ritz method are, respectively, $k_b = 48EI/L^3$ and $k_c = 3EI/h^3$.

The generalized coordinates considered here, are the displacements of the mass at the mid span of the beam M. Using nodal displacements of (1), the kinetic energy is defined in Eq. (2).

$$T = \frac{1}{2}M\left(\dot{u}_1^2 + \dot{v}_1^2\right) + \frac{1}{2}m\left(2\dot{u}_1^2\right) + \frac{1}{2}m_4\dot{q}_4^2 \tag{2}$$

Introducing the generalized coordinates q_1 and q_2, the kinetic energy becomes Eq. (3).

$$T = \frac{1}{2}M\left(\dot{q}_1^2 + \dot{q}_2^2\right) + \frac{1}{2}m\left(2\dot{q}_1^2\right) + \frac{1}{2}m_4\dot{q}_4^2 \tag{3}$$

The potential energy of the system is given by the strain energy of the structure, the cubic nonlinear strain of NES device, the work of the weight of the masses of the beam and columns and by electrical potential part of the piezoelectric circuit with the contribution of the piezoelectric and the capacitor, resulting in Eq. (4).

$$U = \frac{1}{2}k_c\left(u_2^2 + u_3^2\right) + \frac{1}{2}k_b\left(v_1 - \frac{v_2 + v_3}{2}\right)^2 + mg\left(v_2 + v_3\right) + Mgv_1 + \frac{1}{4}k_n\left(q_4 - v_1\right)^4 - \frac{d\left(q_1\right)}{C_p}Q\left(u_2 + v_2\right) + \frac{1}{2}\frac{Q^2}{C_p} \tag{4}$$

Substituting (1) in (4), in terms of the general coordinates q_1, q_2, q_4 and Q, we have the potential energy in Eq. (5).

$$U = \left(k_c - mgA\right)q_1^2 + \frac{1}{2}k_b\left(q_2^2 + Aq_2q_1^2\right) + Mgq_2 + \frac{1}{4}k_n\left(q_4 - q_2\right)^4 - \frac{d\left(q_1\right)}{C_p}Q\left(q_1 + \frac{B}{4}q_2^2\right) + \frac{1}{2}\frac{Q^2}{C_p} \tag{5}$$

Now, we consider energy dissipation of the system, comprising the structural and NES damping defined by a Rayleigh function and the resistor of the electrical circuit. Then it follows in Eq. (6).

$$D = \frac{1}{2}c_1\dot{q}_1^2 + \frac{1}{2}c_2\dot{q}_2^2 + \frac{1}{2}c_n\left(\dot{q}_4 - \dot{q}_2\right)^2 + \frac{1}{2}R\dot{Q}^2 \tag{6}$$

The harmonic excitation force is given by (7).

$$S = F_0\cos(\omega_n t) \tag{7}$$

The Lagrangean function is defined by Eq. (8). Substituting Eqs. (3) and (5) in Eq. (8) we have the Lagrangian of Eq. (9).

$$L\left(q,\dot{q},t\right) = T - U \tag{8}$$

$$L = \frac{1}{2}M\left(\dot{q}_1^2 + \dot{q}_2^2\right) + \frac{1}{2}m\left(2\dot{q}_1^2\right) + \frac{1}{2}m_4\dot{q}_4^2 - \left[\begin{array}{l}\left(k_c - mgA\right)q_1^2 + \frac{1}{2}k_b\left(q_2^2 + Aq_2q_1^2\right) + \ldots \\ Mgq_2 + \frac{1}{4}k_n\left(q_4 - q_2\right)^4 - \frac{d\left(q_1\right)}{C_p}Q\left(q_1 + \frac{B}{4}q_2^2\right) + \frac{1}{2}\frac{Q^2}{C_p}\end{array}\right] \tag{9}$$

Now, using Euler-Lagrange, Eq. (10), we have the equation of motion of the system that are Eqs. (11), (12), (13) and (14).

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_{ext} \qquad i = 1,4 \tag{10}$$

$$\left(2m + M\right)\ddot{q}_1 + 2\left(k_c - mgA\right)q_1 + k_bAq_2q_1 + c_1\dot{q}_1 = \frac{d\left(q_1\right)}{C_p}Q \tag{11}$$

$$M\ddot{q}_2 + k_bq_2 + c_2\dot{q}_2 + Mg + \frac{Ak_b}{2}q_1^2 + k_n\left(q_2 - q_4\right)^3 + c_n\left(\dot{q}_2 - \dot{q}_4\right) = F_0\cos\omega_n t + \frac{d\left(q_1\right)}{C_p}\frac{B}{2}Qq_2 \tag{12}$$

$$m_4\ddot{q}_4 - k_n\left(q_2 - q_4\right)^3 - c_n\left(\dot{q}_2 - \dot{q}_4\right) = 0 \tag{13}$$

$$R\dot{Q} - \frac{d\left(q_1\right)}{C_p}\left(q_1 + \frac{B}{4}q_2^2\right) + \frac{Q}{C_p} = 0 \tag{14}$$

For a better analysis, a dimensionless process is carried out, resulting the dimensionless equations system as follows

$$x_1'' + \mu_1 x_1' + x_1 + \alpha_1 x_1 x_2 = \theta\left(1+\Theta\left|x_1\right|\right)\delta_1 V \tag{15}$$

$$x_2'' + \mu_2 x_2' + \omega_2^2 x_2 + \alpha_2 x_1^2 + G_0 + K_{2n}\left(x_2 - x_4\right)^3 + \mu_{2n}\left(x_2' - x_4'\right) = E_0 \cos\Omega\tau + \theta\left(1+\Theta\left|x_1\right|\right)\delta_2 V x_2 \tag{16}$$

$$x_4'' - K_{4n}\left(x_2 - x_4\right)^3 - \mu_4\left(x_2' - x_4'\right) = 0 \tag{17}$$

$$V' - \theta\left(1+\Theta\left|x_1\right|\right)\left(\delta_3 x_1 + \delta_4 x_2^2\right) + \delta_3 V = 0 \tag{18}$$

where the dimensionless parameters are

$$\begin{gathered}
x_1 = \frac{q_1}{h}, \quad x_2 = \frac{q_2}{L}, \quad x_4 = \frac{q_4}{x_{04}}, \quad V = \frac{Q}{q_0}, \quad \tau = \omega_1 t, \quad \omega_1 = \sqrt{\frac{2\left(k_c - mgA\right)}{2m+M}}, \quad \hat{d}\left(x_1\right) = \frac{h}{q_0} d\left(q_1\right) \\
\mu_1 = \frac{c_1}{\left(2m+M\right)\omega_1}, \quad \mu_2 = \frac{c_2}{M\omega_1}, \quad \mu_{2n} = \frac{c_n}{M\omega_1}, \quad \mu_{4n} = \frac{c_n}{m_4\omega_1}, \quad G_0 = \frac{g}{\omega_1^2 L}, \quad E_0 = \frac{F_0}{M\omega_1^2 L} \\
\omega_2 = \frac{1}{\omega_1}\sqrt{\frac{k_b}{M}}, \quad K_{2n} = \frac{k_n L^2}{M\omega_1^2}, \quad K_{4n} = \frac{k_n L^2}{m_4\omega_1^2}, \quad \alpha_1 = \frac{Ak_b L}{\left(2m+M\right)\omega_1^2}, \quad \alpha_2 = \frac{Ak_b h^2}{2M\omega_1^2 L} \\
\delta_1 = \frac{q_0^2}{\omega_1^2 h^2\left(2m+M\right)C_p}, \quad \delta_2 = \frac{Bq_0^2}{2M\omega_1^2 h C_p}, \quad \delta_3 = \frac{1}{RC_p\omega_1}, \quad \delta_4 = \frac{BL^2}{4RC_p\omega_1 h}, \quad \Omega = \frac{\omega_n}{\omega_1}
\end{gathered} \tag{19}$$

To calculate the harvested power of the considered system, Eqs. (20) and (21) are given as dimensional and dimensionless harvested power, respectively.

$$P = R\dot{Q}^2 \tag{20}$$

$$P = R_0 V'^2 \tag{20}$$

where $R_0 = R(\omega_1 q_0)^2$.

The average power of the system can be calculated by Eq. (22), as in (Triplett and Quinn, 2009; Iliuk, et al. 2013).

$$P_{avg} = \frac{1}{T}\int_0^T P\left(\tau\right)d\tau \tag{21}$$

Now, section 3 will show some numerical simulations with and without the NES device, considering the nonlinear piezoelectric contribution fixed in $\Theta = 1$.

3. NUMERICAL SIMULATIONS AND DISCUSSIONS

The numerical simulations realized in this section used the software MATLAB©. The parameters considered in this work are in Tab. 1

Table 1 - Dimensional System Parameters

Parameters	Values	Means			
g $[m/s^2]$	9.81	Gravity aceleration	k_n $[N/m^3]$	74	Nonlinear NES Stiffness
M $[kg]$	2.00	Beam Mass	L $[m]$	0.52	Beam Length
m $[kg]$	0.50	Column Mass	h $[m]$	0.36	Column Length
m_4 $[kg]$	0.009	NES Mass	$F_0[N]$	40	External Excitation Amplitude
$c_1[Ns/m]$	1.55	Column Damping	R $[k\Omega]$	100	Piezoelectric Resistance
$c_2[Ns/m]$	3.14	Beam Damping	$C_p[\mu F]$	1	Piezoelectric Capacitance
$c_n[Ns/m]$	14.7	NES Damping	ω_n $[rad/s]$	148	External Excitation Frequency
EI $[Nm^2]$	128	Linear Stiffness	θ	0.3	Linear Piezoelectric Coefficient
			Θ	1	Nonlinear Piezoelectric Coefficient

The parameters of the NES (m_4, c_n and k_n), will be varied in order to get the optimal combination of the three values related to the average power. But, initially we will begin with the values of Tab. 1, which are in 5:1 ratio according to (Tusset, et al., 2012; Iliuk, et al., 2013).

The next subsection we will analyze the system without the NES device in order to comparisons.

3.1 Analysis of the sole portal frame system

In this section, we will analyze the energy harvesting from the portal frame structure without the NES device, using saturation phenomenon as a means of vibration energy transferring and the piezoelectric material as a means of vibration energy transduction from the column's vibration. Therefore, using the parameters of Tab. 1 numerical simulations are performed.

Figure 2a shows the time history of the displacements. We can see energy transferring between the two vibration movements at 0 < Time < 400. After that, the system goes to steady state and the highest amplitude is of the horizontal mode, i.e., saturation phenomenon actuated.

Figures 3a and 3b show phase planes (in black) and Poincaré maps (red dots) of the horizontal and vertical movements, respectively. We can see that the system is periodic-2 in horizontal mode and periodic-1 in vertical mode. The periodic behaviour provide us a way to harvest energy.

Figures 2b shows the time history of the harvested power. We can see at steady state that the average harvested power is 35.32, approximately.

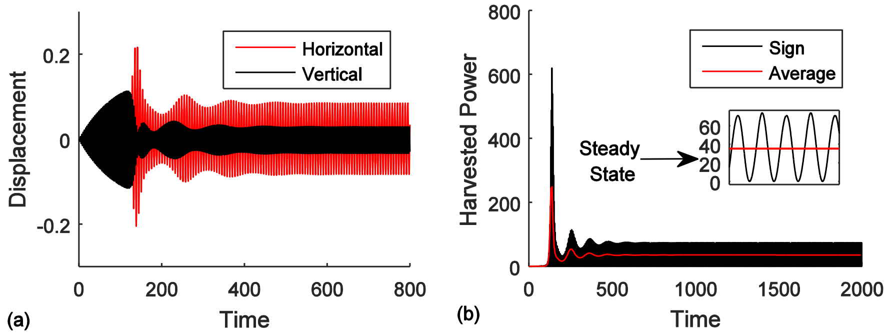

Figure 2 - Time histories, without NES, of (a) horizontal (red) and vertical (black) displacements, (b) harvested power (black) and average harvested power (red).

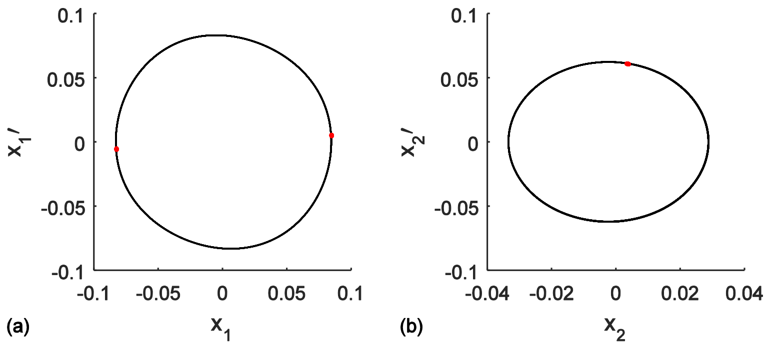

Figure 3 - Phase planes (black) and Poincaré maps (red dots) of (a) horizontal displacement, (b) vertical displacement.

In next subsection, we will show the analysis of the system coupling the NES device at mid span of the beam of the portal frame.

3.2 Analysis with the NES coupling

In this subsection, we will analyze the whole system including the NES device influence. First, we will use the parameters of Tab. 1 including the NES parameters, to the next numerical simulations.

Figure 4a shows the time history of the displacements of the system. The behaviour of the system with NES keeps the same in comparison to without NES.

Figures 5a, 5b and 5c show the phase plane (black) and Poincaré maps (red dots) of the horizontal, vertical and NES displacement, respectively. We can see that, Fig. 5a, the horizontal mode keeps period-2. Figure 5b shows the vertical movement continues to be period-1. The NES behaviour is period-1, as in Fig. 5c.

Figure 4b shows the time history of the harvested power. We see a little gain of power than in the case without the NES. Average power goes from 35.32 to 35.59.

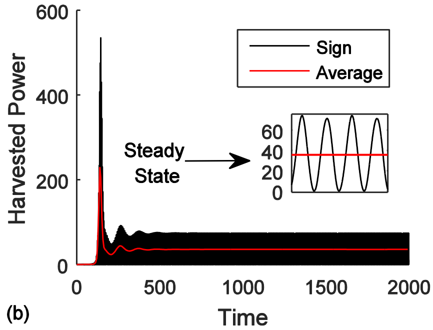

Figure 4 - Time histories, using NES, of (a) horizontal (red) and vertical (black) displacements, (b) harvested power (black) and average harvested power (red).

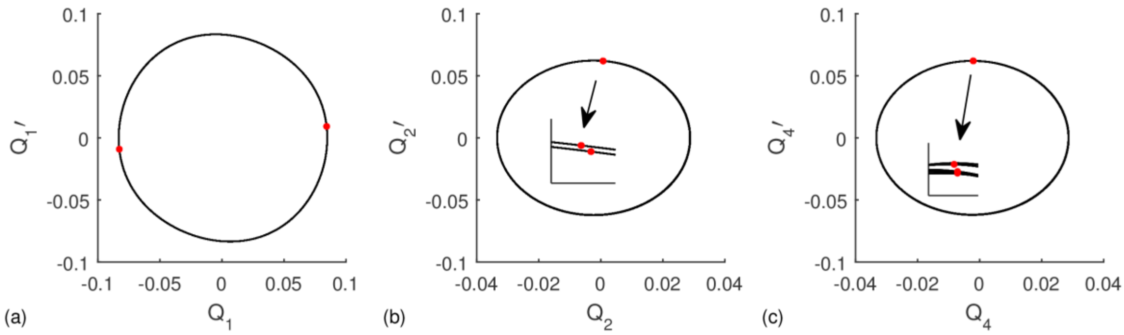

Figure 5 - Phase planes (black) and Poincaré maps (red dots) of (a) horizontal displacement, (b) vertical displacement. (c) NES displacement.

To this case, the NES device almost does not make any difference to the harvested power. Therefore, some analysis of the NES parameters need to be evaluated. The first parameter to be analyzed is the nonlinear stiffness of the NES device k_n.

Figures 6a and 6b present the analysis of the average harvested power related to the nonlinear stiffness and a bifurcation diagram, respectively, varying the nonlinear stiffness of the interval $0 < k_n < 1000\text{N/m}^3$, and keeping the others NES parameters as Tab. 1.

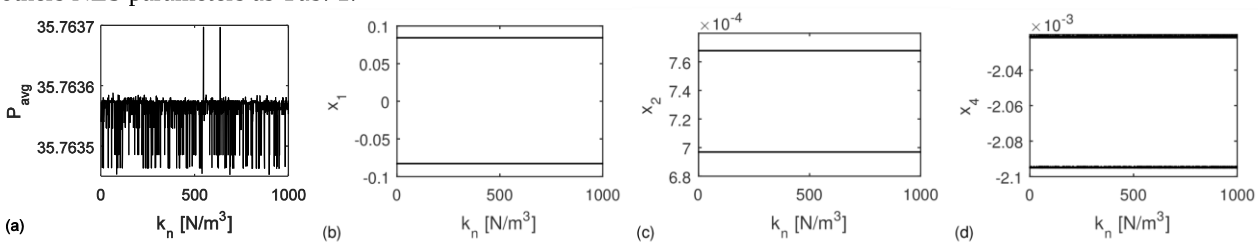

Figure 6 – (a) Analysis of the average harvested power related to the nonlinear stiffness; Bifurcation diagram of (b) Sway mode, (c) Symmetric mode, (d) NES movement

According to the bifurcation diagrams, in Figs. 6b, 6c and 6d, we see that the system keeps period-2 in all the coordinates of the mechanical system. It is very important because of the maintaining of the energy harvesting.

Hence, in Fig. 6a, we see that higher the nonlinear stiffness, the same will be the average harvested power.

Because of that, we will vary the NES mass (m_4) and damping (c_n), keeping $k_n = 74\text{N/m}^3$, in order to find a good relation between both parameters. The results are shown in Figs. 7.

Figure 7a shows a surface, which relate the NES mass with NES damping related to the average harvested power. We can see that, higher the damping higher will be the average power, but it will depend on the NES mass. The most average harvested power is when m_4 = 0.018kg.

With the analysis of the three parameters, some bifurcation diagrams were performed in order to evaluate the behaviour of the system. Figures 7b, 7c and 7d show the behaviour of the sway mode, symmetric mode and NES movement, respectively, varying the NES damping from 0 to 1000 Ns/m. We see the behaviour of the three coordinates are period-2.

Now, setting c_n = 100Ns/m, Figs. 7e, 7f and 7g shows the behaviour of the sway mode, symmetric mode and NES movement, respectively, varying the NESS mass from 0 to 0.200kg. We see some changes of the behaviour with the increase of the NES mass. The system keeps period-2 from 0 to almost 0.094kg (red dotted line), after that, the symmetric mode and NES movement go to period-1, while the sway mode keeps period -2. We see too loss of power as in Fig. 7a.

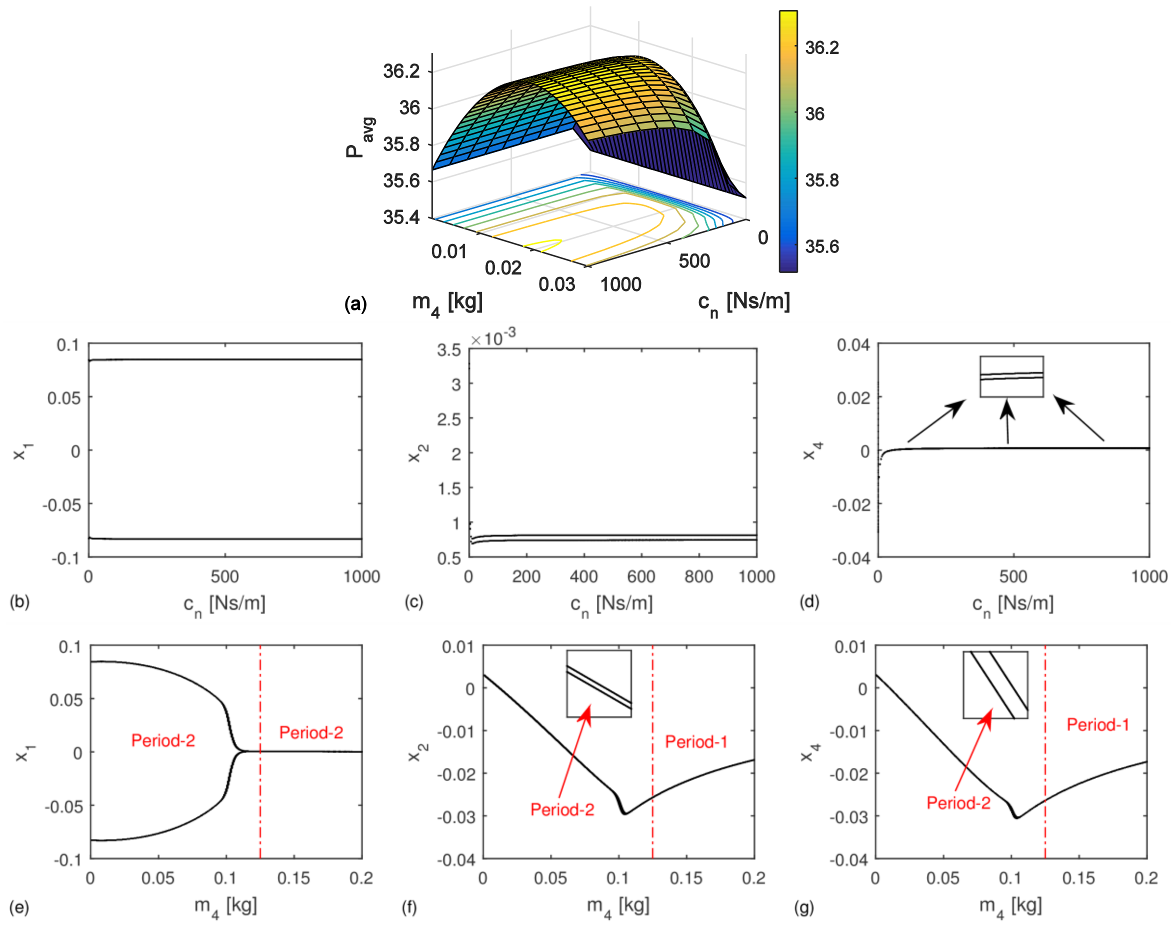

Figure 7 - (a) Surface of c_n and k_n related to the average power. (b), (c) and (d) Bifurcation diagrams of the c_n related to the sway mode, symmetric mode and NES movement, respectively; (e), (f) e (g) Bifurcation diagrams of k_n related to sway mode, symmetric mode and NES movement, respectively.

With the new NES parameters (k_n = 74N/m³, c_n = 100Ns/m, m_4 = 0.018kg), the average power increases from 35.59 to 35.96.

4. CONCLUSIONS

In this work, we showed the analysis of energy harvesting from a portal frame base-excited coupled to a NES device. The device showed that is possible to have some improvement of the energy harvesting, it will depend on its damping and mass parameters. The most important of the NES device in this case is, with NES we guarantee the system will be periodic, breaking unstable possible movements of the system. Table 2 shows a brief of the conclusion about this work.

Table 2 - Resume of the work

NES	k_n $[N/m^3]$	c_n $[Ns/m]$	m_4 $[kg]$	Behaviour			Average Power	Status of energy harvesting
				Sway mode	Symmetric mode	NES movement		
-	0	0	0	Period-2	Period-1	-	35.32	(To be compared)
+	74	14.8	0.009	Period-2	Period-2	Period-2	35.59	Gain
+	74	0	0.018	Period-2	Period-2	Period-2	35.32	-
+	74	100	0.018	Period-2	Period-2	Period-2	35.96	Gain
+	74	100	0.050	Period-2	Period-2	Period-2	32.73	Loss
+	74	100	0.150	Period-2	Period-1	Period-1	$8.46x10^{-4}$	Loss
+	74	1000	0.018	Period-2	Period-2	Period-2	36.14	Max.

5. ACKNOWLEDGEMENTS

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