MECHANICAL DESIGN OF AN EXTERNAL PRESSURE VESSEL OF AN AUV

Artur Siqueira Nobrega de Freitas
Ettore Apolonio de Barros
Unmanned Vehicle Laboratory, Polytechnic School, University of Sao Paulo, Sao Paulo, Sao Paulo, Brazil 05508-010
e-mails: arsnfreitas@gmail.com; eabarros@usp.br

Abstract. *The Autonomous Underwater Vehicles (AUV), as well the Remotely Operated Vehicles (ROV), are unmanned technologies used in oceanographic investigations, offshore oil extraction, military applications, among others. Differently from AUVs, ROVs uses a physical connection with the surface for energy supply e data traffic. The AUVs use batteries and embedded data acquisition systems. This work presents a methodology for external pressure vessel design, responsible for contain and keep the internal components of the vehicle isolated from contact with water, creating a pressure differential between the inner and external regions. The vessel is subject to an external pressure relative to the depth of the operation. This pressure leads to structural instability of the vessel, phenomenon known as buckling. When submitted to buckling the structure may lose its capacity to withstand the external load. In this context, the analysis of elastic instability and alternative constructive solutions are discussed.*

Keywords: *pressure vessel, AUV, buckling, instability*

1. INTRODUCTION

In the past few decades, the AUV's (Autonomous Underwater Vehicles) and ROV's (Remotely Operated Vehicle) technologies have progressed, supported by studies in the areas of robotics, embedded systems, naval engineering, etc. These vehicles are used in oceanographic investigations, offshore oil extraction, military applications, among others Griffiths (2002). Differently from AUVs, ROVs uses a physical connection with the surface for energy supply e data traffic.AUVs use batteries and embedded data acquisition systems that are included in pressure vessels.

Many AUVs are designed as a single vessel where the vehicle hull is the pressure vessel. In this configuration the maintenance tasks, battery changes, and other activities may expose the vessel to increased risk. Another configuration is the free-flooded AUV, with modular subsystems that are contained within their own pressure-compensated or small pressure vessels. These modules are covered by a secondary hull is filled with water when the vehicle is submerged. This configuration allows an easy maintenance and data storage modules, hydrodynamics profiles, and swap payloads and modules faster. Subsystems are also protected from impacts, fouling, and other damage by the secondary hull.

The vessel is submitted to an external hydrostatic pressure and must resist without an expressive deformation, which would imply the loss of functionality, damage to the internal equipment or loss of the vehicle. This kind of compression loading may lead to structural instability of the vessel. This phenomenon is known as buckling and is dependent to the type of the material, boundary conditions and the geometry.

This work proposes a road map to pressure vessel design which could be easily applied to the Auv development. The methodology deals, firstly, on material selection. Some comparison criteria are presented and, taking into account the operational conditions, the most suitable material is suggested. Structural design formulas for buckling prevention of conventional shapes (cylindrical, conical and spherical) are presented and analyzed. Assembling those shapes into a single body is investigated as well. A study is made for concentrating stress in the case of openings in the pressure vessel. Results derived from analytical expressions given by classical papers and books are compared to those ones from finite element analysis.

2. PRESSURE VESSEL DESIGN

The materials used in underwater vehicles must have properties that resist the efforts and submerged operating conditions. An important factor in the selection of materials is the ratio between strength and density, because as it grows more payloads the vehicle can carry. This ratio is known as specific strength. The main materials used in the market are high strength steels, aluminum alloys, titanium alloys, composite materials and in certain cases ceramic and polymeric materials. The following table shows the properties of some of the most used materials.

Table 1: MATERIALS PROPERTIES.

materials	density(g/cm^3)	Yield Stress(Mpa)	Ultimate Stress(Mpa)	Young's Modulus(Gpa)	Poisson's coeficient
aluminium 5456 -H111	2,66	230	320	70,3	0,33
Titanium Ti-6Al-4V	4,43	880	950	113,8	0,342
AISI 316	8	250	565	193	0,3
GFRP pultruded section (tensile)	1,8	-	290	18	-
CFRP unidirecional	1,55	-	840	190	-

Besides specific strength, a common comparison criteria is based on the formula presented by Almendinger (1990), which considers the ratio between strength and density of the material. Comparisons are made to the properties (density ρ, Young's module E) of a reference material, giving:

$$\frac{P}{P_{ref}} = \frac{\rho}{\rho_{ref}}(\frac{E_{ref}}{E})^{1/3} \tag{1}$$

Where P is the pressure and de P_{ref} is the reference pressure.

After material selection, one can begin the structural design. A simple analytical approach is based on the membrane theory (Young and Budynas (2002)), which assumes that the vessel wall is relatively thin, i.e., the wall thickness is less than one tenth of the vessel radius (h < R/10). MOreover, it is assumed that abrupt changes in the thickness, slope or deflection of the walls do not occur, loads are subject to small variations and are axisymmetric. One can then consider the axial stress (σ_1) and circumferential ("hoop", σ_2) uniform along the walls of the structure. The radial stresses (σ_3) and bending stresses assume small values compared to the others may be discarded.

Theory of membrane considers only the normal forces to the hull. When reinforcing rings are used this theory is no longer valid, since the bending and shearing efforts are added to the structure (Allmendinger (1990)).
Timoshenko and Voinovskii-Kriger (1963) have claimed that the membrane theory fails to represent the stresses in the regions of the vessel ends, since membrane stresses do not entirely satisfy the conditions in these regions. A model of the vessel is presented as a cylinder subject to a lateral uniform pressure with the shell theory. It can be said that all resultants in the cylindrical shell are small, with the exception of the effort in the circumferential direction (N_y, figure 2). Therefore the products of these resultants with the derivatives of the displacements are negligible, as well as the bending and torsion moments.

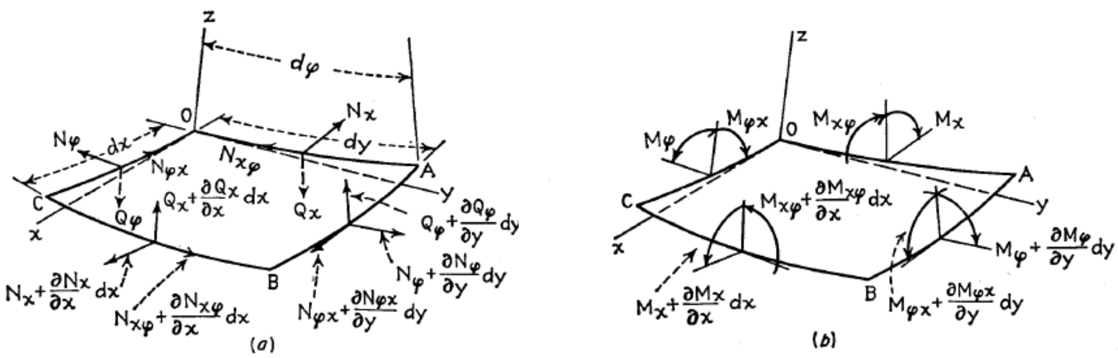


FIGURE 1: FORCES AND MOMENTS IN A CYLINDER SHELL ELEMENT (adapted from,Timoshenko and Voinovskii-Kriger (1963))

Considering θ the circumferential direction, x the longitudinal direction and y the radial direction of the cylinder, a particular solution for the equilibrium equations is the case that the cylindrical shell remains circular and uniformly compressed in the circumferential direction. In this case one can say: $v = 0, w = R^2q/Eh, N_y = -qR$ e $N_x = N_{xy} = M_x = M_y = M_{xy} = 0$. Then the momentum equilibrium equations are stated (Timoshenko and Voinovskii-Kriger (1963)):

$$\begin{aligned} Q_x &= \frac{1}{R}\frac{\partial M_{xy}}{\partial \theta} + \frac{\partial M_x}{\partial x} \\ Q_y &= \frac{1}{R}\frac{\partial M_y}{\partial \theta} - \frac{\partial M_{xy}}{\partial x} \end{aligned} \tag{2}$$

A particular solution for the equilibrium equations is the case that the cylindrical shell remains circular and uniformly compressed in the circumferential direction. For buckling situations, small displacements are considered and some corrections are made about writing the efforts in function of the deformation and slope alterations and writing these in function of the displacements. Also, the stresses are represented in function of the displacements and integrate them to the thickness to get the forces components. So substituting all the corrections, assumptions and knowing that u,v,w are the displacements in the x,y,z directions respectively, the three equilibrium differential equations that represent buckling of a cylinder submitted to a lateral uniform pressure are presented (Timoshenko and Voinovskii-Kriger (1963)).

Considering the edges simply supported, a length l and the x coordinate measured from the mid section of the cylinder, the boundary conditions are $w = 0$ and $\partial^2 w/\partial x^2 = 0$ for $x = \pm l/2$. Then a solution for the equilibrium equation is presented and the equation for the buckling critical pressure is obtained:

$$q_{cr} = \frac{Eh}{R(1-\nu^2)}[\frac{1-\nu^2}{(n^2-1)(1+\frac{n^2l^2}{\pi^2R^2})} + \frac{h^2}{12R^2}(n^2-1) + \frac{2n^2-1-\nu}{1+\frac{n^2l^2}{\pi^2R^2}})] \tag{3}$$

Where E is the Young‘s module, ν is the Poisson‘s coeficient, n is the lobar mode, l is the vessel lenght and R is the vessel Radius

Based in this formula, one can see that the lowest buckling critical pressure is obtained for the lowest value of n. Another interesting fact is that for any longer the cylinder gets, the critical loading does not change. There are many formulations for prevent buckling. Windenburg and Trilling (1934) in their classical work present several equations and conclude that the Von Mises equation gives the best results for buckling.

2.1 Cylindrical Vessel Formulation

In most of the cases, AUV vessels are submited to hydrostatic loads. For those cases, the analytical approach can be based on the shell theory , just presented, considering lateral and axial loads. The most commonly adopted result for estimating the buckling critical pressure, according to this method, is the Von Mises formulation (Windenburg and Trilling, 1934):

$$q_{cr} = \frac{Eh}{R}\frac{1}{n^2-1+\frac{\pi^2R^2}{2l^2}}[\frac{1}{(1+\frac{n^2l^2}{\pi^2R^2})^2} + \frac{h^2}{12R^2(1-\nu^2)}(n^2+\frac{\pi^2R^2}{l^2})^2] \tag{4}$$

The equilibrium equation of a membrane can be used to prevent the yield fail in thin walled cylinder.GROEHS (2002) derives expressions for the the membrane stresses in a revolution body to calculate the longitudinal (σ_l) and circunferential (σ_θ) stresses:

$$\begin{aligned} \sigma_l &= \frac{pR}{2h} \\ \sigma_\theta &= \frac{pR}{h} \end{aligned} \tag{5}$$

2.2 Spherical Vessels Formulation

The membrane equilibrium equations can be applied to a thin-walled spherical vessel as well. It can be shown that circunferential and longitudinal stresses have the same value:

$$\sigma_\theta = \sigma_l = \frac{qR}{2h} \tag{6}$$

Linear instability in spherical vessels has been studied by Von Karman and Tsien Von Karman (2012), The theoretical expression derived for perfect thin-walled shells submitted to a uniform external pressure is given by:

$$p_{cr} = \frac{2Eh^2}{R^2\sqrt{[3(1-\nu^2)]}} \tag{7}$$

Further experiments on these types of vessels have shown that this expression overestimates the critical buckling pressure. Young and Budynas (2002) propose a formula, adopted in this work, that indicates the probable minimum critical pressure:

$$p_{cr} = 0,365E(\frac{h}{R})^2 \tag{8}$$

2.3 Conical Vessels Formulation

Once again the membrane equilibrium equations can be used to thin walled conical vessels. Expressions for estimating circunferential and longitudinal stresses are similar to those ones derived in the cyllinder case, except that the vessel radius is a function of the taper angle, i.e. $R = R\cos(\alpha)$:

$$\sigma_l = \frac{pR}{2h\cos\alpha}$$
$$\sigma_\theta = \frac{pR}{h\cos\alpha} \tag{9}$$

According to Ross (Ross (2011)), one should analyze the taper angle n order to define which type of failure is predominant. . If the angle is small or the vessel is long, it tends to buckling. If the angle is large or the vessel is short, it tends to axysimmetric yield. However, calculating these stresses analytically can be very difficult. A numerical calculation based on finite element methods can be an interesting alternative in this case..

On the other hand,Young and Budynas (2002) states that a good estimate of the critical buckling pressure in a thin-walled cone can be made with the equations of Von Mises. In this case, the length of the cylinder in the formulas should be replaced by the equivalent length of the cone, and the radius of the cylinder should be replaced by an average radius of curvature of the wall of the cone given by $a = (R_a + R_b)/(2\cos\alpha)$. It is also recommended to use eighty percent of critical pressure for conical vessels and the cylinders.

2.4 Combining the vessels cylindrical/spherical/conical

The Auv hull geometry, specially in the case of axysimmetric bodies, is usually resultant of a combination of elements such as ogives, cylinders, semi-spheres, cones, etc. Considering the the hydrodynamic hull the same as the pressure hull, those combinations of shapes affect the structural design. Unions between different shapes represent geometric discontinuities, and act as stress concentrators. The ideal design should produce the same deformations along the transition between vessels in order to avoid stress concentration. For example, the conical and spherical vessels tend to present lower stresses than a cylindrical one, indicating that the former vessels should have lower thickness than the cyllinder. However, its recommended to use the thickness of the worst case for both vessels that are joined in order to reduce the bending moments in the union (Martinez (1987)).

The solution presented by Young and Budynas (2002) assumes small radial deflections (of lower order than the thickness) and long thin-walled vessels. In their book several tables are presented inclunding the main cases of shell union. The tables list the different types of vessels, loads and the characteristics necessary for problem identification. As example, formulation related to the cylinder and cone combination are presented figure 2 for a internal pressure case. Attention should be paid to the signals used as reference in the tables presented, they must be changed for the case of external pressure vessels as in the footnotes in figure 2.

The cylindrical shape has a critical role in such combination. If the length is small relative to the diameter and the thickness, the vessel behaves like a thin ring, introducing radial deflections and rotations relative to original meridian. However, it does not change its shape after deformation. Therefore, for a longer cylinder (lower than the critical length), the cross sections suffer deformations in shape due to the influence that one end causes to the other. Finally, for the cylinder longer than the critical length, the ends do not influence one another, therefore the stresses at each end can be calculated as two independent problems.

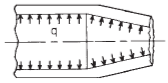
2.5 Openings in the hull

Openings in the hull are important because they allow to use communication cables, pressure sensors, drive shafts for rudders, among others. They are considered discontinuities in the hull and interfere with the stresses distribuition, causing high gradients and the peak may exceed the permissible limit. The publications on this subject are very general and therefore there is some difficulty in finding specific information. Therefore, a proposal is analyzed: an adaptation of the method proposed by Faupel and Harris (1957) for long cylinders.

This criterion uses the Lame formulations, which assume an infinite cylinder with constant thickness and submited to an internal and external pressure. Thus, the generated displacements will be radial, symmetrical and uniform in relation to the longitudinal axis. Then the plate with a hole theory is applied and a coefficient of stress concentration is estimated.

For a submerged vessel, under external hydrostatic pressure, the inner surface is the most requested (the inner radius is Ri, Faupel and Harris (1957)). Assuming the hole diameter is much smaller than the dimensions of the vessel, it can be considered as a plate with a hole under traction. Since the hole as an ellipse where the largest dimension is a and the lower b, a coefficient can be estimated from the circumferential tensions:

Loading and case no.	Load terms	Selected values
2a. Internal* pressure q	see below	see below



Load terms

$$LT_{A1} = \frac{b_1 R_1}{t_1^2}$$

$$LT_{A2} = \frac{-b_2 E_1 R_2}{E_2 t_1 t_2 \cos\alpha_2}$$

$$LT_{AC} = \frac{t_2 \sin\alpha_2}{2t_1} C_{AA2} + \frac{t_2^2 - t_1^2}{8t_1^2} C_{AB2} + \frac{E_1 \nu_2 R_1 (t_1 - t_2\cos\alpha_2)}{2E_2 t_1 t_2 \cos\alpha_2} \dagger$$

$$LT_{B1} = 0$$

$$LT_{B2} = \frac{-2E_1 b_2 \tan\alpha_2}{E_2 t_2 \cos\alpha_2}$$

$$LT_{BC} = \frac{t_2 \sin\alpha_2}{2t_1} C_{AB2} + \frac{t_2^2 - t_1^2}{8t_1^2} C_{BB2} + \frac{E_1 \sin\alpha_2}{E_2 t_2 \cos^2\alpha_2}(t_1 - t_2\cos\alpha_2) \dagger$$

At the junction of the cylinder and cone

$$V_1 = qt_1 K_{V1}, \qquad M_1 = qt_1^2 K_{M1}, \qquad N_1 = 0$$

$$\Delta R_A = \frac{qt_1}{E_1}(LT_{A1} - K_{V1}C_{AA1} - K_{M1}C_{AB1})$$

$$\psi_A = \frac{q}{E_1}(K_{V1}C_{AB1} + K_{M1}C_{BB1})$$

Selected values

For internal pressure, $R_1 = R_2$, $E_1 = E_2$, $\nu_1 = \nu_2 = 0.3$, and for $R/t > 5$. (*Note:* No correction terms are used)

$$\Delta R_A = \frac{qR_1^2}{E_1 t_1} K_{\Delta RA}, \qquad \psi_1 = \frac{qR_1}{E_1 t_1} K_{\psi A}, \qquad \sigma_2 = \frac{qR_1}{t_1} K_{\sigma 2}$$

		R_1/t_1 = 20			R_1/t_1 = 40		
	$\frac{t_2}{t_1}$	α_2 = −30	30	45	α_2 = −30	30	45
K_{V1}	0.8	−0.3602	−0.3270	−0.5983	−0.5027	−0.4690	−0.8578
	1.0	−0.1377	−0.1419	−0.4023	−0.1954	−0.1994	−0.5633
	1.2	0.0561	0.0212	−0.2377	0.0725	0.0374	−0.3165
	1.5	0.3205	0.2531	−0.0083	0.4399	0.3715	0.0225
	2.0	0.7208	0.6267	0.3660	1.0004	0.9040	0.5628
K_{M1}	0.8	0.4218	−0.1915	−0.4167	0.6693	−0.2053	−0.4749
	1.0	0.3306	−0.3332	−0.6782	0.4730	−0.4756	−0.9638
	1.2	0.3562	−0.3330	−0.7724	0.5175	−0.4696	−1.1333
	1.5	0.5482	−0.1484	−0.6841	0.8999	−0.1011	−0.9487
	2.0	1.0724	0.4091	−0.1954	1.9581	0.9988	0.0084
$K_{\Delta RA}$	0.8	1.2517	1.1314	1.2501	1.2471	1.1612	1.2969
	1.0	1.1088	1.0015	1.0942	1.1060	1.0293	1.1368
	1.2	1.0016	0.9078	0.9840	1.0008	0.9335	1.0225
	1.5	0.8813	0.8050	0.8667	0.8830	0.8281	0.9000
	2.0	0.7378	0.6823	0.7323	0.7426	0.7026	0.7594
$K_{\psi A}$	0.8	1.9915	0.7170	1.1857	2.5602	1.2739	2.1967
	1.0	1.0831	−0.1641	0.0411	1.2812	0.0201	0.5668
	1.2	0.4913	−0.7028	−0.6817	0.4554	−0.7544	−0.4765
	1.5	−0.0178	−1.1184	−1.2720	−0.2449	−1.3635	−1.3488
	2.0	−0.3449	−1.2939	−1.5808	−0.6758	−1.6457	−1.8483
$K_{\sigma 2}$	0.8	1.2517	1.1314	1.2501	1.2471	1.1612	1.2969
	1.0	1.1088	1.0015	1.0942	1.1060	1.0293	1.1368
	1.2	1.0016	0.9078	0.9840	1.0008	0.9335	1.0225
	1.5	0.8813	0.8050	0.8667	0.8830	0.8281	0.9000
	2.0	0.7378	0.6823	0.7323	0.7426	0.7026	0.7594

*For external pressure, substitute $-q$ for q, a_1 for b_1, and a_2 for b_2 in the load terms.
†If α_2 approaches 0, use the correction term from case 1a.

FIGURE 2: A CASE OF FORMULAS FOR DISCONTINUITY STRESSES AND DEFORMATIONS ANT THE JUNCTIONS (adapted from,Young and Budynas (2002))

$$k = \frac{\sigma_\theta(1 + 2.a/b)}{\sigma_\theta} \tag{10}$$

For small circular holes distant from each other (relative to diameter) a and b coefficients of the above equation are equal, and $k = 2.5$. If the hole diameter becomes larger or there are holes diametrically opposed, the coefficient k loses its precision. Since diametrically opposed holes occur in most of the cases, the formula for the cirfunferential stress is presented

$$\sigma_\theta = \frac{p_o \pi R_e^2}{\pi(R_e^2 - R_i^2) - 2n_h \pi L} \tag{11}$$

Where $n_h = 2$ for holes diametrically opposed. Afterwards, the method uses a compilation of data by Peterson that gives two coeficients α and γ. These coeficients are taken in graphics using the side hole ratio (hole radius divided by the vessel radius, Faupel and Harris (1957)). A new stress concentration factor, K, is finally calculated:

$$k = \frac{\sigma_\theta \alpha + \sigma_l \gamma}{\sigma_\theta} \tag{12}$$

3. Comparison between Analytical and Numerical Formulation in a Case Study

The method proposed in this work is applied to a case study, and the generated results are compared to those ones produced by a numerical simulation based on the finite element method.

The body under study is the cylindrical vessel of an underwater vehicle developed in Unmanned Vehicles Laboratory of the Polytechnic School of USP, the AUV Pirajuba, as a reference. In order to fulfill the vehicle design constraints, this vessel has fixed length, L = 1200 mm and internal diameter $D_i = 188mm$ are fixed. Therefore, the methodology should define thickness and material for this problem.

The cylindrical vessel alone and its union with a conical vessel were treated. The conical vessel geometrical parameters are: maximum internal diameter $D_a = 188mm$, minimum internal diameter $D_b = 50mm$ and a taper angle of $\alpha = 13$.

At the other extreme, the cylinder is to be joined to a spherical vessel. Both vessels should have the same internal diameter, Di = 188mm.

As suggested by Allmendinger (1990) for submersibles design, three materials are considered: aluminum -H111 5456, Titanium Ti-6Al-4V and stainless steel AISI 316, represented by the data in table 1

An avaliation of the safety criteria is required, so that an equivalent stress needs to be calculated. One of the criteria commonly used is the equivalent stress of Von Mises. Assuming that the radial and the face shear stresses are neglible compared to the stress at the inner surface, the Von Mises stress can be simplified to:

$$\sigma_v = \sqrt{\sigma_\theta^2 - \sigma_\theta \sigma_l + \sigma_l^2} \tag{13}$$

At first, the cylinder design was considered. Table 2 shows the thickness calculated by the Von Mises formula (4) referent a depth of 100 meters, the mass of each vessel and equivalent stresses for each material.

Table 2: ANALYTICAL CALCULATION.

	Cilindrical Vessel		
materials	Von Mises thickness(mm)	Von Mises Equivalent Stress(Mpa)	Mass (kg)
aluminium 5456 -H111	4,6	19,587	8,89
Titanium Ti-6Al-4V	3,9	23,103	12,36
AISI 316	3,2	28,158	18,49
	Sphrerical Vessel		
alumínio 5456 -H111	1,4	35,72	0,407
Titânio Ti-6Al-4V	0,86	58,14	0,424
AISI 316	1,2	41,67	1,123
	Conical Vessel		
alumínio 5456 -H111	3,8	42,43	1,33
Titânio Ti-6Al-4V	3,2	50,38	1,865
AISI 316	2,7	59,72	2,821

In comparison, finite element simulations were made using the software ANSYS. Some simulations were made to optimize the mesh, considering the symmetr of an axisymmetric solid. It was used tetraedrical and hexaedrical elements and was concluded that, for all simulations, a element dimension displayed in table 3 could produce satisfatory results. As boundary conditions simply supported edges were assumed in all cases simulated. The following table resumes the final parameters used in simulations. The differences found between the analytical and simulated results, are within of 10% and are considered satisfatory.

Table 3: SIMULATION RESULTS BASED ON FEM.

Cilindrical Vessel	element size	number of elements	number of nodes	Von Mises Stress
al5456	2,5	116411	6022994	17,54
TI-6Al-4V	2	131889	688266	20,45
inox 316	2	215783	1115924	25,016
Spherical Vessel				
al5456	1	450619	760044	36,7
TI-6Al-4V	2	115136	233316	59,15
inox 316	2	115549	234182	42,67

The finite element method allows calculating the eigenvalues and eigenvectors associated with the form and magnitude of the applied load. The eigenvalue problem can be defined as:

$$\lambda[K][\phi] = [k][\phi] \tag{14}$$

Where λ is the eigenvalue corresponding to the load, K is the geometric stiffness matrix, k is the global elastic stiffness matrix corresponding to the nodes and ϕ is the nodal displacements. The critical buckling pressure is the multiplication of the eigenvalue for the applied pressure. As the simulation gives the critical pressure, for comparison with the analytical results, a uniform unitary external pressure is applied. So the eigenvalue obtained will be the exact critical pressure. The following image shows the linear buckling simulation of the cilindrical vessel.

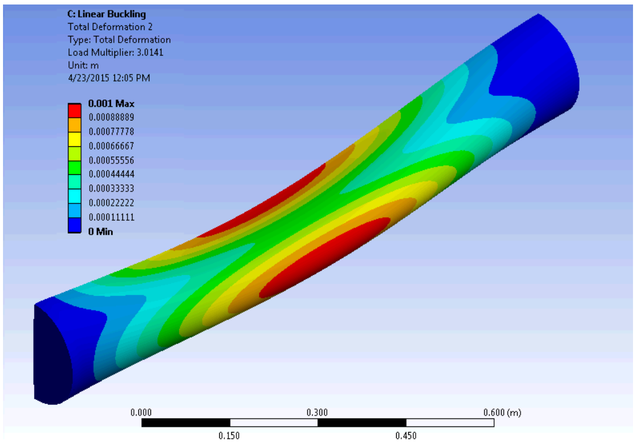


FIGURE 3: BUCKLING SIMULATION OF A CYLINDRICAL VESSELMADE IN AL5456(n=2).

The critical pressure simulated is about three times greater than the expected pressure, showing the consistency of the formulations because it was used a safety factor of three. Therefore, the collapse pressure is three times the critical pressure obtained.

A similar procedure was carried out to demonstrate the calculations about the stress concentrators. Another vessel from the Pirajuba AUV was taken for study. It contains the actuators from the manoeuvre system, and 4 holes in the vessel are spaced by 90 degrees from each other, in order to provide the connection between actuators and rudders in a cruciform configuration. Therefore, the dimensions are length L = 150 mm, inner diameter Di = 188 mm and thickness = 6 mm.

The following table shows the estimated coefficients and the equivalent stresses (MPa) generated by concentrators.

TABLE 4: ANALYTICAL CALCULATION FOR STRESS CONCENTRATORS.

Analytical	
Circunferential Stress (Mpa)	2
Longitudinal Stress (Mpa)	9,035
alpha	2,9
gamma	-0,8
k (faupel)	6,51
Von Mises Stress(Mpa)	53,54

For the "vessel with holes" problem, the finite element model was also applied to generated numerical results that were compared to the analytical ones. The element types and boundary conditions were the same in the previous cases. The following figure shows the simulated stresses. It can be seen that the Faupel method provides a good approximation to the FEM results ($\sigma_{eq} = 51,196 MPa$).

Significant stresses are also found around the region of contact between the end caps and the cyllindrical body, as expected. This happens because when there is deformation, the contact of the bodies makes superficial stresses, similar to contact in gears. However, the related plastic deformation is too small, so that those stresses can be negligible and does not affect the vessel functionality.

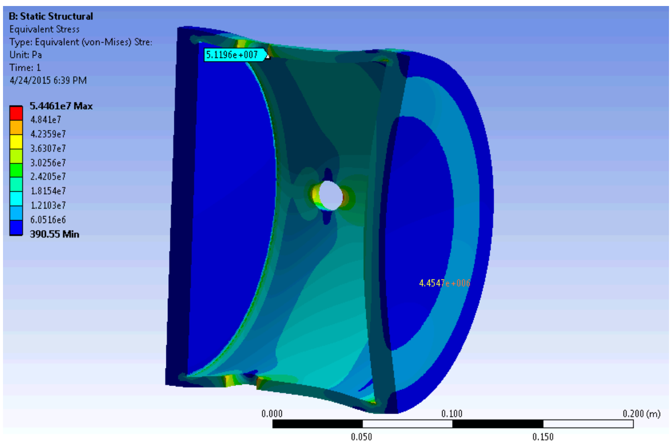


FIGURE 4: SIMULATION FOR STRESS CONCENTRATORS.

The case of cylinder and cone combination was investigated as well. The tabulated values from Budynas and Young were applied to produce the analytical results. Both the cylinder and cone have the dimensions based on the AUV Pirajuba ($Di = 188mm, e = 4,6mm, \alpha = 13$). The analytical solution was produced in two steps: calculation of shear forces and moments in the Union, and addition of stresses so that the equivalent stress is calculated. Table 5 presents the results from such method.

Table 5: ANALYTICAL CALCULATION FOR THE UNION CILYNDRICAL AND CONICAL.

Union calculation	
Circunferential Stress (Mpa)	18,03
Longitudinal Stress (Mpa)	27,63
Von Mises Stress(Mpa)	24,30

The finite element method produced a close result to the value found by the analytical formulation. Figure 5 presents results from the numerical simulation.

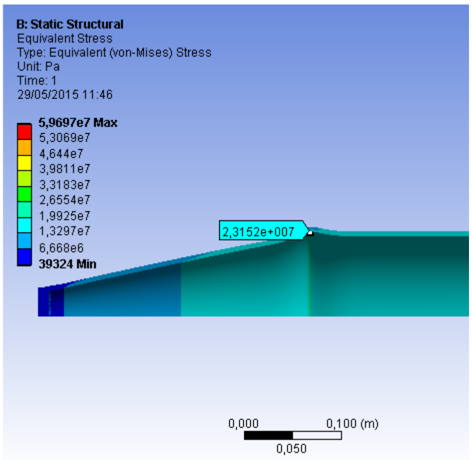


FIGURE 5: SIMULATION FOR THE CYLINDER AND CONE COMBINATION.

4. Conclusions

This work presented a method to design external pressure vessels with the commonly adopted geometry in the AUVs. The method is based on the fusion of a number of analytical results presented in the literature.

The case studies show that the proposed method produce results close to those ones based on the application of the finite element method. Therefore, a numerical tool based on a less intensive computation than that of FEM can be developed and customized to the AUV designer needs.

5. REFERENCES

Allmendinger, E.E., 1990. *Submersible vehicle systems design*, Vol. 96. Society of Naval Architects &.

Faupel, J. and Harris, D., 1957. “Stress concentration in heavy-walled cylindrical pressure vessels-effect of elliptic and circular side holes”. *Industrial & Engineering Chemistry*, Vol. 49, No. 12, pp. 1979–1986.

Griffiths, G., 2002. *Technology and applications of autonomous underwater vehicles*, Vol. 2. CRC Press.

GROEHS, A., 2002. “Resistência dos materiais e vasos de pressão”. *UNISINOS-São Leopoldo*.

Martinez, M., 1987. “Projeto do casco cilindrico resistente de submersiveis”. *Tese de Livre DocÃłncia apresentada a Escola PolitÃĺ’cnica da USP*.

Ross, C.T., 2011. *Pressure vessels: external pressure technology*. Elsevier.

Timoshenko, S.P. and Voinovskii-Kriger, S., 1963. “Plates and shells”. *Fizmatgiz, Moscow*, p. 635.

Von Karman, T., 2012. “The buckling of spherical shells by external pressure”. *Journal of the Aeronautical Sciences (Institute of the Aeronautical Sciences)*, Vol. 6, No. 2.

Windenburg, D.F. and Trilling, C., 1934. “Collapse by instability of thin cylindrical shells under external pressure”. *Trans. Asme*, Vol. 11, pp. 819–825.

Young, W.C. and Budynas, R.G., 2002. *Roark’s formulas for stress and strain*, Vol. 7. McGraw-Hill New York.