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Flow Simulation of a Free Plane Jet

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Abstract. *Simulations using the two-equation k-ε and one-equation Spalart-Allmaras turbulence models were performed for a free, incompressible plane jet with a Reynolds number 10000. Results were compared with experiments from the literature through flow velocity profiles and jet spreading rates. Although the k-ε model provided good agreement with the experiments, the simulations with the Spalart-Allmaras model failed to provide satisfactory results for the spreading rate, and it is believed that the reason for this deficiency is related to the way in which the turbulent length scale is defined in the one-equation turbulence model. We discuss probable alternatives to enhance the ability of the Spalart-Allmaras model to predict free jet flows.*

Keywords: *free plane jet, turbulence, incompressible flow, CFD simulation, turbulence models*

1. INTRODUCTION

Due its great importance in many areas of engineering, the study of turbulent jets has been the subject of both numerical and experimental studies. The first investigations were limited only to the measurement of pressures and speeds but the availability of hot wire anemometers allowed measurements that provided greater insight into the structure of the plane jets. Later, through the use of the Laser Doppler Anemometry (LDA) it was discovered the existence of reverse flows and more information about jet boundaries was provided (Namer, 1988).

In this work it was made a comparative study between the experimental results from Bradbury (1965) for a free plane jet and simulations using the standard k-ε two-equation (Launder and Spalding, 1972) and standard Spalart-Allmaras one-equation (Spalart and Allmaras, 1992) turbulence models. From the experimental velocity profiles it can be concluded that the jet spreading rate S is around 0.110. According to Fig. 1, if the jet has a velocity U_1 at the centerline, the spreading rate S is defined as the dimensionless distance y/x for which the velocity drops to $U_1/2$. The simulations were carried out for a Reynolds number $\mathrm{Re} = U_j H / \nu = 10000$, where U_j and H are the velocity and width of the jet when it emerges in a stagnant surrounding and ν is the kinematic viscosity. Flow was considered steady and incompressible.

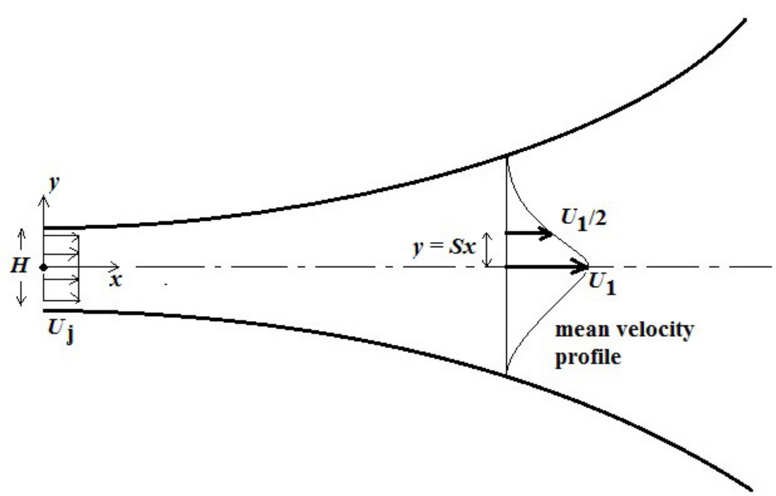

Figure 1. Schematic representation of the free plane jet.

The velocity profiles obtained from the simulations were compared with a dimensionless velocity profile that Bardina at al. (1997) compiled from the experiments of Bradbury (1965):

y/x = [0.000; 0.009; 0.018; 0.026; 0.034; 0.035; 0.044; 0.053; 0.061; 0.062; 0.070; 0.079. 0.088; 0.096; 0.097; 0.106. 0.110; 0.110; 0.114; 0.123; 0.123; 0.132; 0.141; 0.142; 0.150; 0.158; 0.167; 0.169; 0.176; 0.185; 0.188; 0.194; 0.202; 0.211; 0.220]

U/U_1= [1.000; 0.996; 0.983; 0.962; 0.916; 0.933; 0.898; 0.856; 0.784; 0.809; 0.758; 0.703; 0.646; 0.553; 0.588; 0.529; 0.500; 0.468; 0.471; 0.400; 0.414; 0.358; 0.305; 0.284; 0.256; 0.210; 0.168; 0.174; 0.131; 0.099; 0.084; 0.072; 0.050; 0.033; 0.020]

2. DESCRIPTION OF NUMERICAL PROCEDURE

The two-dimensional steady and incompressible turbulent flow can be represented by the continuity and Reynolds Averaged Navier-Stokes (RANS) equations:

$$\frac{\partial U_j}{\partial x_j}=0 \tag{1}$$

$$\frac{\partial (U_j U_i)}{\partial x_j}=-\frac{1}{\rho}\frac{\partial P}{\partial x_i}+\frac{\partial}{\partial x_j}\left[\nu\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)-\overline{u_i' u_j'}\right] \tag{2}$$

The Reynolds stress tensor is treated through the Boussinesq approximation, as can be seen in Wilcox (1993). For an incompressible flow:

$$-\overline{u_i' u_j'}=-\frac{2}{3}k\,\delta_{ij}+\nu_t\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right) \tag{3}$$

Substitution of Eq. (3) in Eq. (2) results:

$$\frac{\partial (U_j U_i)}{\partial x_j}=-\frac{1}{\rho}\frac{\partial P_{eff}}{\partial x_i}+\frac{\partial}{\partial x_j}\left[\nu_{eff}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\right] \tag{4}$$

In the last equation, the effective values of pressure and viscosity are given by $P_{\mathrm{eff}} = P + 2/3\rho\, k$ and $\nu_{\mathrm{eff}} = \nu + \nu_{\mathrm{t}}$. The eddy viscosity ν_{t} is calculated through the transported turbulence variables, k and ε in the two-equation k-ε model or $\tilde{\nu}$ in the one-equation Spalart-Allmaras model.

In the standard k-ε two-equation model (Launder and Spalding, 1972), the eddy viscosity is given by:

$$\nu_t=C_\mu\frac{k^2}{\varepsilon} \tag{5}$$

In the last equation, C_μ is a constant and the turbulence kinetic energy k and its dissipation rate ε are given by two transport equations:

$$\frac{\partial (U_j\, k)}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]+P_k-\varepsilon \tag{6}$$

$$\frac{\partial (U_j\, \varepsilon)}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]+C_{\varepsilon 1}\frac{\varepsilon}{k}P_k-C_{\varepsilon 2}\frac{\varepsilon^2}{k} \tag{7}$$

The production of kinetic energy P_k is given by:

$$P_k = -\overline{u'_i u'_j} \frac{\partial U_i}{\partial x_j} \tag{8}$$

And, finally, the model is closed by the constants:

$$C_\mu = 0.09 \qquad \sigma_k = 1 \qquad \sigma_\varepsilon = 1.3 \quad C_{\varepsilon 1} = 1.44 \qquad C_{\varepsilon 2} = 1.92 \tag{9}$$

In the Spalart-Allmaras one-equation model(Spalart and Allmaras, 1992) the kinetic energy k is dropped from the expression of the Boussinesq approximation, Eq. 3, and the eddy viscosity is given by:

$$\nu_t = f_{v_1} \tilde{\nu} \tag{10}$$

The eddy viscosity is linked to the transported turbulent viscosity through a damping function, f_{v1}, and the only transport equation of the model is given by:

$$\frac{\partial (U_j \tilde{\nu})}{\partial x_j} = c_{b1} \tilde{S} \tilde{\nu} - c_{w1} f_w \left(\frac{\tilde{\nu}}{d} \right)^2 + \frac{1}{\sigma} \frac{\partial}{\partial x_j} \left[(\nu + \tilde{\nu}) \frac{\partial \tilde{\nu}}{\partial x_j} \right] + \frac{c_{b2}}{\sigma} \frac{\partial \tilde{\nu}}{\partial x_j} \frac{\partial \tilde{\nu}}{\partial x_j} \tag{11}$$

In the above equation the destruction term of the turbulent viscosity is related to a length scale that is simply the distance to the wall d. The Spalart-Allmaras model uses the Prandtl Mixing Length κd, where κ is the Von Kármán constant, as the characteristic turbulence length scale. The other auxiliary expressions and constants of the model are:

$$\tilde{S} = S + \frac{\tilde{\nu}}{\kappa^2 d^2} f_{v2} \qquad S = \sqrt{2 \Omega_{ij} \Omega_{ij}} \qquad \Omega_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i} \right) \tag{12}$$

$$\chi = \frac{\tilde{\nu}}{\nu} \qquad g = r + c_{w2} (r^6 - r) \qquad r = \frac{\tilde{\nu}}{\tilde{S} \kappa^2 d^2} \tag{13}$$

$$f_{v_1} = \frac{\chi^3}{\chi^3 + c_{v1}^3} \qquad f_{v2} = 1 - \frac{\chi}{1 + \chi f_{v1}} \qquad f_w = g \left[\frac{1 + c_{w3}^6}{g^6 + c_{w3}^6} \right]^{\frac{1}{6}} \tag{14}$$

$$c_{b1} = 0.1355 \qquad c_{b2} = 0.622 \qquad c_{v1} = 7.1 \qquad \sigma = 2/3 \tag{15}$$

$$c_{w1} = \frac{c_{b1}}{\kappa^2} + \frac{1 + c_{b2}}{\sigma} \qquad c_{w2} = 0.3 \qquad c_{w3} = 2 \qquad \kappa = 0.41 \tag{16}$$

The open source Finite-Volume CFD package OpenFOAM® was used to perform the numerical simulations. Pressure and velocities were coupled through a SIMPLE (Patankar and Spalding, 1972) segregated approach. A second-order linear upwind interpolation was used for the advective fluxes of the transport equations of momentum and turbulence quantities.

When using the OpenFOAM® Spalart-Allmaras model, the reader should be aware that, up to version 2.3.0 of the package (the version used here), the model programmed in the software is not the original model proposed by Spalart and Allmaras (1992). To carry out the present simulations the source code had to be corrected and the binaries recompiled.

The simulation domain was chosen to extend 20H in the y direction and 40H in the x direction. Due to the symmetric nature of the jet, a symmetry plane boundary condition was used on the bottom boundary in order to have half the number of cells. A symmetry plane boundary condition was used on the upper boundary in order to have a non viscous free stream. On the left boundary it was chosen to have an outlet and on the right boundary a wall and an inlet with specified values for velocity and turbulence variables. Since H=1m, a velocity U_j=1m/s was imposed on the inlet boundary. The kinematic viscosity was set as ν = 0.0001 m^2/s, resulting in a Reynolds number Re=10000. For the k-ε

simulations, the values at the inlet were set as k_{inlet} = 0.0038 m^2/s^2 and $\varepsilon_{\text{inlet}}$ = 0.00056 m^2/s^3. The wall was treated through a wall function approach based in the log law. For the Spalart-Allmaras simulations, the value of the turbulence kinematic viscosity was set as $\tilde{\nu}_{inlet}$ = 0.0023 m^2/s at the inlet.

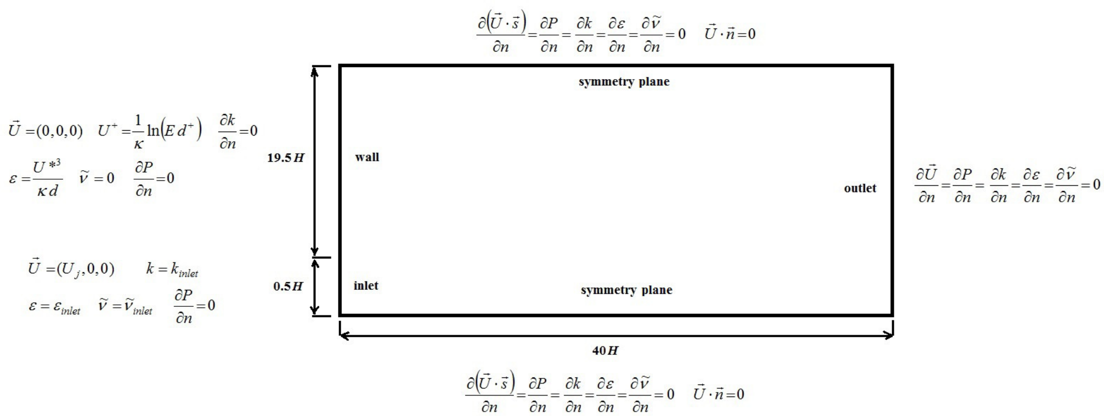

Figure 2. Geometry and boundary conditions.

The inlet boundary was discretized with ten uniform cells, and ninety cells were used in the remaining *y* extension of the domain, with an expansion factor of 1.028 away of the inlet. On the *x* direction the domain was discretized with two hundred cells with an expansion factor of 1.012 away of the inlet, resulting in a mesh with 20000 quadrilateral cells. The mesh can be seen in Fig. 3.

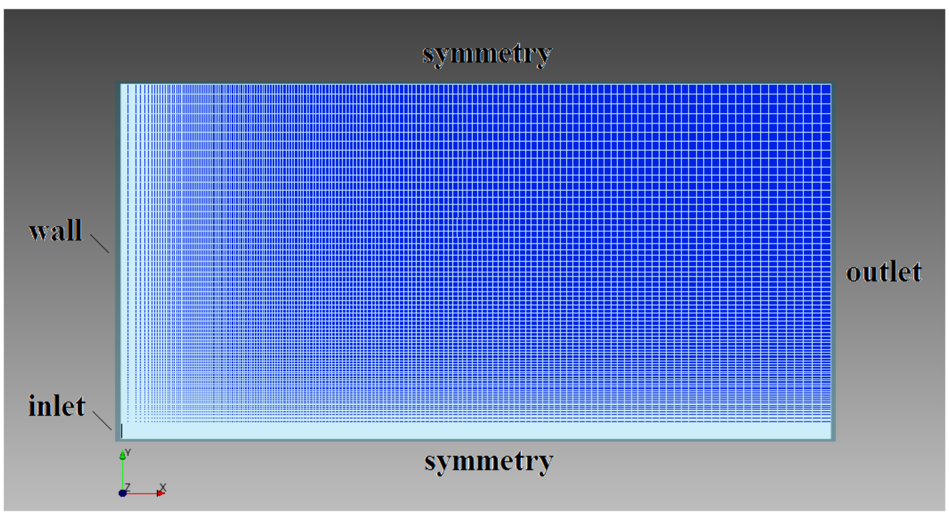

Figure 3. A view of the mesh. The concentration of cells is greater in the inlet region.

3. SIMULATION RESULTS

Figure 4 shows a comparison between the dimensionless velocity profile of the experimental results discussed in the introduction and the obtained simulation profiles with the k-ε model for planes *x/H*=20, *x/H*=30 and *x/H*=40. It is possible to see the closeness of agreement between them. Figure 5 shows a comparison of the dimensionless experimental velocity profile and the results obtained with the Spalart-Allmaras model for the same planes. One can see the large discrepancy between the experimental and simulation results. While in the case of the k-ε model all profiles obtained along the *x* direction of the simulation domain closely agreed with the dimensionless experimental profile, in the case of the Spalart-Allmaras model a strong dependency of the distance from the wall can be noticed.

The spreading rate *S* obtained with the *k-ε* model is approximately 0.10 for all *x*/*H* positions, as can be seen in Tab. 1. When using the Spalart-Allmaras model the value of the spreading rate *S* is far larger then the experimental result for all *x*/*H* positions and decreases with the distance from the wall, resulting in *S* = 0.171 for *x*/*H* = 40.

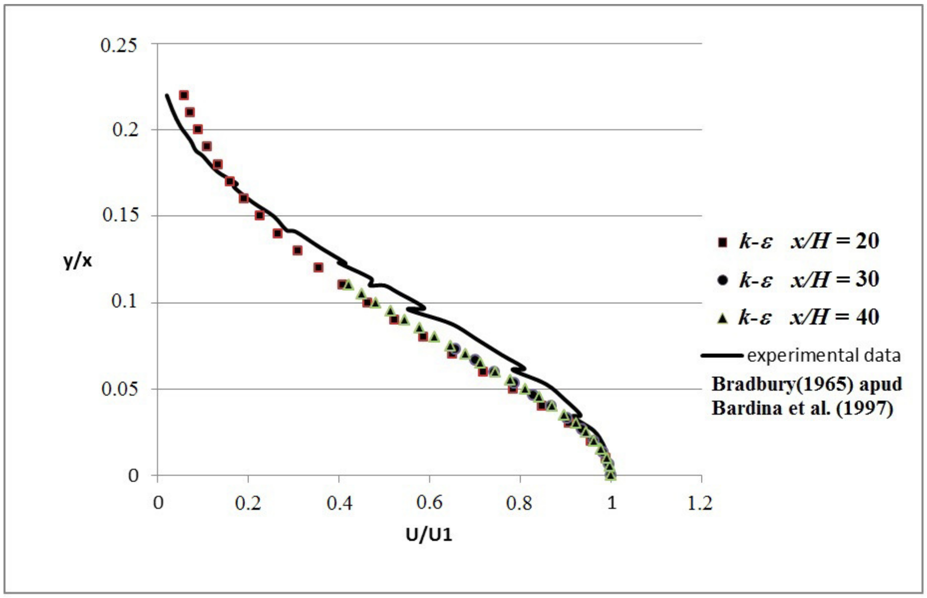

Figure 4. Velocity profiles for *x*/*H*=20, *x*/*H*=30 and *x*/*H*=40 obtained using the *k-ε* turbulence model and compared to the experimental velocity profile from Bradbury (1965) apud Bardina et al. (1997).

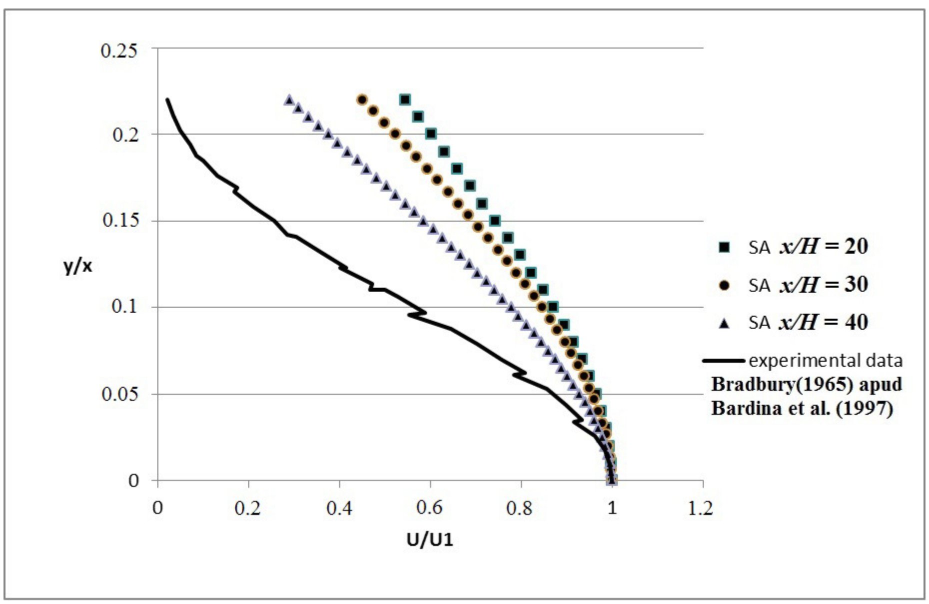

Figure 5. Velocity profiles for *x*/*H*=20, *x*/*H*=30 and *x*/*H*=40 obtained using the Spalart-Allmaras turbulence model and compared to the experimental velocity profile from Bradbury (1965) apud Bardina et al. (1997).

Table 1. Comparison of the results for the spreading rate S.

Model	Spreading rate *S*	*x*/*H*=20	*x*/*H*=30	*x*/*H*=40
Experimental	~ 0.110	-	-	-
k-ε	0.095 – 0.097	0.095	0.099	0.097
Spalart-Allmaras	0.236 – 0.171	0.236	0.206	0.171

4. CONCLUSION

The one-equation Spalart-Allmaras turbulence model failed to correctly predict the spreading rate of a free plane jet, while the k-ε two-equation model provided much better results. Moreover, not only the Spalart-Allmaras model failed to predict the spreading rate, but presented a dependence of the results on the distance from the wall.

These results are not completely unexpected. As a two-equation model, the k-ε model is related to a turbulence length scale $L = C_\mu^{3/4}\ k^{3/2}\ \varepsilon^{-1}$. The turbulence length scale is an output of the flow simulation. The Spalart-Allmaras model, however, as a one-equation model, is related to a grid-dependent turbulence length scale given by the Prandtl Mixing Length, $L = \kappa d$. Moreover, this expression for the mixing length was derived for equilibrium boundary layers with negligible pressure gradients, so it was devised for flow parallel to a wall, not away from it.

Although the Spalart-Allmaras model failed to predict the behavior of the free plane jet, it would be interesting to study if Detached Eddy Simulation (DES) or Scale Adaptive (SAS) unsteady three-dimensional simulations with the model can better predict this type of flow, or other similar flows as the axial jet. Although in the DES formulation the length scale is still strongly influenced by the mesh distribution through the dimensions of the individual cells, in the SAS technique the Von Kármán length scale is obtained through first and second order derivatives of the velocity field, overcoming the Spalart-Allmaras Model dependence from the wall distance.

5. ACKNOWLEDGEMENTS

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