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CHAOTIC BEHAVIOR OF A MEMS SYSTEM CHARACTERIZED BY NONLINEAR TOOLS

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Abstract. *In this work, nonlinear tools to characterize whether a nonlinear system is periodic or non-periodic, were studied. Classical tools that characterized the behavior of the nonlinear system are, for example, Lyapunov exponents, FFT, among others. We compared these techniques, with others that recently appeared in the literature like 0-1 test and the Scale index. In this context, we applied these techniques in a MEMS system that present a rich nonlinear dynamics. Numerical results show that the Scale index can detect the behavior of the system when the signal is periodic or non-periodic, and the results are compared with 0-1 test and Lyapunov exponents.*

Keywords: *0-1 test, Scale index, chaos, nonlinear dynamics*

1. INTRODUCTION

Microelectromechanical systems (MEMS) are devices or systems built in small sizes, which combine both electrical and mechanical components. The machines with MEMS components have many functions, that can be included, for example, sensing and actuation (Tusset, *et al.*, 2012). Currently, a great deal of research has been performed to report nonlinear dynamic phenomena in MEMS resonators (Braghin, *et al.*, 2007, Mestrom, *et al.*, 2007). We remarked that nonlinearities, in MEMS, including nonlinear springs and damping mechanisms (Roukes, M., 2001), nonlinear resistive, inductive and capacitive circuit elements (Xie and Fedder, 2002) and nonlinear surface, fluid, electric, and magnetic forces (Younis and Nayfeh, 2003). As we know, nonlinearities may lead to chaotic behaviors, and some works studies the existence of chaotic motion in MEMS . The appearance of chaotic motion, was also reported in (Luo, *et al.*, 2002, Liu, *et al.*, 2004). Tusset, *et al.* (2012) applied three different control strategies to control the trajectory of the system, and the controls proved effective in controlling the trajectory of the system studied and robust in the presence of parametric errors.

Most dynamical systems among them the MEMS, have a nonlinear nature that leads to observe special behaviors such as chaotic motions. According to Strogatz (1994) chaos in deterministic nonlinear system exhibits two important characteristics : (1) sensitive dependence on initial conditions and (2) non-periodic long-term behavior (or it does not converge to a periodic orbit). Along the years, two classical tools were developed to characterize the deterministic chaos behavior. On criterion (1), The Lyapunov exponent can be used as a criterion to detected the sensitive dependence on initial conditions, albeit cannot determine non-periodicity (Benítez *et al.*, 2010). On the other hand, as to criterion (2), Fourier analysis can be used in order to study non-periodicity. More recently, these studies have been refined, and new techniques to confirm if the dynamical system has a chaotic behavior, has been developed. Two of the new tools to identify the chaos are: 0-1 test and scale index.

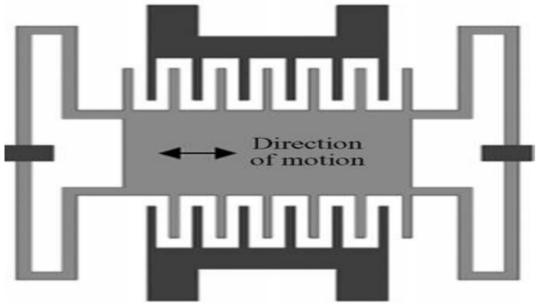
The 0-1 test is used to characterize the behavior dynamical system. The 0-1 test uses the temporal series data to characterize the dynamics of the system, analyzing its spectral and statistical properties based on the asymptotic properties of a Brownian motion. The 0-1 test is very useful in systems where the Lyapunov exponents are very difficult to calculated.. The 0-1 test have been successfully used in the analysis of the chaotic regimes of the SMA hysteretic oscillators (Bernardini *et al.*, 2013; Litak *et al.*, 2009; Litak *et al.*, 2013).

As mentioned, the Fourier analysis, can be usel to identify the periodicy, however chaotic signals may be highly non-stationary, which makes wavelets more suitable (Chandre , *et al.*, 2003). Moreover, compactly supporte wavelets are a useful tool in the analysis of non-periodicity in compactly supported signals. Therefore, Benítez *et al.* (2010) developed a tool called, scale index technique, to identify the non-periodicity based on Continuous Wavelet Transform (CWT) and the wavelet Multiresolution Analysis (MRA) of a signal. The scale index were usefull to identify the behavior of several system, like, logistic map, the Henon map, the Bonhoeffer-van der Pol oscillator (Benítez *et al.*, 2010), the Rössler system (Akhshani *et al.*, 2014), the heartbeat dynamics (Behnia *et al.*, 2013), and the SMA hysteretic oscillators (Piccirillo *et al.*, 2015).

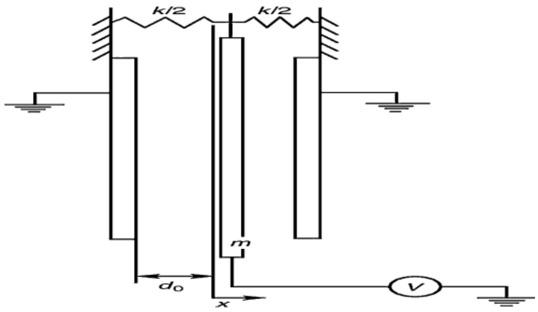
In this work, the 0-1 test and scale index theory are explored to investigate the nonlinear dynamics of the MEMS and the results are compared with a classical Lyapunov exponent.

2. MICRO ELECTROMECHANICAL SYSTEM

The behaviour of the microelectromechanical system represented by an electrostatic generator of energy, shown in figure 1 was studied previously in Tusset, et. al., 2012. According to Moon (1998), the physical system represented by Fig. 1, may be considered as a set of micro-beams. These works considered fractional order effects and analysed of the chaotic behaviour. In the present study the periodic responses at the resonances are closely investigated. The considered dynamical system as shown in figure 1, and this device is consisting of two fixed plates and a movable plate between them (see Fig. 1b), which it is applied a voltage $V(t)$ composed of a polarization voltage (DC) V_p, and an alternating voltage (AC) $V_i \sin(wt)$ (Moon, 1998)). The (DC) voltage applies an electrostatic force on the beam and usually changes the equilibrium position. The plates have the function of providing electrodes to form a capacitor or storage of electrical energy, and provide elasticity or mechanical strength.



(a)



(b)

Figure 1: (a) In-plane gap closing, (b) Micro electromechanical system

The equations of motion of the plates is as follows

$$m\ddot{x} = -F_K - F_C + F_e \tag{1}$$

where F_K is the conservative force of the spring, F_C is the damping force of the elastic term, and F_e the electric force. As can be seen from Fig. 1(b), that the distance between the fixed and movable plates depends on the initial distance between the plates d_0 and the displacement x of the movable plate. According to Tusset, el. al. (2012) the total electric energy stored, in the system, can be obtained from:

$$W^* = \frac{\varepsilon_0}{2} V^2 A\left[\left(\frac{1}{d_0 - x}\right) - \left(\frac{1}{d_0 + x}\right)\right] \Rightarrow W^* = \varepsilon_0 V^2 A\left(\frac{d_0}{d_0^2 - x^2}\right) \tag{2}$$

where ε_0 is the permittivity of vacuum, A is the plate area, and $V = V_p + V_i \sin(wt)$. The electric force F_e is a nonlinear function of displacement in x and a quadratic function of voltage, namely:

$$F_e = \frac{\partial W^*}{\partial x} = 2\varepsilon_0 d_0 V^2 A \frac{x}{(d_0^2 - x^2)^2} \tag{3}$$

In this work, the considered systems has one type of non-linearity, associated with the spring stiffness terms. In this way the conservative spring force F_K, and the non-conservative force F_C, can be written as follows:

$$F_K = k_1 x + k_3 x^3 \tag{4}$$

$$F_C = c\dot{x} \tag{5}$$

Then, the equation of motion for the microelectromechanical system can be written as follows:

$$m\ddot{x} + c\dot{x} + k_1 x + k_3 x^3 = 2\varepsilon_0 d_0 V^2 A \frac{x}{\left(d_0^2 - x^2\right)^2} \tag{6}$$

In order to carry out effective numerical calculations, the equation (6) was non-dimensionalized, using the non-dimensional variables and parameters as follows:

$$b = \frac{c}{m\omega_0}, \alpha_1 = \frac{k_1}{m\omega_0^2}, \alpha_3 = \frac{k_3 x_0^2}{m\omega_0^2}, \beta = \frac{2\varepsilon_0 d_0 A}{m\omega_0^2 x_0^4}, \Omega = \frac{w}{\omega_0}, d = \frac{d_0}{x_0}, u = \frac{x}{x_0}, t = \tau\omega_0$$

Then the non-dimensional system of equations for the microelectromechanical oscillator can be written in the following form:

$$\begin{cases} \dot{u}_1 = \dot{u}_2 \\ \dot{u}_2 = \beta V^2 \dfrac{u_1}{\left(d_0^2 - u^2\right)^2} - bu_2 - \alpha_1 u_1 - \alpha_3 u_1^3 \end{cases} \tag{7}$$

where, $V = V_p + V_i \sin \Omega\tau$

3. 0-1 TEST

The 0-1 test, proposed by Gottwald and Melbourne (2004) is directly applied to the time series data, based on the statistical properties of a single coordinate (herein, the variable u_1 (see Eq. 7)). Basically, the 0-1 test consists of estimating a single parameter $\bar{K}$. The test considers a system variable x_j, where two new coordinates (p,q) are defined as follows

$$p(n,\bar{c}) = \sum_{j=0}^{n} x(j)\cos(j\bar{c}) \tag{8}$$

$$q(n,\bar{c}) = \sum_{j=0}^{n} x(j)\sin(j\bar{c}) \tag{9}$$

where, $\bar{c} \in (0,\pi)$ is a constant. The mean square displacement of the new variables $p(n,\bar{c})$ and $q(n,\bar{c})$ is given by

$$M(n,\bar{c}) = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \left[\left(p(j+n,\bar{c}) - p(j,\bar{c})\right)^2 + \left(q(j+n,\bar{c}) - q(j,\bar{c})\right)^2 \right] \tag{10}$$

where $n \ll N$ and, therefore, we obtain the parameter $K_{\bar{c}}$ in the limit of a very long time

$$K_{\bar{c}} = \frac{\operatorname{cov}\left(Y, M(\bar{c})\right)}{\sqrt{\operatorname{var}(Y)\operatorname{var}\left(M(\bar{c})\right)}} \tag{11}$$

where vectors $Y = [1, 2, \ldots, n_{\max}]$ and $M(\bar{c}) = \left[M(1,\bar{c}), M(2,\bar{c}), \ldots, M(n_{\max},\bar{c})\right]$.

Given any two vectors x and y, the covariance $\operatorname{cov}(x,y)$ and variance $\operatorname{var}(x)$, of $n_{\max}$ elements, are usually defined as (Litak *et al.*, 2013):

$$\operatorname{cov}(x,y)=\frac{1}{n_{\max}}\sum_{n=1}^{n_{\max}}\left(x(n)-\bar{x}\right)\left(y(n)-\bar{y}\right) \tag{12}$$

$$\operatorname{var}(x)=\operatorname{cov}(x,x) \tag{13}$$

where $\bar{x}$ and $\bar{y}$ are the average of $x(n)$ and $y(n)$, respectively. As a final result, the value of the searched parameter $\bar{K}$ is obtained taking the median of 100 different values of the parameter $\bar{c}\in(0,\pi)$ in equation (11). If the $\bar{K}$ value is close to 0 the system is periodic, on the other hand, if $\bar{K}$ value is close to 1 the system is chaotic. In all simulations we have chosen $n=10000$ and $j=\frac{n}{100},\dots,\frac{n}{10}$.

4. A WAVELET-BASED SCALE INDEX

The wavelet transform of a one-dimensional (1D) signal consists of its expansion with respect to a basis constructed via solutions-like functions called wavelet, by means of various internal transformations and shifts (Permann and Hamilton,1992; Awrejcewicz, *et al.*, 2011) .

Non-periodic behavior means that there are trajectories that do not settle down to periodic or quasi periodic orbits as $t\to\infty$ (Strogatz, 1994).

Given $f\in L^2(\mathbb{R})$, the Continuous Wavelet Transform (CWT) of f at time u and scale s is defined as

$$Wf(u,s):=\left\langle f,\psi_{u,s}^{*}\right\rangle=\int_{-\infty}^{+\infty}f(t)\psi_{u,s}^{*}(t)\,dt \tag{14}$$

where

$$\psi_{u,s}^{*}=\frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right),u\in\mathbb{R},s>0 \tag{15}$$

and $Wf(u,s)$ provides the frequency component (or relevant details) of the signal of f at time u and scale s with respect to the analyzing wavelet $\psi_{u,s}$ (Benítez *et al.*, 2010).

The scalogram of f, $\wp$, is defined as follows (Benítez *et al.*, 2010):

$$\wp(s):=\left\|Wf(u,s)\right\|=\left(\int_{-\infty}^{+\infty}\left|Wf(u,s)\right|^2du\right)^{1/2} \tag{16}$$

$\wp(s)$ is the energy of the CWT of f at scale s. The scalogram is a useful tool to study a signal, since it offers the possibility to detect its most representative scales (or frequencies).

Then, the innerscalogram of f at scale s can be defined by (Benítez *et al.*, 2010):

$$\wp^{inner}(s):=\left\|Wf(u,s)\right\|_{J(s)}=\left(\int_{c(s)}^{d(s)}\left|Wf(u,s)\right|^2du\right)^{1/2} \tag{17}$$

where $J(s)=\left[c(s),d(s)\right]\subseteq I$ is the maximal subinterval in I for which the support of $\psi_{u,s}$ is included in I for all $u\in J(s)$, where I is a finite time interval [1]. The length of $J(s)$ depends on the scale s, so that the values of the inner scalogram at different scales cannot be compared. Therefore, the inner scalogram should be normalized as follows (Benítez *et al.*, 2010).

$$\bar{\wp}^{inner}(s) = \frac{\wp^{inner}(s)}{\left(d(s) - c(s)\right)^{1/2}} \tag{18}$$

The Scale index in the scale interval $\left[s_0, s_1\right]$ can be defined by the quotient (Benítez *et al.*, 2010).

$$i_{scale} := \frac{\wp(s_{\min})}{\wp(s_{\max})} \tag{19}$$

where $s_{\max}$ and $s_{\min}$ are the smallest scales such that $\wp(s_{\min}) \le \wp(s) \le \wp(s_{\max})$ for all $s \in \left[s_0, s_1\right]$. Note that for compactly supported signals only the normalized inner scalogram will be considered (Benítez *et al.*, 2010).

From its definition, the Scale index i_{scale} is such that $0 \le i_{scale} \le 1$ and it can be interpreted as a measure of the degree of non-periodicity of the signal: the Scale index will be zero or close to zero for periodic sequences and close to one for highly non-periodic sequences (Benítez *et al.*, 2010).

5. NUMERICAL RESULTS

The values of the parameters used in the current numerical investigations were taken from Tusset *et al.* (2012) and they are as follows: $\alpha_1 = 1$, $\beta = 69141.6$, $\Omega = 6.28$, $d = 25$, $V_i = 10$, $u_1 \ 0 = 10$ and $u_2 \ 0 = 5$. These system parameters they are kept constant, all the time, which allows us concentrate on the effects caused by the variations of the damping parameter b, the polarization voltage parameter V_p, and the nonlinear stiffness term α_3.

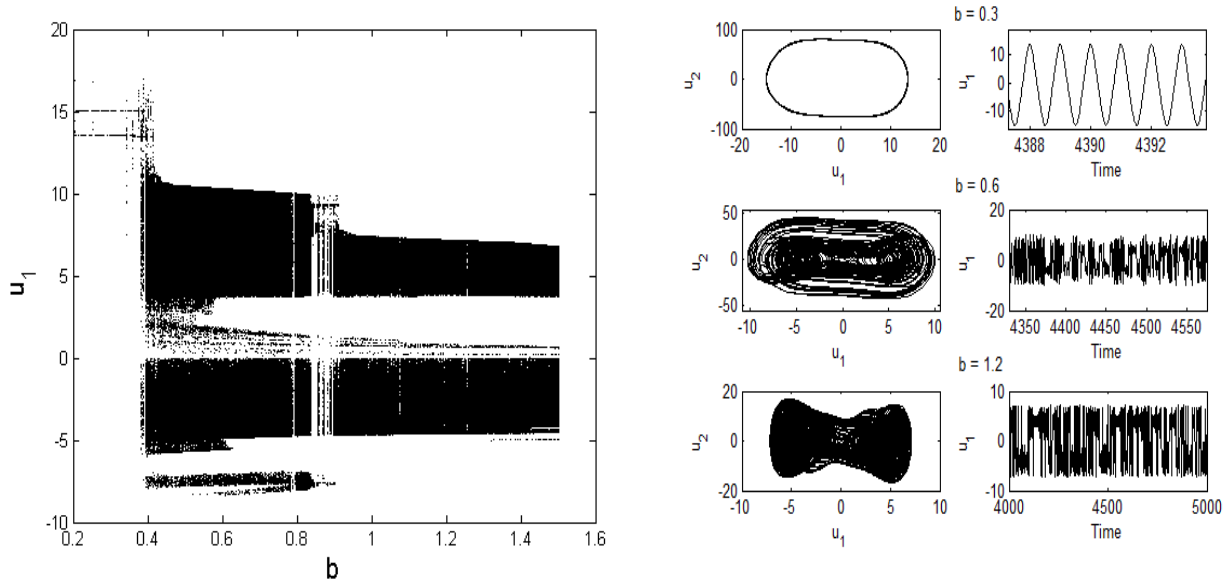
In order to investigate the different parameters of the MEMS model, in Eq. 7, the bifurcation diagram of the local maximum displacement is constructed with the transient being omitted (Souza, *et al.*, 2005; Balthazar *et al.*, 2012).

In this paper, the computation of the scale index was carried out as follows: using the Daubechies eight-wavelet (db8) function and the scale interval from $s_0 = 1$ to $s_1 = 512$, the value of scale index was evaluated for a given control parameter.

5.1 Damping effect of the MEMS

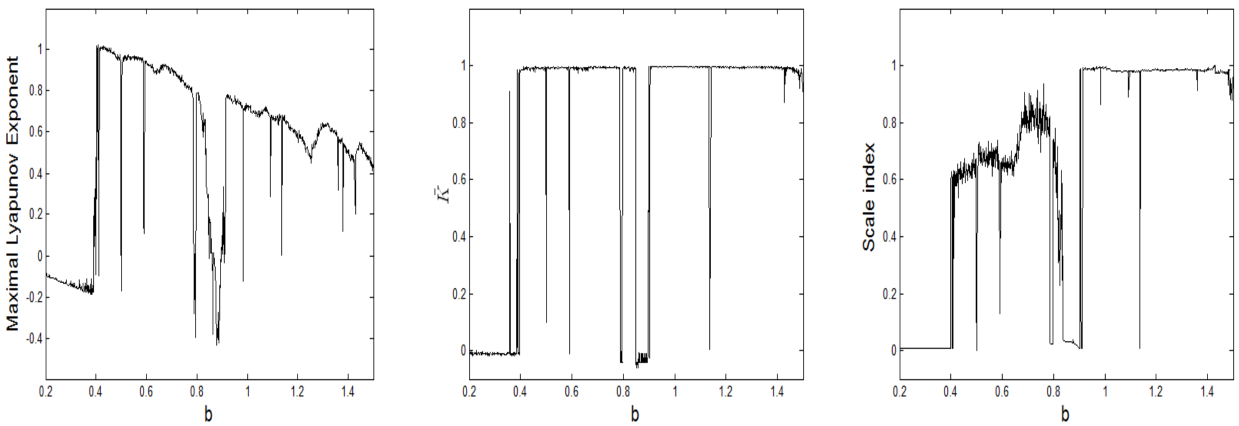
Our numerical study of the MEMS shows that the damping effect can be very significant. Figure 2(a) shows the bifurcation diagram changing the damping parameter b, however we consider $V_p = 2$ and $\alpha_3 = 0.445$. As can be seen, that both kinds of orbits (periodic and nonperiodic) can be achieved with changing the damping parameter $b \in \left[0.2, 1.5\right]$. Phase portrait and time history of these periodic and nonperiodic responses are shown in Fig, 2(b) for the damping values $b = 0.3$, $b = 0.6$, $b = 1.2$.

Figure 3(b) show the maximal Lyapunov exponent of the MEMS for different values of the control parameter $b \in \left[0.2, 1.5\right]$. This Lyapunov exponents are calculated by the process presented by Wolf *et al.* (1985). As we know, if the values of the maximal Lyapunov exponent is negative the system is periodic and, on the other hand, the system is considered chaotic if the maximal Lyapunov exponent is positive. In order to show the effectiveness of the 0-1 test $\left(\bar{K}\right)$ and the scale index $\left(i_{scale}\right)$, we compare the bifurcation diagram, the maximal Lyapunov exponent, the $\bar{K}$ and i_{scale}. Figure 3(a), (b) and (c) depict the comparison between the three methods mentioned above. Note the high level of agreement among the maximal Lyapunov exponent, the 0-1 test and the scale index: the values of b for which the maximal Lyapunov exponent is negative are also the values for which 0-1 test and the scale index are close to 0. On the other hand, when the maximal Lyapunov exponent is positive, the attractor is chaotic, and the values of the 0-1 test and the scale index are different to zero. Note that in this case, the most of the values of the 0-1 test is one (indicating the chaotic attractor), and the scale index values is always larger than 0.2, clearing indicating the chaotic attractor.



(a) (b)

Figure 2: (a) Bifurcation Diagram and (b) phase portraits and time history for different values of b

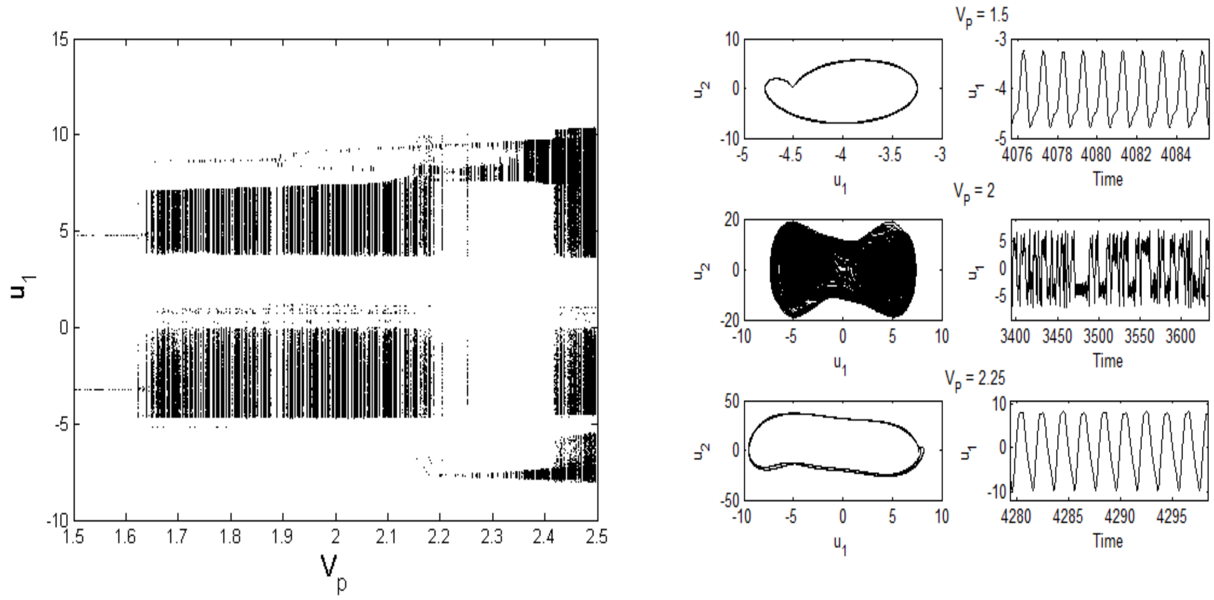


(a) (b) (c)

Figure 3: (a) Lyapunov exponent vs b, (b) $\bar{K}$ vs b (0-1 test) and (c) Scale index vs b

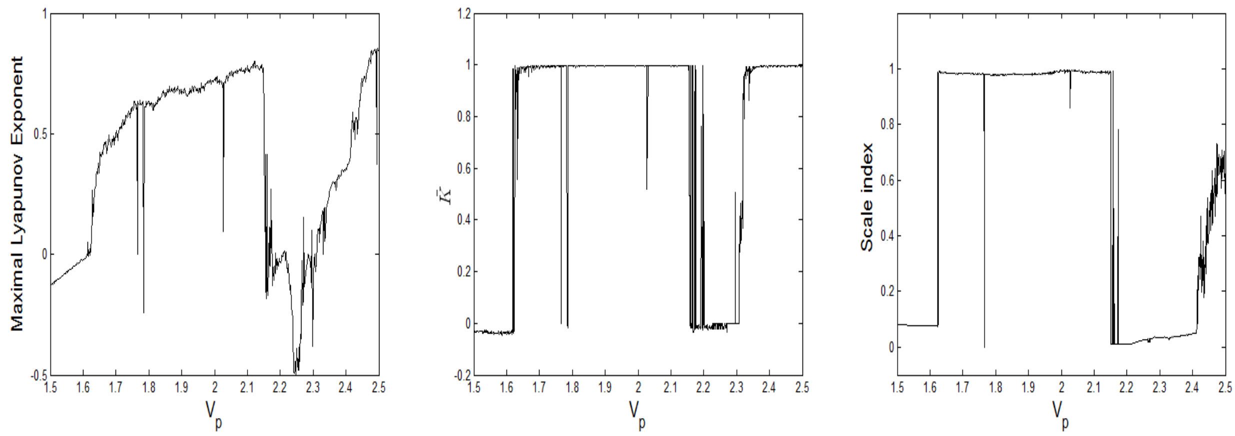
5.2 Influence of the Polarization Voltage

In this section, the nonlinear equations of MEMS will be investigated by maximal Lyapunov exponent, 0-1 test and scale index considering the variation of the V_p (Polarization Voltage). Initially, in order to investigate the dynamics of the MEMS, we present bifurcation diagram in Figure 4(a) for the parameter V_p. Fig. 4(b) shows three phase portrait and time history of periodic and nonperiodic responses, namely, $V_p = 1.5$, $V_p = 2$, $V_p = 2.25$. Firstly, to characterize the nature of the behavior observed, we calculate the maximal Lyapunov exponents, as shown in Figure 5(a), using the Wolf algorithm described in the (Wolf et al., 1985). If the largest Lyapunov exponent is positive the attractor is chaotic, if not the attractor is periodic. Figure 5 shows the comparison among the maximal Lyapunov exponent, the 0-1 test and the scale index. Again, here, we can also observe a good convergence among the three techniques.



(a) (b)

Figure 4: (a) Bifurcation Diagram and (b) phase portraits and time history for different values of V_p

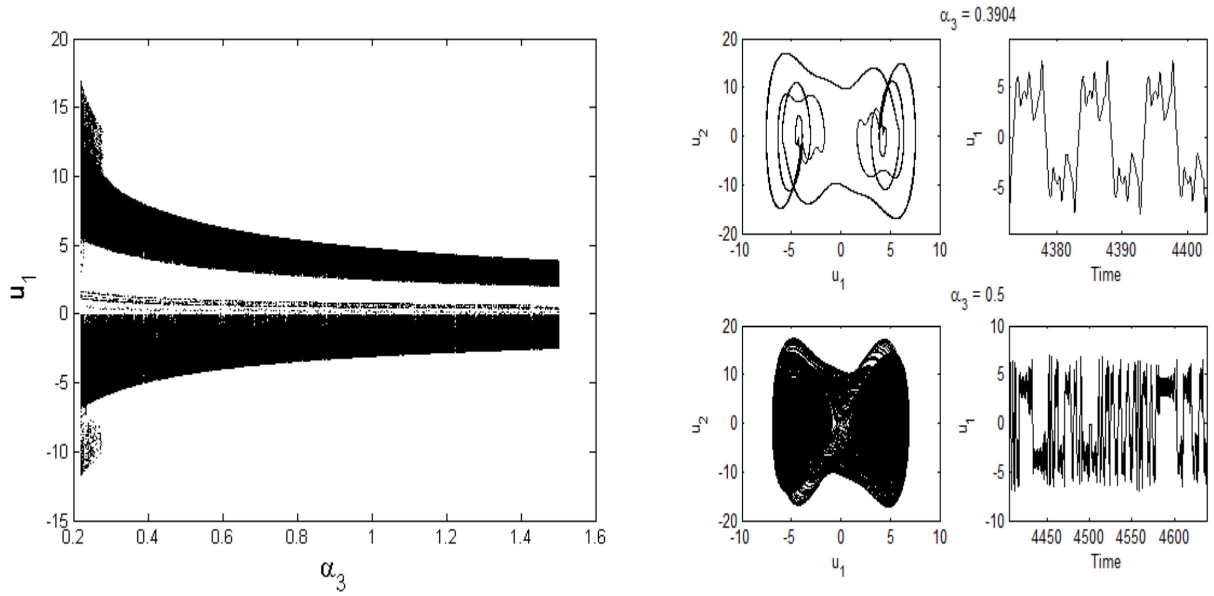


(a) (b) (c)

Figure 5: (a) Lyapunov exponent vs V_p, (b) $\bar{K}$ vs V_p (0-1 test) and (c) Scale index vs V_p

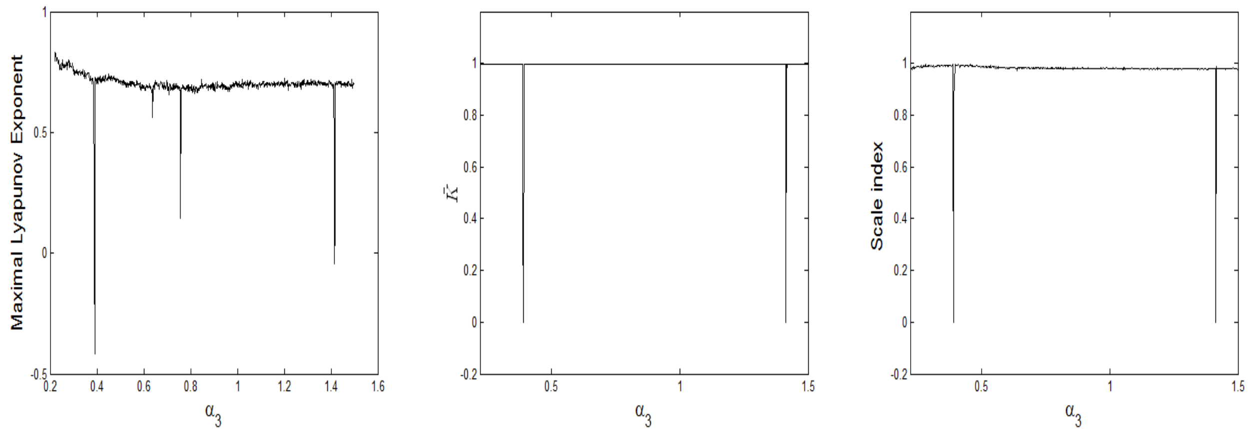
5.3 Influence of the nonlinear stiffness term

Now, we consider the variation of the parameter α_3 (nonlinear stiffness term), and again we compare the maximal Lyapunov exponent, 0-1 test and scale index. The bifurcation diagram is shown in Figure 6(a) for the parameter α_3. Fig. 6(b) shows two phase portraits and time histories of periodic and nonperiodic responses, namely, $\alpha_3 = 0.3904, \alpha_3 = 0.5$. Figure 7 shows the comparison among the maximal Lyapunov exponent, the 0-1 test and the scale index. Again, here, we can also observe a good convergence among the three techniques.



(a) (b)

Figure 6: (a) Bifurcation Diagram and (b) phase portraits and time history for different values of α_3



(a) (b) (c)

Figure 7: (a) Lyapunov exponent vs α_3, (b) $\bar{K}$ vs α_3 (0-1 test) and (c) Scale index vs α_3

6. CONCLUSION

In this work, we investigated the influence of some parameters on the dynamic behaviour of a MEMS. The MEMS model proposed in Tusset *et al.* (2012) was adopted here. In this way, based in the bifurcation structure, the effect of three parameters on the MEMS were examined. In terms of the system dynamics our analysis shows that, as the control parameters (damping , voltage and nonlinear terms) are varied, a complex MEMS system undergoes a variety of dynamic transitions which change its stability. To characterize the regularity and irregularity (order or chaos) of the MEMS we compare the classical Wolf's method to find the Lyapunov exponents with two new tools, namely, 0-1 test and scale index. In particular, both 0-1 test and scale index are a good complements to the Lyapunov exponent, since the 0-1 test can be applied in systems where the Lyapunov exponent are difficult to estimation for example, non-continuous and hysteretic system. On the other hand, scale index gives a measure of the degree of nonperiodicity of the signal, while the Lyapunov exponent gives information about the sensitivity to initial conditions.

7. ACKNOWLEDGEMENTS

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