23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

AUTOMATIC GENERATION OF DYNAMIC MODELS OF CABLES TO UNDERWATER APPLICATIONS

Thiago A. Pouzada
Sebastião C. P. Gomes
Mathematics, Statistics and Physics Institute (IMEF), Federal University of Rio Grande (FURG), CEP 96201-900 , Rio Grande-RS, Brazil
thiago.pouzada@gmail.com; sebastiaogomes@furg.br

Abstract. *Dynamic modeling of cables is a research theme of great importance and practical application nowadays. However, due to non-linearity behavior present on cable dynamics, as well as the need to work with many degrees of freedom, to develop dynamic models for these systems becomes a hard task. To develop dynamic models of cables, this article uses the Lumped Mass Approach, which supposes the cable consisting of rigid links connected by fictitious elastic joints. The cables were considered with one of its endings articulated to a floating structure (ship or platform), while on the other ending were considered two cases: one free terminal loading; ending fixed to the ground. The cables are submersed and the hydrodynamic drag was considered from a simple model, proportional to the square of the relative velocity between the structure and the fluid. Using the Euler-Lagrange formalism, the dynamic models for cables considering two, three and four links were developed. From these models algorithms have been developed to automatically generate dynamic models for any number of links considered. Simulations were performed, which showed the results agreeing qualitatively with the physical expected. A software has been developed to display animation showing frames each 0.005 s. This animation, based on data from numerical simulation, allowed us show that the developed dynamic model gives a great sense of reality.*

Keywords: *dynamic modeling, generic algorithms, flexible structures, cables, underwater applications.*

1. INTRODUCTION

The importance of cable modeling is directly associated to a growing development sector, linked to oil and naval industries. Umbilical cables used in ROVs (Remotely Operated Vehicles), risers, or yet towing cables, mooring lines or anchoring to floating structures require special attention from the companies working on these sectors. Thus, the faithful modeling of these cables, putting them on the oceanic context and approaching them to real situations becomes attractive.

Mostly of the found scientific articles about cable modeling uses the finite elements technique and the final interest generally lies on the determination of the forces on static equilibrium, important for the dimensioning. However, there are a few papers that study the cable modeling.

Recently, Bamdad (2013) has proposed an analytical solution to solve the problem of cable vibration, using an algorithm to simulate and validate the obtained results. Bi, *et al.* (2013) applies the Finite Elements Method to study the tensions present in umbilical underwater cables. Chang, *et al.* (2008) investigates the dynamic behavior of a cable, considering the non-linearities present due to sagging effect, based on the catenary cable theory. Chatjigeorgiou and Mavrakos (2010) propose the solution to non-linear dynamic modeling of marine structures, especially risers, using the finite differences technique. Escalante, *et al.* (2011) uses the Finite Elements Method applied to cables to achieve a reduced order model by means of the Proper Orthogonal Decompositions (POD). Fang, *et al.* (2012) represents the bending and torsional effects for towing cables, also using the Finite Elements Method. Gobat and Grosenbaugh (2001) analyze the shortcomings on temporal integration, defined by the Finite Difference Method, proposing the Generalized-α Method for spatial discretization, applied to cable dynamics. Grosenbaugh, *et al.* (1993) uses a numerical technique to calculate the two-dimensional motions of a tethered underwater vehicle.

Guo, *et al.* (2007) investigates the wind-induced vibration control of a single column tower of a cable-stayed bridge with a multi-stage pendulum mass damper (MSPMD) using the Finite Elements Method. Liu, *et al.* (2013) studies the sliding characteristics when a cable is connected with other rods in transmission line structures, based on updated Lagrangian formulation. Luongo, *et al.* (2008) uses methods of solutions such as finite differences and the Galerkin Method, to study nonlinear multimodal galloping of suspended cables. Masciola, *et al.* (2012) studies a solution method for the static configuration of a cable discretized. Nahon (1999) presents the model of a triple-tethered aerostat proposed to support a receiver in a very large radio telescope antenna, using the Lumped Mass Approach. Pathote (2014) uses the b-spline curve on a new technique of mathematical modeling for prestressing cables. Pereira (2010) investigates the fluid-structure interaction of a submersed cable, from the coupling of the cable dynamics with the fluid motion, using the Lumped Mass Approach on the cable dynamics.

Seo, *et al.* (2010) presents a simplified dynamics analysis of a suspended bridge, subjected to ground motions, simulating seismic events, using the Finite Elements Method. Srivastava, *et al.* (2011) presents a numerical approach by the Finite Differences Method, capable to predict the dynamic behavior of underwater towed cables, when the towing ship has its velocity linearly changed. Vaz and Patel (1995) present a numerical solution to the transient motion of towed marine cables, being towed of a ship with its velocity changing. Wolfschluckner and Jodin (2013) compare the Finite Elements Method with numerical and analytical methods, to describe the dynamic behavior of a cable. Zanela (2013) uses the Lumped Mass Approach for three-dimensional modeling of cable type structures. Zhao, *et al.* (2005) proposes the Finite Differences Method to simulate transverse vibrations on an axially moving viscoelastic string.

All these contributions prove the growing importance of this theme, especially in terms of dynamic modeling. Thus, it was considered two different situations: the cable articulated to a mobile platform in one of its endings and free on the other one; cable articulated to the mobile platform on one of its endings and fixed to the ground on the other one. The mobile platform (which can be a ship or a vessel) was considered with the possibility to move itself only on the vertical direction, simulating the swings that occur on the sea surface. The Euler-Lagrange formalism was used to obtain the dynamic model of the cable, whose flexible structure was approached as a discrete equivalent one formed by rigid links connected by elastic fictitious joints. Dynamic models were developed for the cases of the cable divided in two, three and four links, to analyze and investigate the growth patterns present on them. Once obtained these patterns, generic algorithms were developed, to automatically generate the equations for the dynamic models of cables considering any number of links in which the cable can be divided. Simulations were performed, in order to verify qualitatively if the results agreed with the physical expected.

From the dynamic model of a cable, it is possible to simulate several situations, considering external forces present on the oceanic context, which have influence in the motion of the cable.

2. DYNAMIC MODELS

The cables considered on the research that originated this article are articulated to a platform or ship on water surface and, on the other ending, were considered two situations: terminal loading with a free mass m_c; cable fixed to the ground. On both cases, it is supposed that the vibrating movement of the cable is inside a single plan. These situations allow approximating several real applications, such as anchorage lines, risers, etc., as illustrated on Fig. 1. Yet, it was considered that the movement from the ending of the cable fixed to the platform or ship happens on only one degree of freedom, in the vertical.

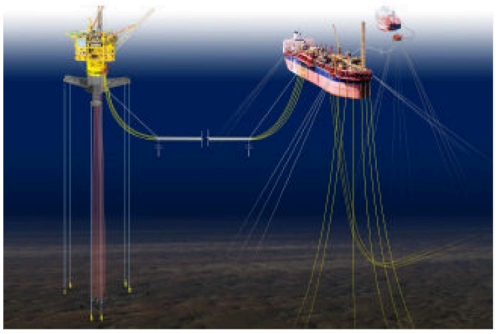

Figure 1. Cables in underwater applications (font: http://diariodopresal.wordpress.com/petroleo-e-gas)

The Lumped Mass Approach was used, supposing the cable divided in rigid links connected by elastic fictitious joints, as illustrated on Fig. 2. Each fictitious joint has its elastic constant and damping coefficient, which are known.

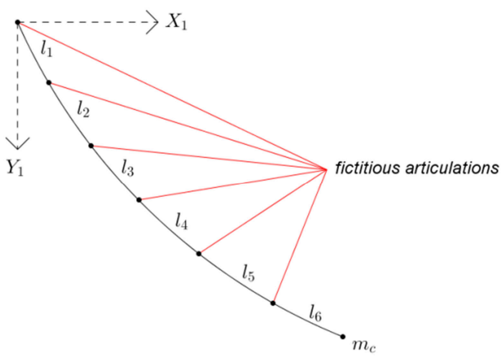

Figure 2. Discrete approach to the cable type structure

Dynamic models considering two, three and four links were analytically developed. The Fig. 3 shows an illustration for the cable divided in three links. The upthrust (Archimedes' principle) cancels the weight on the case of the ship or platform and the imposed dynamics from the water surface was approached by a mass-spring-damper system.

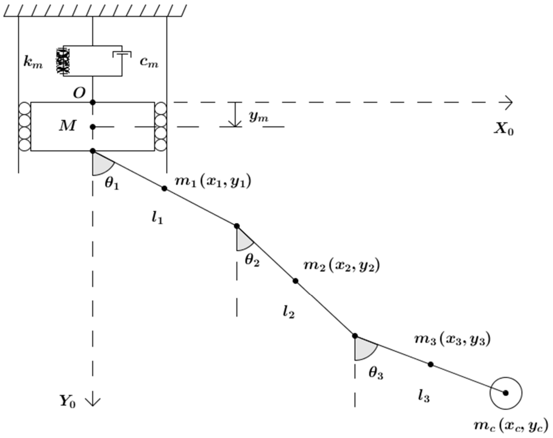

Figure 3. Discrete approach for the cable divided in three links

The position coordinate vector for the structure with three links has the form:

$$\vec{y} = [y_m \quad \theta_1 \quad \theta_2 \quad \theta_3]^T \tag{1}$$

Thus, the dynamics with n links has $n+1$ degrees of freedom. y_m is the vertical position of the ship or platform, related to a inertial reference originated on the average surface of the water. $\theta_i, i = 1, \ldots, n$, are the angles of the links related to the vertical direction.

The translation kinetic energy of each link is written by:

$$E_{C_{T_i}} = \frac{1}{2} m_i (\dot{x}_i^2 + \dot{y}_i^2) \tag{2}$$

where: m_i are the masses on each link; x_i and y_i are the horizontal and vertical positions, respectively, of the mass center on each link, related to the same inertial reference adopted to the ship or platform, with $i = 1, \ldots, n$.

The rotational kinetic energy of each link, related to the link's spin over a cross axis passing through its mass center, has the form:

$$E_{C_{R_i}} = \frac{m_i}{12} l_i^2 \dot{\theta}_i^2 \tag{3}$$

where l_i are the lengths of each link.

The total kinetic energy of the structure is obtained as:

$$E_C = \frac{1}{2} M \dot{y}_m^2 + \sum_{i=1}^{n} \left(E_{C_{T_i}} + E_{C_{R_i}} \right) \tag{4}$$

The elastic potential energy on each joint can be written in the form:

$$E_{P_{E_i}} = \frac{1}{2} k_i (\theta_{i+1} - \theta_i)^2 \qquad (i = 1, \ldots, n-1) \tag{5}$$

where k_i are the elastic constants of each joint.

The gravitational potential energy of each link has the form:

$$E_{P_{G_i}} = m_i g h_i \tag{6}$$

where: h_i corresponds to the high of the mass center of each link related to its position when the structure finds itself in rest and in the vertical; g corresponds to the gravitational acceleration constant.

The total potential energy of the structure can be obtained from the following equation:

$$E_P = \frac{1}{2}k_m y_m^2 + m_c g h_c + \sum_{i=1}^{n-1} E_{P_{E_i}} + \sum_{i=1}^{n} E_{P_{G_i}} \tag{7}$$

where: m_c e h_c correspond to the mass and the high of the terminal loading, respectively when this is free. k_m corresponds to the elastic constant considered on the ship or platform.

Once determined the kinetic and potential energies of the structure, the Lagrangian has the form:

$$L = E_C - E_P \tag{8}$$

Applying the Euler-Lagrange equations to each of the $n + 1$ degrees of freedom, the dynamic model of the system is obtained, in the form:

$$I(\vec{y})\ddot{\vec{y}} + C\dot{\vec{y}} + K\vec{y} + \vec{F}(\vec{y}, \dot{\vec{y}}) + \vec{G}(\vec{y}) = \vec{T}_m \tag{9}$$

$I(\vec{y})$ is the inertia matrix, C the damping coefficients matrix, K the elastic constants matrix and $\vec{F}(\vec{y}, \dot{\vec{y}})$ the vector of the Coriolis-centripetal forces kind. $\vec{G}(\vec{y})$ is the vector of gravitational forces, in which are included the forces due to upthrust in the links and in the terminal loading. These upthrusts are considered acting on the mass center of each element, in the opposite direction of the g-force. $\vec{T}_m$ is the vector with the external forces and the forces due to hydrodynamic drag are included on it. The fluid was considered in rest (absence of underwater currents) and in each link, the drag force acts on its mass center and is proportional to its linear velocity's square, in the form:

$$a_i = -\gamma_i v_i^2 sign(v_i) = -\gamma_i \frac{l_i^2}{4} \dot{\theta}_i^2 sign(\dot{\theta}_i) \tag{10}$$

where: γ_i is the drag coefficient of the link i and v_i its linear velocity.

3. GENERIC ALGORITHMS

The developed analytical equations of the models for cables divided in two, three and four links allowed the identification of generic growth patterns for its different matrixes and vectors. Thus, general forms for each one of these were developed, considering any number n of links in which the cable can be divided. With these general forms, generic algorithms were also developed, enabling simulations to be performed and results to be analyzed.

The inertia matrix is symmetrical, so the algorithm expressed in Eq. (11) determines the elements of its main diagonal and the ones above it. M is the floating structure mass, while l_i and m_i, with $i = 1, \dots, n$, correspond, respectively, to the lengths and masses of each link. m_c corresponds to the terminal loading mass.

The damping coefficients matrix C is symmetrical and of order $n + 1$, being c_m the damping coefficient in the floating structure, while c_i, with $i = 1, \dots, n$, correspond to the damping coefficients in each fictitious joint. The elastic constants matrix obey to the same formation rule of the damping coefficients matrix, so the same algorithm is used, only changing C for K, c_m for k_m and c_i for k_i. The algorithm expressed in Eq. (12) determines the elements of the damping coefficients matrix.

For the Coriolis-centripetal vector, the first element has a different generation from the others, as shown in Eq. (13). The other elements $(f_2, \dots, f_{n+1})$ are obtained from the algorithm expressed in Eq. (14). In this equation, β is a parameter depending on the values for i and j, in the form shown in Eq. (15).

For the gravitational forces vector, the first component is null, as expressed in Eq. (16). It is related to the mass of the floating structure and is null because its weight equals its upthrust. The other components are determined from the algorithm shown in Eq. (17).

$$
\begin{array}{l}
for\ \ i = 1{:}n+1,\\
\quad for\ \ j = 1{:}n+1,\\
\quad\quad if\ i = 1,\\
\quad\quad\quad if\ j = i,\\
\quad\quad\quad\quad I(i,j) = M + m_c + \sum_{k=1}^{n} m_k\ ;\\
\quad\quad\quad end\ if,\\
\quad\quad\quad if\ j > i,\\
\quad\quad\quad\quad I(i,j) = -\left(\frac{m_{j-1}}{2} + m_c + \sum_{k=1}^{n} m_k\right) l_{j-1}\sin(\theta_{j-1})\ ;\\
\quad\quad\quad end\ if,\\
\quad\quad else\\
\quad\quad\quad if\ j = i,\\
\quad\quad\quad\quad I(i,j) = \left(\frac{m_{i-1}}{4} + m_c + \sum_{k=1}^{n} m_k\right) l_{i-1}^2 + I_{ri}\ ;\\
\quad\quad\quad end\ if,\\
\quad\quad\quad if\ j > i,\\
\quad\quad\quad\quad I(i,j) = \left(\frac{m_i}{2} + m_c + \sum_{k=1}^{n} m_k\right) l_{i-1} l_{j-1}\cos(\theta_{i-1} - \theta_{j-1})\ ;\\
\quad\quad\quad end\ if,\\
\quad\quad end\ if,\\
\quad\quad I(j,i) = I(i,j);\\
\quad end,\\
end,
\end{array}
\tag{11}
$$

$$
\begin{array}{l}
for\ i = 1{:}n,\\
\quad for\ j = 1{:}n,\\
\quad\quad a(i,j) = 0;\\
\quad\quad if\ \ i = j\ \ and\ \ i < n,\\
\quad\quad\quad a(i,j) = c_i + c_{i+1};\\
\quad\quad end\ if,\\
\quad\quad if\ \ i = j\ \ and\ \ i = n,\\
\quad\quad\quad a(i,j) = c_i;\\
\quad\quad end\ if,\\
\quad\quad if\ \ j = i+1,\\
\quad\quad\quad a(i,j) = -c_j;\\
\quad\quad\quad a(j,i) = a(i,j);\\
\quad\quad end\ if,\\
\quad end,\\
end,\\
for\ i = 1{:}n+1,\\
\quad for\ j = 1{:}n+1,\\
\quad\quad C(i,j) = 0;\\
\quad\quad if\ \ i = 1\ \ and\ \ j = 1,\\
\quad\quad\quad C(i,j) = c_m;\\
\quad\quad end\ if,\\
\quad\quad if\ \ i > 1\ \ and\ \ j > 1,\\
\quad\quad\quad C(i,j) = a(i-1, j-1);\\
\quad\quad end\ if,\\
\quad end,\\
end,
\end{array}
\tag{12}
$$

$$
f_1 = -\sum_{k=1}^{n}\left[\left(\frac{m_k}{2} + m_c + \sum_{g=k+1}^{n} m_g\right) l_k \cos(\theta_k)\,\dot{\theta}_k^2\right]
\tag{13}
$$

$for\ i = 1{:}n,$

$$f_{i+1} = l_i \sum_{\substack{j=1 \\ j\neq i}}^{n} \left[\beta l_j \begin{pmatrix} \frac{m_{\max(i,j)}}{2} + m_c + \\ + \sum_{g=\max(i,j)+1}^{n} m_g \end{pmatrix} \begin{pmatrix} -\frac{1}{2} sign(\beta)\cos(\theta_i)\sin(\theta_j) + \\ + \frac{1}{\beta}\sin(\theta_i)\cos(\theta_j) \end{pmatrix} \dot{\theta}_j^2 \right]; \tag{14}$$

$end,$

$if\ i > j,$
$\quad \beta = 2;$
$else\ if\ i < j,$ (15)
$\quad \beta = -2;$
$end\ if,$

$$G(1) = 0 \tag{16}$$

$for\ i = 2{:}n+1,$

$$G(i) = \left(\frac{m_{i-1}}{2} + m_c + \sum_{k=i}^{n} m_k \right) g l_{i-1} \sin(\theta_{i-1}); \tag{17}$$

$end,$

4. SIMULATION RESULTS

The simulations were performed considering the two cases previously nominated: cable articulated to the platform in one ending and free on the other; articulated to the platform on one ending and fixed to the ground on the other. The dynamic model is the same. The difference is that on the second case, where the cable is considered fixed to the ground, the terminal loading mass is treated as being huge (considered as $m_c = 8 \times 10^8\ kg$) and its upthrust equal to its weight. The parameters adopted for the simulations performing were the following:

$l = 240\ m$ (cable length);
$l_i = l/n$ (length for each link);
$r_i = 0.01m$ (radius of the link, considered constant on all its extension);
$\rho_c = 7850\ \ kg/m^3$ (specific mass of the cable);
$m_c = 60\ kg$ (mass of the terminal loading);
$m_c = 8 \times 10^8\ kg$ (mass of the terminal loading when the cable is fixed to the ground);
$M = 6000\ kg$ (mass of the platform);
$c_m = 237.6\ Ns/m$ (damping coefficient of the platform);
$c_i = 2.8\ Ns/rd$ (damping coefficient of each fictitious joint);
$k_i = 104.7\ N/rd$ (elastic constant of each fictitious joint);
$k_m = 1200\ N/m$ (elastic constant of the platform);
$\gamma_i = 200\ Ns^2/(rd\ m)^2$ (hydrodynamic drag coefficient).

The simulation results showed here are in the form of a sequence of frames, related to the spatial configuration of the cable. 60 and 80 links were considered, to represent the continuous flexibility ($n = 60$ and $n = 80$).

The first simulation consists on the cable divided in 60 links, loose on its free ending, from a spatial initial configuration, leaving the initial state of rest to a free fall situation. The cable is subjected to hydrodynamic drag and upthrust, as well as g-force, to simulate it underwater. The Fig. 4 illustrates the movement for the cable in this simulation, considering a time of $30s$ for it and frames shown each $2s$.

The second simulation consists on the cable divided in 80 links and fixed to the ground on its terminal loading. The cable is also subjected to hydrodynamic drag and upthrust, as well as g-force, to simulate it underwater. The Fig. 5 illustrates the movement for the cable in this simulation, considering a time of $15s$ for it and frames shown each $1s$.

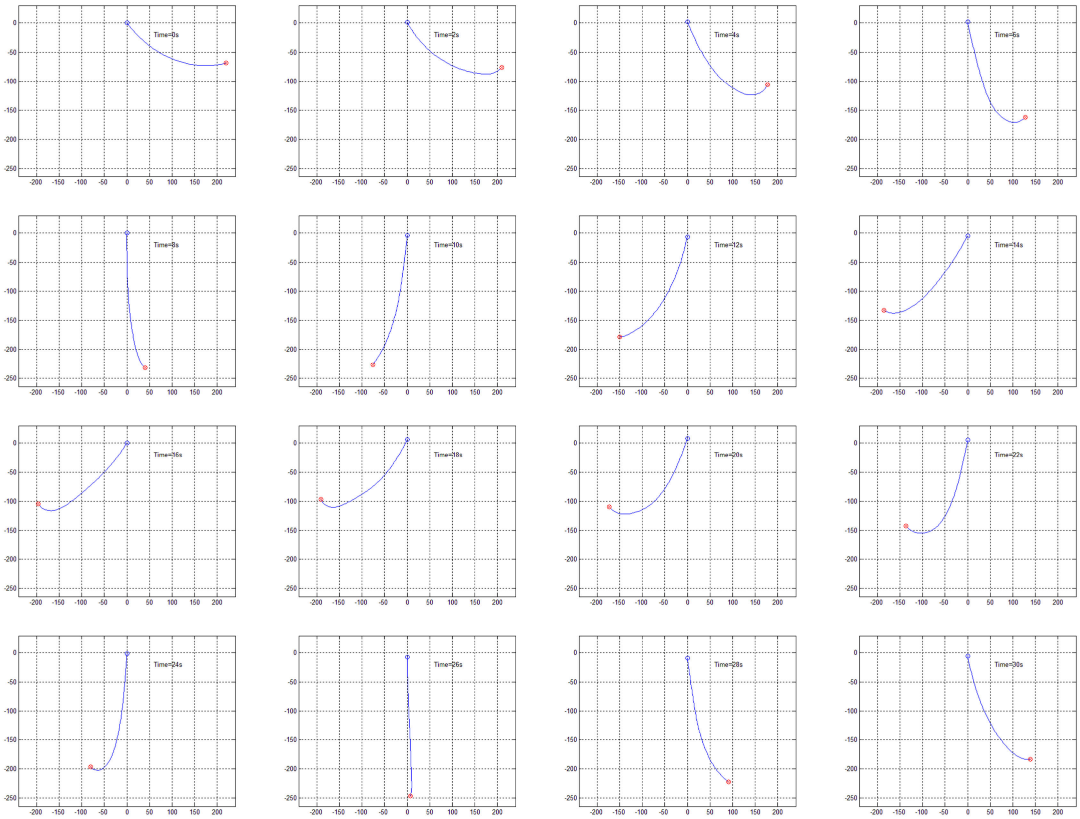

Figure 4. First simulation - cable divided in 60 links, loose on its second ending, 30s and frames each 2s

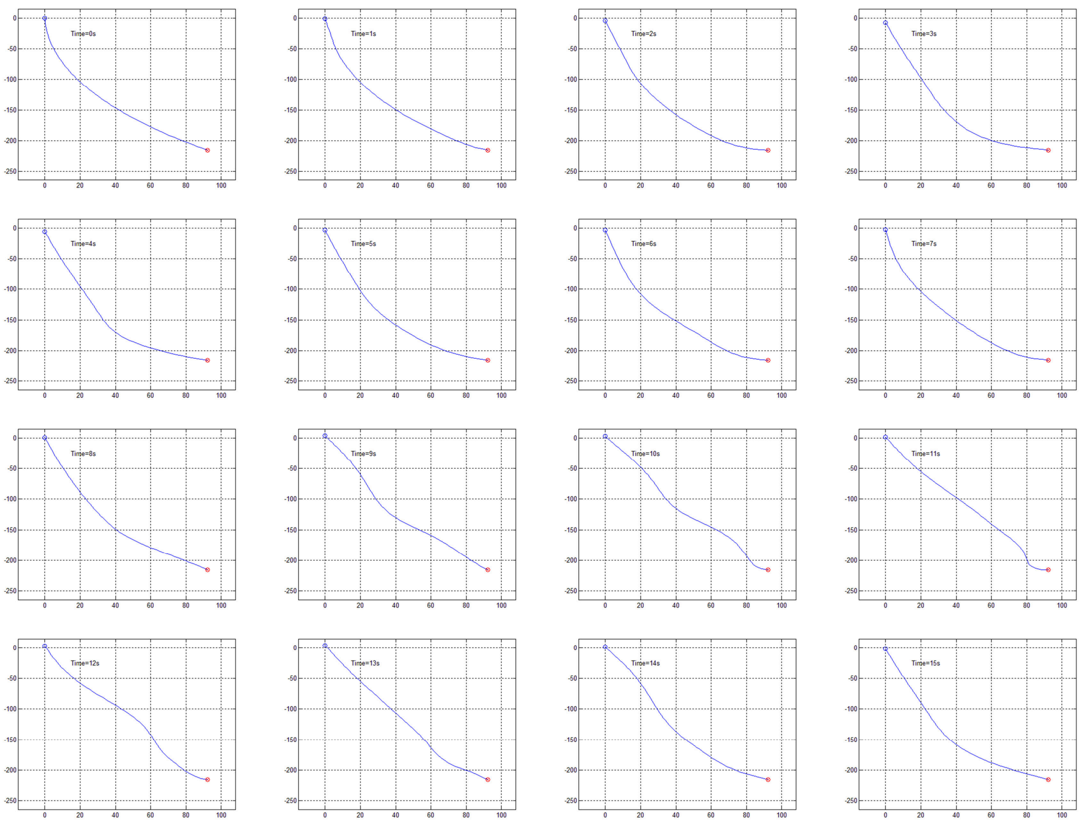

Figure 5. Second simulation - cable divided in 80 links, terminal loading fixed to the ground, 15s and frames each 1s.

5. CONCLUSIONS

The developed dynamic models showed simulation results in accordance with the physical expected, in both cases, i.e. the cable with free terminal loading and the cable fixed to the ground. This last one is similar to applications related to anchoring cables or mooring lines used in platforms or ships, which constitute important underwater applications. The proposed generic algorithms allow the automatic generation of dynamic models considering any number of links to approximate the continuous cable's flexibility. This is the most important contribution of this article. It was developed a software for graphic animation, which allowed to show that the results of simulations add a great sense of physical reality. The next phase of research will be to expand the modeling formalism to the 3D space as well as to include cable's terminal loads with own dynamics, as a ROV, for example.

6. REFERENCES

Bamdad, M. "Analytical Dynamic Solution of a Flexible Cable-suspended Manipulator". In: *Front. Mech. Eng.,* v. 8(4), p. 350-359, May 2013.

Bi, G.; Zhu, S.; Liu, J.; Fang, X.; Wang, L. "Dynamic Simulation and Tension Compensation Research on Subsea Umbilical Cable Laying System". In: *J. Marine Sci. Appl.*, v. 12, p. 452-458, September 2013.

Chang, S.; Park, J.; Lee, K. "Nonlinear Dynamic Analysis of Spatially Suspended Elastic Catenary Cable with Finite Element Method". In: *KSCE Journal of Civil Engineering*, v. 12, no. 2, p. 121-128, March 2008.

Chatjigeorgiou, I. K.; Mavrakos, S. A. "The 3D Nonlinear Dynamics of Catenary Slender Structures for Marine Applications". In: *Nonlinear Dynamics. Croatia: INTECH*, January 2010. 366 p. ISBN 978-953-7619-61-9.

Escalante, M. R.; Sampaio, R.; Rosales, M. B.; Ritto, T. "A Reduced Order Model of a 3D Cable Using Proper Orthogonal Decomposition". Rosario, Argentina. In: *Mecánica Computacional*, v. XXX, p. 1143-1158, Nov. 2011.

Fang, Z.; He, Q.; Xiang, B.; Xiao, H.; He, K.; Du, Y. "A Finite Element Cable Model and Its Applications Based on the Cubic Spline Curve". In: *China Ocean Eng.*, v. 27, n. 5, p. 683-692, December 2012.

Gobat, J. I.; Grosenbaugh, M. A. "Application of the Generalized-α Method to the Time Integration of the Cable Dynamics Equations". In: *Computer Methods in Applied Mechanics and Engineering*, v. 190, Issues 37-38, p. 4817-4829, June 2001.

Grosenbaugh, M. A.; Howell, C. T.; Moxnes, S. "Simulating the Dynamics of Underwater Vehicles with Low-tension Tethers". In: *International Journal of Offshore and Polar Engineering*, v. 3, no. 3, ISSN 1053-5381, September 1993.

Guo, A.; Xu, Y.; Li, H. "Dynamic Performance of Cable-stayed Bridge Tower with Multi-stage Pendulum Mass Damper Underwind Excitations – I: Theory". In: *Earthquake Engineering and Engineering Vibration*, v. 6, Issue 3, p. 295-306, September 2007.

Liu, Y.; Qian, Z.; Xia, K.; "Mechanical Response of Transmission Lines Based on Sliding Cable Element". In: J. Cent. South Univ., v. 21, p. 3370-3377, August 2013.

Luongo, A.; Zulli, D.; Piccardo, G. "Analytical and Numerical Approaches to Nonlinear Galloping of Internally-resonant Suspended Cables". In: *Journal of Sound and Vibration, Elsevier*, 315(3), p. 375-393, 2008.

Masciola, M. D.; Nahon, M.; Driscoll, F. R. "Static Analysis of the Lumped Mass Cable Model Using a Shooting Algorithm". In: *J. Waterway, Port, Coastal, Ocean Eng.*, v. 138(2), p. 164-171, March-April 2012.

Nahon, M. "Dynamics and Control of a Novel Radio Telescope Antenna". In: *The American Institute of Aeronautics and Astronautics*, AIAA-99-4120, p. 214-222, 1999.

Pathote, P. "Efficient Mathematical Model for Prestressing Cables and Its Implementation in Computer Analysis". In: *IOSR Journal of Mathematics (IOSR- JM)*, e-ISSN: 2278-5728, p-ISSN: 2319-765X, v. 10, Issue 3, Ver. V, p. 32-38, May-June 2014.

Pereira, A. E. L. *O Método da Decomposição de Adomian Aplicado à Interação Fluido-estrutura de um Cabo*. Ph.D. thesis, Federal University of Rio Grande do Sul, Porto Alegre.

Seo, Y.; Jeong, U.; Cho, J. "Simplified Finite Element Dynamic Analysis of Cable-supported Structures Subjected to Seismic Motions". In: *KSCE Journal of Civil Engineering*, v. 14(4), p. 579-587, 2010.

Srivastava, V. K.; Yvss, S.; Tamsir, M. "Dynamic Behavior of Underwater Towed-cable in Linear Profile". 2011. In: *International Journal of Scientific & Engineering Research*, v. 2, Issue 7, ISSN 2229-5518, July 2011.

Vaz, M. A.; Patel, M. H. "Transient Behavior of Towed Marine Cables in Two Dimensions". In: *Applied Ocean Research, Elsevier*, v. 17(1995), p. 143-153, August 1995.

Wolfschluckner, A.; Jodin, D. "Modelling Cable Dynamics Exemplified by Load Transposition of Insulator Strings on Overhead Lines". In: *Proceedings in Manufacturing Systems*, v. 8, Issue 3, ISSN 2067-9238, 2013.

Zanela, E. B. *Modelagem Analítica de Estruturas do Tipo Cabo para Aplicações Subaquáticas*. Master's dissertation, Federal University of Rio Grande, Rio Grande.

Zhao, W.; Chen, L.; Jean, W. Z. "A Finite Difference Method for Simulating Transverse Vibrations of an Axially Moving Viscoelastic String". In: *Applied Mathematics and Mechanics (English Edition), Editorial Committee of Appl. Math. Mech.*, ISSN 0253-4827, 27(1), p. 23-28, September 2005.

7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.