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VORTICAL STRUCTURES: SOME ASPECTS OF THE RECENT IDENTIFICATION SCHEMES

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Abstract. *The contribution deals with the recent vortex-identification schemes (proposed by the present authors) compared on the background of some widely used methods from the literature. It concentrates on their physical and kinematic aspects and presents some comparison results based on a straightforward application of these schemes to the 3-D outcome of numerical simulation (including a transonic flow past a turbine cascade and a flow around an impulsively started flat plate). The main aspects of vortex identification discussed are: the local corotation of line segments near a point, vorticity decomposition and the so-called residual vorticity, the effect of shear, the role of the axial-stretching rate, pressure aspects, applicability of identification schemes to compressible flows, the need for a local vortex intensity with an easy kinematic interpretation and linear response, and limited applicability of integral vortex strength in terms of the circulation along the vortex boundary.*

Keywords: *vortical structures, vortex identification, vortex-identification schemes, decomposition of motion, corotation*

1. INTRODUCTION

An extensive discussion of different vortex-identification methods can be found in the literature focusing both on region-type methods (e.g. Jeong and Hussain, 1995; Chakraborty *et al.*, 2005; Jiang *et al.*, 2005; Kolář, 2007), line-type methods (Roth, 2000, review and analysis), and various fundamental issues like material objectivity and directional tendencies of stretching material elements (Haller, 2005; Thompson *et al.*, 2009; Bacchi and Thompson, 2010). Though many vortex-identification schemes have proved (more or less successfully) to be useful for the identification of vortical structures in transitional and turbulent flows, the longstanding identification problem is not fully resolved and therefore continuously attracts attention. Anyhow, some of these methods have become popular and are widely used nowadays. The present paper aims at an advanced discussion on vortex identification, focusing especially on the recently proposed easy-to-interpret kinematic (corotational) schemes and their comparison with the widely used criteria Q, λ_2 and λ_{ci}. Two corotational methods proposed by the present authors are discussed: (i) the average-corotation method (Kolář et al., 2013) and (ii) the Q_{M}-criterion (Kolář and Šístek, 2015) representing a modification of the Q-criterion. All considered methods are pointwise region-type vortex-identification methods sharing a basis in the velocity-gradient tensor $\nabla\mathbf{u}$ as briefly summarized in the Appendix.

Regarding widely used discriminative criteria Q, λ_2 and λ_{ci} determining 3-D vortex regions, there is a natural need to interpret their criterial quantities Q, λ_2 and λ_{ci} as local (i.e. near a point) vortex intensities *inside* a vortex region. Both Q and λ_2 behave, even for the simplest 2-D case of local rigid-body rotation of a fluid element, in a nonlinear (quadratic) manner with respect to the local angular velocity of rigid-body rotation of a fluid element. Moreover, not much kinematic insight can be revealed in 2D as well as in 3D for Q and λ_2. On the contrary, the quantity λ_{ci} is related to a local frequency of revolving streamlines and, considered as a local vortex intensity, is therefore more promising due to its physical interpretation and kinematic insight. The criteria Q and λ_2 are applicable to incompressible flows only, while the λ_{ci}-criterion is extendable to compressible flows. It should be emphasized in this context that the new easy-to-interpret kinematic corotational schemes discussed below are applicable to compressible flows as their physical reasoning — corotation of material line segments near a point — relates to divergence-free (deviatoric) part of motion.

The next section deals with the recently proposed corotational schemes. The application of the average-corotation scheme and the Q_{M}-criterion to the outcome of numerical simulation is performed in direct comparison with the widely used criteria Q, λ_2 and λ_{ci} in this section. Relevant vortex-identification aspects are treated in a later section (vortex stretching, compressibility, pressure aspects, and integral vortex strength).

2. LOCAL COROTATION OF MATERIAL LINE SEGMENTS AND VORTEX IDENTIFICATION

The triple-decomposition method (Kolář, 2007) offers a certain qualitative "comeback" of vorticity to vortex identification: specific portion of vorticity labelled *residual* vorticity, obtained after the elimination of a local pure shearing motion near a point, represents a local vortex intensity. For an arbitrary planar cross section of a 3-D flow, the *residual* vorticity permits a straightforward kinematic interpretation in terms of the least-absolute-value angular velocity of all (infinitesimal) material line segments going through the examined point as described in Fig. 1 with the help of extremal angular velocities $\boldsymbol{\Omega}_{\mathbf{HIGH}}$ and $\boldsymbol{\Omega}_{\mathbf{LOW}}$. Note that vorticity itself, expressing an average angular velocity of fluid elements, is represented in Fig. 1 by the average angular velocity $\boldsymbol{\Omega}_{\mathbf{AVERAGE}}$ (which can be assigned to a specific line segment). The least-absolute-value angular velocity is just a corotation measure, it turns out to be zero for the contrarotation case where deviatoric strain rate dominates over vorticity.

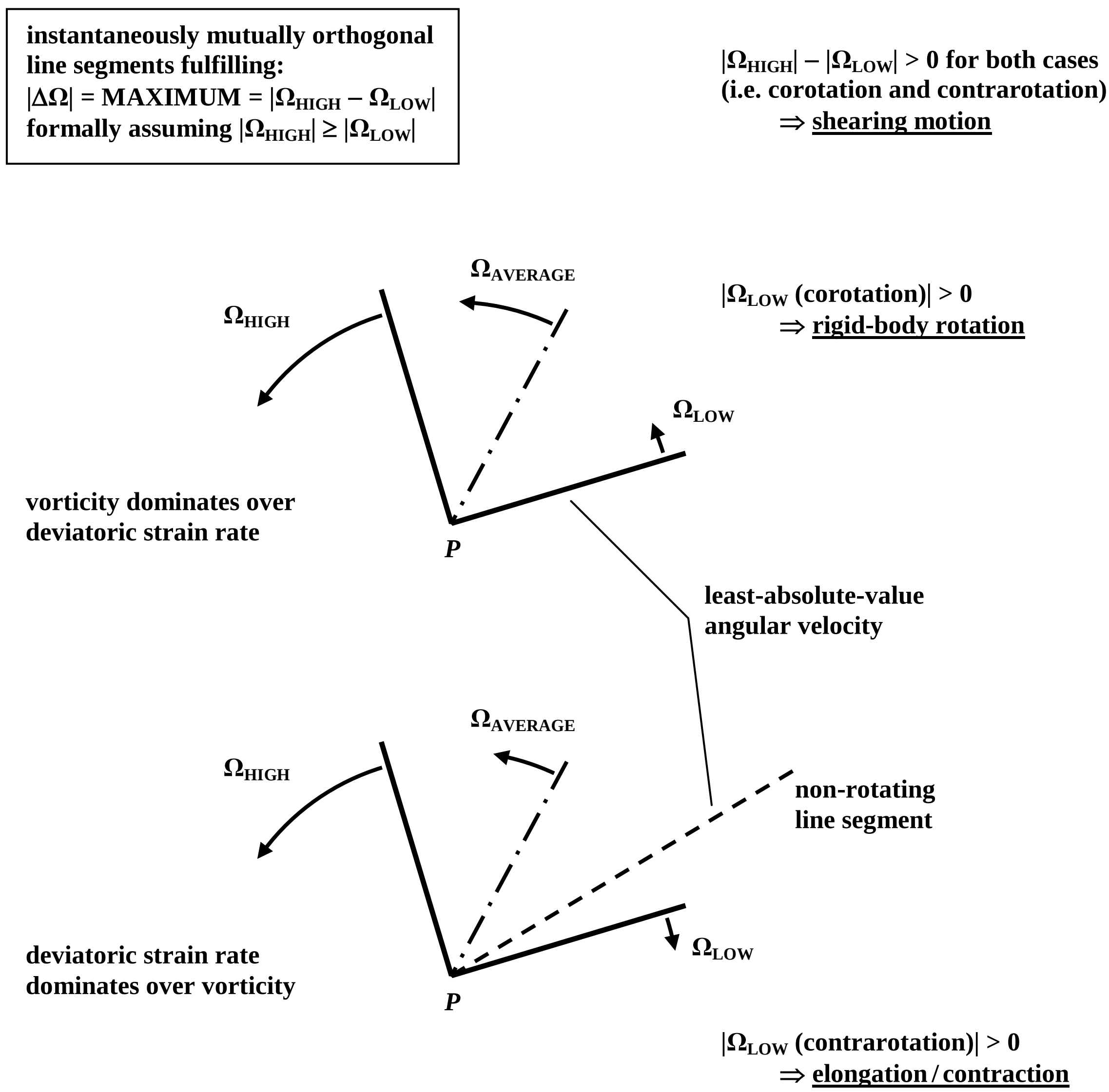

Figure 1. Corotation of line segments: The *residual* vorticity in terms of the least-absolute-value angular velocity.

The planar corotation of line segments has become a basis for the concept of two recent "corotational" methods presented here with appplications. The first one is the average-corotation method (averaging process is applied to all planar cross sections going through the examined point to determine the average-corotation vector $\boldsymbol{\omega}_{\mathrm{RAVG}}$) and the second one represents nothing but a modification of the Q-criterion (the criterial quantity Q_{M} is inferred from the corotational state in principal planes, though the quadratic nature of Q-criterion remains), for details see the Appendix.

Let us have a look at an application of the two corotational methods (with criterial quantities $|\boldsymbol{\omega}_{\mathrm{RAVG}}|$ and Q_{M}) in comparison with the criteria Q, λ_2 and λ_{ci}. In this comparison, in order to make it as objective as possible, the standard λ_2-criterion provides reference results for the following matching procedure described in Kolář *et al.* (2013). After choosing a sufficiently low threshold level for the “reference criterion” λ_2, we determine the corresponding threshold values for the other four methods by minimizing the characteristic volumetric ratio V_N/V_O over all possible thresholds. Here V_N corresponds to the volume where methods do not overlap in terms of vortex regions (i.e. one method detects a vortex while the other does not) and V_O denotes the overlapping volume (i.e. the volume representing a vortex according to both methods). Two different flow situations are examined: a 3-D transonic flow past a turbine cascade, and a flow around an impulsively started flat plate at an angle of attack.

3-D transonic flow past a turbine cascade: the examined dataset deals with a 3-D transonic flow past a prismatic turbine cascade with Ma = 1.2 at the blade cascade outlet and $Re = 1.34 \cdot 10^6$ according to Šimurda *et al.* (2013). Only one half of the channel is considered due to spanwise symmetry, and blade-to-blade periodicity is assumed. The obtained results are in Fig. 2, conventional methods on the first line, corotational methods on the second line. Figure 2 shows a complex vortex structure: the dominant passage vortex, the associated horseshoe vortex, and the trailing vortex. The following conclusion can be drawn: the identification results of corotational methods are much less noisy near the surfaces, more specifically, much less biased by shear than the results of the criteria Q, λ_2 and λ_{ci}. Also note that the criteria Q and Q_{M} are influenced in a similar manner by the existence of shock waves as the tubular vortices are broken into pieces just at the shock-wave locations (verified independently in terms of the trace of $\nabla\mathbf{u}$, see Kolář and Šístek, 2015), while λ_2, λ_{ci} and $|\boldsymbol{\omega}_{\mathrm{RAVG}}|$ seem much less sensitive in this regard.

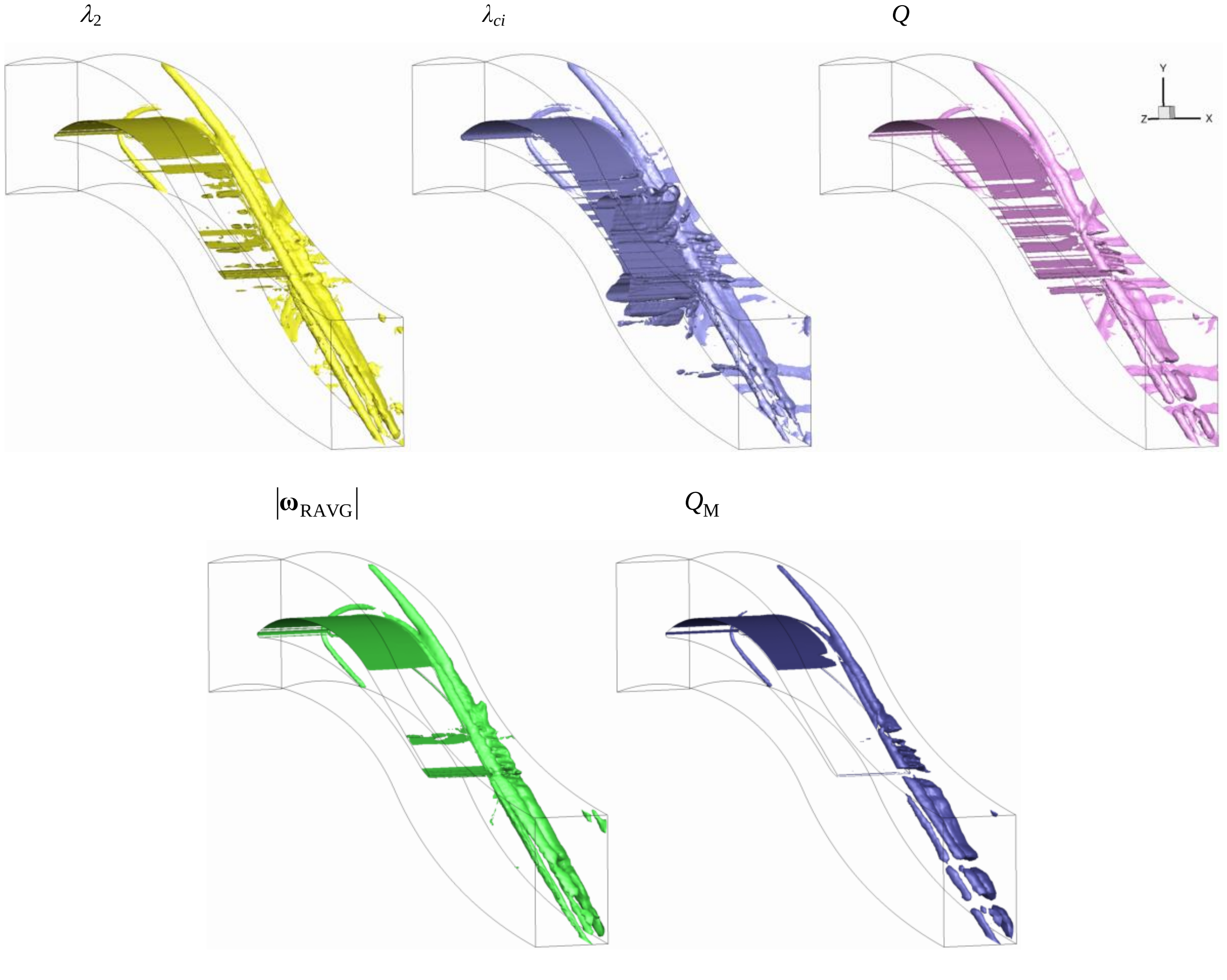

Figure 2. Transonic flow past a turbine cascade.

Flow around an impulsively started flat plate at an angle of attack: the investigated datasets describe the impulsively started incompressible flow around a flat plate (aspect ratio 2) at an angle of attack of 30 deg solved numerically for Reynolds numbers Re = 300 and Re = 1200. The obtained results are shown in Fig. 3 (Re = 300) and Fig. 4 (Re = 1200). From here one can conclude that — unlike corotational methods — the methods Q, λ_2 and λ_{ci} fail to reasonably correctly interpret the high shear zones of a very near wake (behind plate edges), which are interpreted in a biased manner as vortices. Both corotational schemes more adequately capture the vortical structures both in close proximity of the flat plate and in the wake downstream.

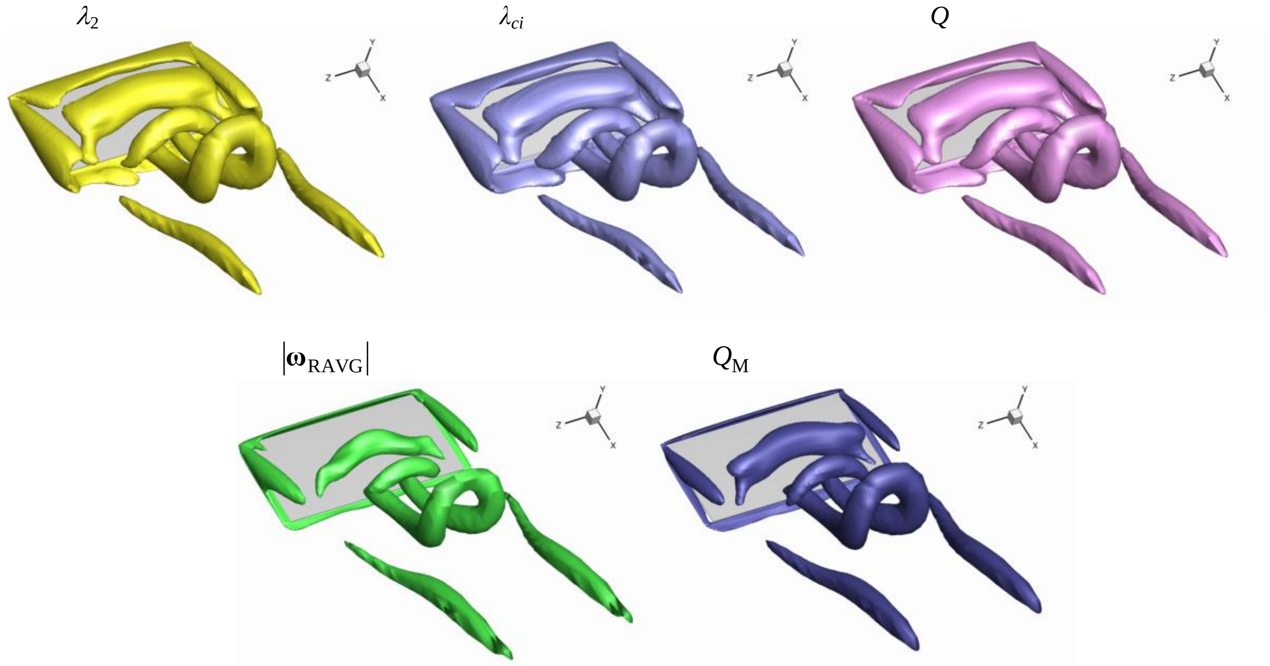

Figure 3. Flow around an impulsively started flat plate for *Re* = 300.

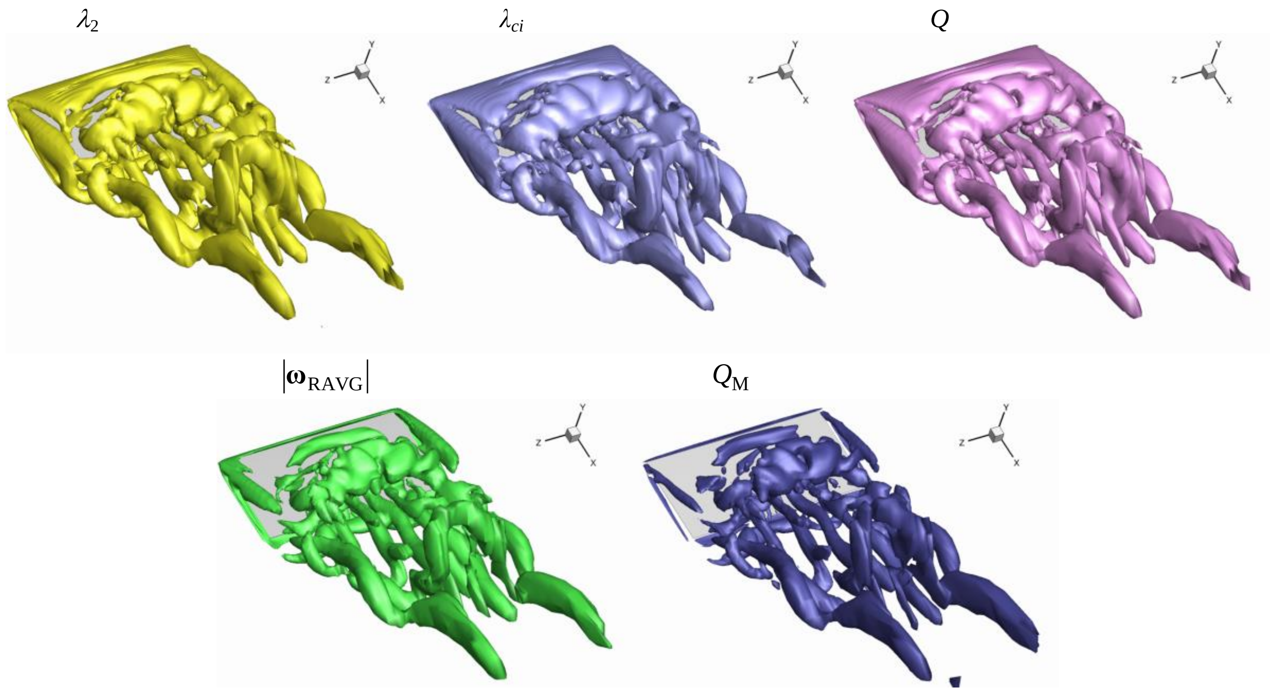

Figure 4. Flow around an impulsively started flat plate for *Re* = 1200.

Let us add a few remarks. Interestingly, the λ_{ci}-criterion with a straightforward physical interpretation somewhat fails in the examined flow cases. Comparing the corotational schemes, $|\boldsymbol{\omega}_{\mathrm{RAVG}}|$ vs. Q_{M}, the results are quite similar. The quantity Q_{M} is easy to evaluate, while the average corotation $\boldsymbol{\omega}_{\mathrm{RAVG}}$ is a more informative vectorial quantity.

3. VORTEX STRETCHING, COMPRESSIBILITY, PRESSURE, AND VORTEX STRENGTH

The sensitivity of vortex-identification outcome towards the axial stretching rate in the direction perpendicular to the "plane of swirling" has been already studied in the literature. The allowance for an arbitrary axial stretching rate of Wu *et al.* (2005) has become a subject of debate as this requirement does not conform to the orbital compactness proposed in Chakraborty *et al.* (2005). Here we limit to the simple pointwise tensor analysis of the employed schemes.

Let us consider the local uniaxial isochoric stretching (with uniform radial contraction) coupled with the vorticity vector aligned with the stretching axis. Figure 5 shows the results for the criterial quantities (for comparison purposes normalized by the stretching-free value) discussed in this paper, including the residual vorticity tensor, see Appendix.

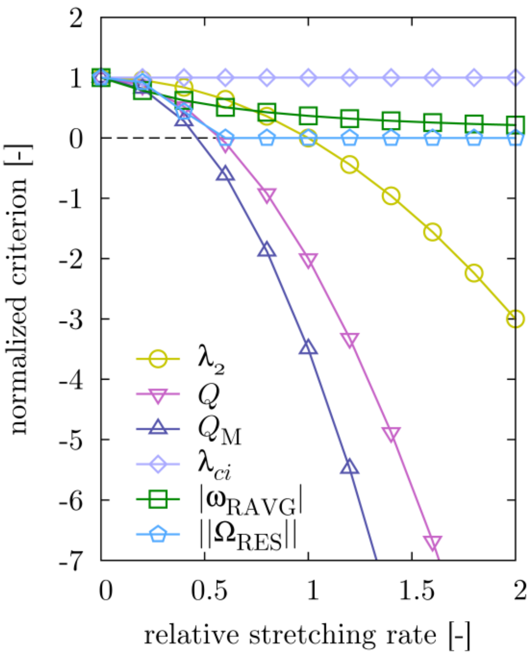

Figure 5. Dependence of various vortex-identification criteria on relative stretching rate.

Only positive values in Fig. 5 identify a vortex region. Figure 5 clearly illustrates that different criteria behave in a quite different manner, from the stretching-insensitive one (λ_{ci}) to the most stretching-sensitive ones (Q and Q_M) — the similar behavior of Q and Q_M at the shock-wave locations where the vortices are broken into pieces can be recalled.

Compressibility presents another aspect which is worth paying attention. Some criteria, though not all, can be applied to compressible flows. It has been shown by Kolář (2009) that from the most popular schemes (Q, Δ, λ_2, and λ_{ci}) only the Δ-criterion and the associated λ_{ci}-criterion are directly extendable to compressible flows. Both corotational methods relate to divergence-free (deviatoric) part of motion and, therefore, are applicable to compressible flows. The residual vorticity is not affected by a non-zero divergence of $\nabla\mathbf{u}$ as the behavior of off-diagonal elements is decisive.

The quantity Q is applicable to incompressible flows only, as discussed in detail in Kolář and Šístek (2015), and provides a direct relation to pressure expressed through the right-hand side of the Poisson equation for pressure, $\nabla^2 p = 2\rho Q$, and can be interpreted as the source term in the Navier-Stokes equations (Jeong and Hussain, 1995). On the contrary, the quantity Q_M has a less direct relation to pressure by substituting Q by Q_M from (A.2) in the Poisson equation for pressure. However, the quantity Q_M is directly applicable to compressible flows as mentioned above.

Finally, it should be emphasized that the term "vortex" and "vortical structure" is more general and not necessarily coinciding with a vorticity tube (a portion of fluid limited by the vorticity lines), the integral strength of which can be characterized by the circulation Γ, the key quantity related to Helmholtz's theorems and Kelvin's circulation theorem. Due to Green's theorem, the strength of the vorticity tube is frequently determined as the surface integral of vorticity over the planar cross section. The use of this basic fluid-mechanical integral characteristic as a vortex strength (in a real flow) may be more or less misleading. Vorticity cannot distinguish between shearing motion and swirling motion, hence it is biased for characterizing the strength of swirling motion, that is, the strength of a vortex. However, the *residual* vorticity of any planar cross section of a vortex can be integrated (inside a vortex region) to provide an unbiased vortex strength in terms of the "residual circulation" Γ_{RES}.

4. CONCLUSIONS

Recently proposed corotational vortex-identification schemes $|\boldsymbol{\omega}_{RAVG}|$ and Q_M have been applied and discussed on the background of widely used criteria Q, λ_2 and λ_{ci}. The corotational criteria are of an easy-to-follow kinematic interpretation with direct applicability to compressible flows. Two different flow situations have been examined: a 3-D transonic flow past a turbine cascade, and a flow around an impulsively started flat plate at an angle of attack. Relevant aspects like vortex stretching, compressibility, pressure aspects, and integral vortex strength have been treated as well.

5. APPENDIX: VORTEX-IDENTIFICATION METHODS (DISCUSSED IN THIS PAPER)

Q-criterion (Hunt *et al.*, 1988): Vortices of an *incompressible* flow are identified as connected fluid regions with a positive second invariant of the velocity-gradient tensor $\nabla\mathbf{u}$, $\nabla\mathbf{u} = \mathbf{S} + \mathbf{\Omega}$, $\mathbf{S}$ is the strain-rate tensor, $\mathbf{\Omega}$ is the vorticity tensor (in tensor notation below, the subscript comma denotes differentiation),

$$Q = -\frac{1}{2} u_{i,j} u_{j,i} = \frac{1}{2}\left(\|\mathbf{\Omega}\|^2 - \|\mathbf{S}\|^2\right) > 0 . \tag{A.1}$$

This is fulfilled in the regions where the vorticity magnitude prevails over the strain-rate magnitude. The additional pressure condition (Hunt *et al.*, 1988) requiring that the pressure tends to a minimum inside the region $Q > 0$ is arguable (Jeong and Hussain, 1995; Chakraborty *et al.*, 2005; Cucitore *et al.*, 1999) and usually omitted.

Modification of the *Q*-criterion (Kolář and Šístek, 2015): This new criterion — modification of the *Q*-criterion — is based on corotational and compressibility arguments. The criterion employs a modified criterial quantity Q_{M} given by

$$Q_{\mathrm{M}} = Q + \mathrm{II}_{\mathrm{S}} / 2 = Q_{\mathrm{D}} + \mathrm{II}_{\mathrm{SD}} / 2 , \tag{A.2}$$

where II_{S} denotes the second invariant of the strain-rate tensor $\mathbf{S}$, and the same holds for deviatoric quantitites (denoted by the subscript D). The quantity Q_{M} is closely associated with the corotational state of infinitesimal line segments near a point in principal planes. For further details see Kolář and Šístek (2015).

λ_2-criterion (Jeong and Hussain, 1995): This criterion is formulated on dynamic considerations, namely on the search for a pressure minimum across the vortex. The quantity $\mathbf{S}^2 + \mathbf{\Omega}^2$ is employed as an approximation of the pressure Hessian after removing the unsteady irrotational straining and viscous effects from the strain-rate transport equation for *incompressible* fluids. A vortex region is defined as a connected fluid region with two negative eigenvalues of $\mathbf{S}^2 + \mathbf{\Omega}^2$, that is, if the eigenvalues are ordered, $\lambda_1 \geq \lambda_2 \geq \lambda_3$, by the condition $\lambda_2 < 0$.

λ_{ci}-criterion (Zhou *et al.*, 1999; Chakraborty *et al.*, 2005): This scheme is based on the Δ-criterion (Dallmann, 1983; Vollmers *et al.*, 1983; Chong *et al.*, 1990), defining vortices as the regions in which the eigenvalues of $\nabla\mathbf{u}$ are complex. The Δ-criterion, using discriminant of the characteristic equation for the eigenvalues of $\nabla\mathbf{u}$, has motivated the so-called swirling-strength criterion. This criterion, denoted as λ_{ci}-criterion, is based directly on the imaginary part of the complex conjugate eigenvalues of $\nabla\mathbf{u}$ written as $\lambda_{cr} \pm \mathrm{i}\lambda_{ci}$. The criteria Δ and λ_{ci} are equivalent only for zero thresholds ($\Delta=0$ and $\lambda_{ci}=0$). The time period for completing one revolution of the streamline on the plane spanned by the complex eigenvectors is given by $2\pi/\lambda_{ci}$.

Residual vorticity (Kolář, 2007): This criterial quantity is associated with the triple-decomposition method (TDM). TDM is expressed through the corresponding triple decomposition of a local motion near a point. As a result, $\nabla\mathbf{u}$ consists, unlike the double decomposition $\nabla\mathbf{u} = \mathbf{S} + \mathbf{\Omega}$, of three parts so that the strain-rate tensor $\mathbf{S}$ and vorticity tensor $\mathbf{\Omega}$ are cut down in magnitudes to share their portions through the third term $(\nabla\mathbf{u})_{\mathrm{SH}}$ representing a local shearing motion. Consequently, in terms of the *residual* parts of $\mathbf{S}$ and $\mathbf{\Omega}$ it reads

$$\nabla\mathbf{u} = \mathbf{S}_{\mathrm{RES}} + \mathbf{\Omega}_{\mathrm{RES}} + (\nabla\mathbf{u})_{\mathrm{SH}} . \tag{A.3}$$

The first term on the right-hand side of (A.3) stands for an irrotational straining, the second one represents a rigid-body rotation. The third term of the triple decomposition $(\nabla\mathbf{u})_{\mathrm{SH}}$ is described by a purely asymmetric tensor form — labelled *shear* tensor — fulfilling in a suitable reference frame (again, the subscript comma below denotes differentiation)

$$u_{i,j} = 0 \quad OR \quad u_{j,i} = 0 \quad \text{(for all } i, j) . \tag{A.4}$$

From the viewpoint of the double decomposition, $\nabla\mathbf{u} = \mathbf{S} + \mathbf{\Omega}$, the extracted term $(\nabla\mathbf{u})_{\mathrm{SH}}$ itself is responsible for a specific portion of vorticity and for a specific portion of strain rate.

The TDM is closely associated with the so-called “basic reference frame” (BRF). In this frame, the decomposition is performed in a clearly recognizable manner. However, the TDM results generated (i.e. separated) in the BRF are valid for all frames of reference rotated with respect to the BRF under an orthogonal transformation. The search for BRF, which makes the triple-decomposition method computationally expensive, presents an optimization problem for each point in the flow domain as briefly described in the following.

In the BRF: (i) the most effective shearing motion is described by the tensor form (A.4) under the defining condition that (ii) the effect of extraction of a *shear* tensor is maximized within the following decomposition scheme applicable to an arbitrary reference frame

$$\nabla \mathbf{u} \equiv \begin{pmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{pmatrix} = \begin{pmatrix} residual \\ tensor \end{pmatrix} + \begin{pmatrix} shear \\ tensor \end{pmatrix} \tag{A.5}$$

where the *residual* tensor is defined as

$$\begin{pmatrix} u_x & (\operatorname{sgn} u_y)\,\mathrm{MIN}(|u_y|,|v_x|) & \bullet \\ (\operatorname{sgn} v_x)\,\mathrm{MIN}(|u_y|,|v_x|) & v_y & \bullet \\ \bullet & \bullet & w_z \end{pmatrix}. \tag{A.6}$$

The following notation is used in (A.5) and (A.6): *u*, *v*, *w* are velocity components, subscripts *x*, *y*, *z* stand for partial derivatives. The remaining non-specified pairs of off-diagonal elements of the *residual* tensor in (A.6) are constructed analogously as the specified one, each pair — if considered separately — being either symmetric or antisymmetric. The effect of extraction of the *shear* tensor is maximized by changing the reference frame under an orthogonal transformation so that the absolute tensor value of the *residual* tensor is minimized, or the closely related scalar quantity $|S_{12}\Omega_{12}| + |S_{23}\Omega_{23}| + |S_{31}\Omega_{31}|$ is maximized (Kolář, 2007). This extremal condition guarantees that an effective shearing motion is recognized in the BRF as a third elementary part of the triple decomposition and can be extracted from $\nabla \mathbf{u}$ following (A.5) and (A.6). For details and the qualitative description of the flow kinematics near a point adopted in the TDM, see Kolář (2007).

The *residual* vorticity tensor $\mathbf{\Omega}_{\mathrm{RES}}$ representing a rigid-body rotation in 3D is assumed to provide an "unbiased shear-free measure" of the actual swirling motion of a vortex. Finally, according to TDM, non-zero magnitude (i.e. Frobenius norm) of $\mathbf{\Omega}_{\mathrm{RES}}$, $\|\mathbf{\Omega}_{\mathrm{RES}}\| > 0$, identifies the examined point as part of a vortex. Let us mention a few recent applications of the given method. An analysis of cross sections of vortices in turbulent flows using TDM is presented in Maciel *et al.* (2012), and another application of the planar residual vorticity has been recently described in Peltier *et al.* (2014). Some 3-D applications of this method can be found in Kolář and Šístek (2014).

<u>The average-corotation method (Kolář *et al.*, 2013):</u> The scheme is based on the two following steps: Firstly, the local corotation of material line segments at a point is defined on a 2-D planar cross section of a 3-D flow. This planar concept is directly related to the residual vorticity in 2D according to the above-mentioned TDM as the residual vorticity is a measure of local planar corotation of infinitesimal material line segments. Secondly, a proper averaging process is applied to all planar cross sections going through the given point. The average corotation is defined as a surface integral over a unit sphere expressed using the spherical coordinates *r*, φ, ϑ as

$$\boldsymbol{\omega}_{\mathrm{RAVG}}(\mathbf{x}_0) = \frac{\alpha}{4\pi}\int_0^{2\pi}\int_0^{\pi} \boldsymbol{\omega}_{\mathrm{RES}}(\mathbf{x}_0, \mathbf{n}(\varphi,\vartheta)) \sin\vartheta \,\mathrm{d}\vartheta\,\mathrm{d}\varphi. \tag{A.7}$$

Here, $\boldsymbol{\omega}_{\mathrm{RAVG}}$ denotes the average-corotation vector, $\boldsymbol{\omega}_{\mathrm{RES}}$ is the planar residual vorticity determined according to TDM and oriented as the plane normal **n**, and $\mathbf{x}_0$ denotes the examined point. A natural choice $\alpha = 3$ is derived from the requirement that average-corotation vector equals to vorticity vector, $\boldsymbol{\omega}_{\mathrm{RAVG}} = \boldsymbol{\omega}$, for a pure rotational motion $\nabla\mathbf{u} = \mathbf{\Omega}$. In our computations, the integral in (A.7) is approximated using the efficient numerical quadrature from Hannay and Nye (2004).

To evaluate the contribution of shearing motion to vorticity separately, the vector of average-shear vorticity $\boldsymbol{\omega}_{\mathrm{SAVG}}$ is given simply by the resulting formula

$$\boldsymbol{\omega}_{\mathrm{SAVG}} = \boldsymbol{\omega} - \boldsymbol{\omega}_{\mathrm{RAVG}} \tag{A.8}$$

Note that the expression (A.8) is obtainable by the analogous averaging procedure as $\boldsymbol{\omega}_{\mathrm{RAVG}}$.

The magnitude of the vector $\boldsymbol{\omega}_{\mathrm{RAVG}}$ can be employed in 3-D vortex identification as shown in Kolář *et al.* (2013) and Kolář and Šístek (2014) where $|\boldsymbol{\omega}_{\mathrm{RAVG}}|$ has been employed in a similar manner as presented here.

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