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PROBABILISTIC ANALISYS OF REDUNDANT SYSTEM BY THE IDENTIFICATION OF MULTIPLE FAILURE MODES

Adelano Esposito
Marcos Daniel de Freitas Awruch
Mechanical Engineering Department, Federal University of Rio Grande do Sul, Av. Sarmento Leite, 90050-170, Porto Alegre, Brazil
adeesp_@hotmail.com; marcos.dfa@gmail.com

Herbert Martins Gomes
Mechanical Engineering Department, Federal University of Rio Grande do Sul, Av. Sarmento Leite, 90050-170, Porto Alegre, Brazil
herbert@mecanica.ufrgs.br

Abstract. *The failure of structural system can be described by a set of interlinked failures or by the failure of independent components. So far, this type of problem has been handled in a simple fashion, not taking into account several existing failure modes, especially in redundant systems. The simplification of the real problem, overlooking some failure modes, may hinder the calculation of reliability. Bearing that in mind, a method for the probabilistic analysis of the system is proposed, using an improved version of the Hyperspace Division Method (HDM) platform, which allows the detection of multiple failure modes. Besides, the use of a chi-squared distribution allows for a directional integration procedure to calculate the failure probability of each mode in a more accurate manner than in conventional approximation and simulation methods. The key objectives of this study are: to identify the sequence of failure events that make up failure modes at ultimate and serviceability limit states in redundant structures by considering nonlinearities and main structural aspects; to calculate the failure probabilities for each mode; and to compare the efficiency and accuracy of the proposed method with other existing methods and with Monte Carlo simulations. Results demonstrate that the proposed method is efficient at identifying failure modes, including those neglected by the existing approaches.*

Keywords: *reliability, failure modes, redundant systems, structural analysis*

1. INTRODUCTION

The identification of failure modes is one of the most important tasks in the reliability analysis of large structural systems. Many research efforts have been made to estimate the failure probabilities of structural systems. So far, major attention has been paid to component reliability analysis, which deals with a failure event described by a single limit state function in the space of random variables (Kim *et al.*, 2013). However, it is widely accepted that system-level failure of a structure, e.g. collapse, often requires system reliability analysis, in which the failure event is described by a Boolean (or logical) function consisting of multiple limit state functions. Such system reliability analysis aims to compute the probability of the system failure event E_{sys}, which is often described as a *cut-set system*, i.e. (Kim *et al.*, 2013):

$$E_{sys} = \bigcup_{k=1}^{N_{cut}} C_k = \bigcup_{k=1}^{N_{cut}} \left[\bigcap_{i \,\epsilon\, I_{C_k}} E_i \right] \tag{1}$$

where E_i is the ith component event representing the failure at a location or member, $i = 1, \cdots, N_{comp}$; C_k is the kth failure mode, $k = 1, \cdots, N_{cut}$ and I_{ck} is the index set of components that appear in the failure mode C_k. Therefore, the approach developed in this paper based on the formulation in Eq. (1) can be used to compute the system failure probability for a series, parallel and hybrid (or series-parallel) systems.

2. STRUCTURAL RELIABILITY

In the reliability analyses of a structural system, one investigates the probability of a structure not failing under a given load. In a limit state context, the failure of a structural element occurs when the effect of the load (S) exceeds the resistance (R). The reliability can be estimated by the failure probability (P_f) or reliability index (β), i.e.

$$P_f = \Phi(-\beta) = P(\mathrm{S} > \mathrm{R}) = P[g(\mathbf{X}) < 0] = \int_{g(\mathbf{X})<0} f(\mathbf{X})\, \mathrm{d}\mathbf{X} = \int_0^{\infty} F_{\mathrm{R}(r)}\, f_{\mathrm{S}}(r)\, \mathrm{d}r \tag{2}$$

where Φ is the standard normal distribution function, $\mathbf{X}$ represents the vector of stochastic variables of the reliability problem, $g(\mathbf{X})$ is the limit state function defining a safe state when $g(\mathbf{X}) > 0$ and a failure state when $g(\mathbf{X}) < 0$ – the hyper-surface separating the safe from the failure domain $g(\mathbf{X}) = 0$ is called the limit state –, $f_{\mathrm{S}}(r)$ is the probability density function of the load, and $F_{\mathrm{R}}(r)$ is the cumulative probability density function of the resistance. In general, the evaluation of the integral in Eq. (2) for an arbitrary failure region may not be possible. Therefore, approximation methods are obviously needed (Tran and Staat, 2014). In this study, the hyperspace division method (HDM) (Katsuki and Frangopol, 1994), is used to evaluate the failure probability. The key idea of the proposed method in the present paper and of the HDM is the same; however, the proposed method improves the previous and uses different tools to identify multiple points on the limit state surface. Once the limit state surface is detected, the method identifies the existing multiple failure modes, the range of failure probability for each failure mode and the overall failure probability of the system.

3. FUNDAMENTAL PROCEDURE

All directional methods require identification of directions along which the integration is performed in closed form, by simulation or numerical means. In the independent standard normal space, the integral along each direction is obtained exactly by using the χ^2 distribution (Nie and Ellingwood, 2000). The proposed method consists of five basic steps: (a) distribution points on the hypersphere; (b) search by intersection points Ω_i on the failure surface; (c) failure probability associated with the chi-square distribution; (d) contribution of each subdomain P_{fi} to the total failure probability and (e) search technique for multimode failure.

3.1 Even Distribution of Points on the Hypersphere

For the purpose of distributing an arbitrary number of points uniformly on a hypersphere, six steps could be used: (a) the first step consists in generating n-points randomly; the points are generated on the surface of a hypersphere with a unit radius; (b) the respective coordinates are calculated for each point in the n-dimensional space; (c) the Euclidean distance of each point from its neighbors ($\vec{r_{m_i}}$) is then calculated; (d) the vector of repulsive force $\vec{f}$ of each point towards its neighbors is calculated. This repulsive force allows calculating the directional coordinates (with an increment of distance s, such that s ranges from 0.0 to 1.0) of the resulting vector $\vec{f_n}$ indicating the update direction of point positions (i.e, $\vec{f_n}$ is interpreted as a move of point). After the move of point, the point coordinates are projected again onto the unit hypersphere coordinates. The process is iterative and is interrupted when the forces become balanced, i.e., when the repulsive force is not strong enough and therefore does not change the position of any of the points.

3.2 Search by Intersection Points Q_i on the Failure Surface

Polar coordinates of equidistant positions indicate the directions by which one conducts a one-dimensional search for the limit state function roots, which will be later used in the directional integration for each reliability analysis. The proposed method carries out this search through the bisection method. Since bisection permits finding a single root on a given interval, the technique was adapted so that it could be applied to the proposed method, thus allowing it to handle problems involving multiple roots or multiple limit state functions. The technique is summarized in the following steps:

a) Each unit direction vector $\mathbf{e}_i$ of length β_{max} is divided into identical k subintervals such as $\Delta\beta = \beta_{max}/k$;

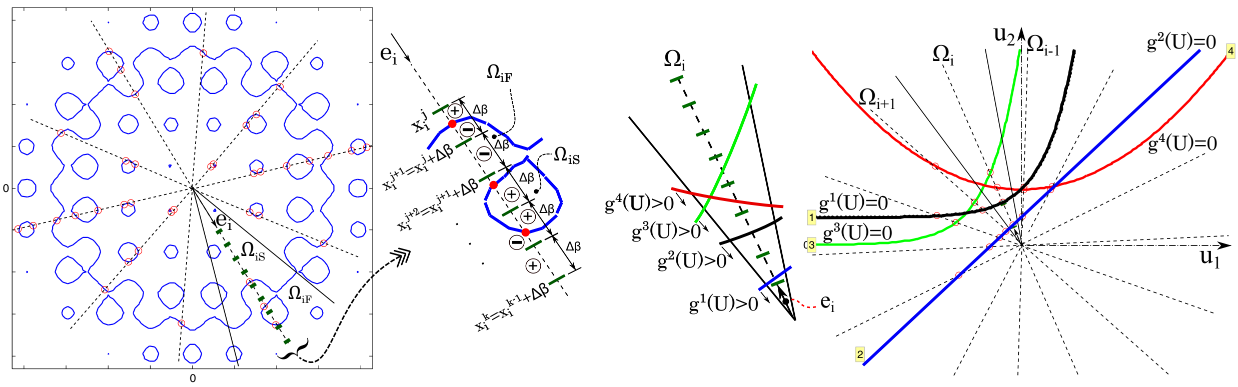
b) The bisection method is performed as shown in Fig. 1a for each subinterval $\Delta\beta$. The lower and upper bounds of $\Delta\beta$ for a given direction i are, respectively, x_i^j and x_i^{j+1}, such that: $j = 1, 2, \cdots, k$;

c) After convergence of the bisection method to a given $\Delta\beta$ (i.e., $[x_i^j, x_i^{j+1}]$), the method is performed in the subsequent subinterval ($[x_i^{j+1}, x_i^{j+2}]$) of the current direction $\mathbf{e}_i$. If the root is located, the polar coordinates of this position are kept, which correspond to point Q_i. The process is repeated until all subintervals have been assessed, i.e., $[x_i^{k-j}, x_i^k]$;

d) Once the method is applied along the whole length of direction $\mathbf{e}_i$,the following condition is checked: if any root is found, the process is repeated along the current direction $\mathbf{e}_i$; otherwise, the process takes place along the direction $\mathbf{e}_{i+1}$. In case of reset in the same direction $\mathbf{e}_i$, the limit state functions already intercepted along the current direction are deactivated, and hence, the method will search for the root(s) of another limit state function. It should be highlighted that only one limit state function is active during a given search in a given direction $\mathbf{e}_i$;

e) The process is repeated for a given direction $\mathbf{e}_i$ until all existing roots along this direction are found. After this condition is met, the method is carried out along the direction $\mathbf{e}_{i+1}$.

f) The search for the roots of the limit state function is made for all directions $\mathbf{e}_i$,such that: $i = 1, 2, \cdots, m$.



(a) Illustration of the failure surface and intersection points Q_i (b) Illustration of the multiple limit states and intersection points Q_i

Figure 1. Illustration of the intersection points Q_i

3.3 Failure Probability Associated with the Chi-square Distribution

After identifying the direction vectors $\mathbf{e}_i$, as well as the points of intersection between the centerline of a subdomain Ω_i and the limit state function Q_i in the independent standard normal space, the integral along each direction is performed in closed form utilizing the χ^2 distribution. If the limit state function is a hypersphere of radius R in the hyperspace Ω, then (Nie and Ellingwood, 2000).

$$g_n(\mathbf{U}) = u_1^2 - u_2^2 - \cdots - u_n^2 + R^2 = -z^2 + R^2 = 0 \tag{3}$$

and the failure probability associated with this hypersphere can be obtained exactly (Ang, 1984).

$$P_f = P[g_n(\mathbf{U} < 0)] = 1 - \chi_n^2(R^2) \tag{4}$$

where R = hypersphere radius and χ_n^2 = cumulative distribution function of the chi-square distribution with n degrees of freedom.

Figure 2a, shows a two-dimensional standard normal space (u_1, u_2) with a circular limit state function (the hypersphere limit state is associated with a two-dimensional space: $g_n = -u_1^2 - u_2^2 + R^2 = 0$). The space is radially and equally split onto 16 subdomains that determine 16 equal arcs (hypersphere segments) on the limit state (Katsuki and Frangopol, 1994). If the hypersphere is radially divided evenly, the contribution of each subdomain (P_{fi}) is the same. Therefore:

$$P_{fi} = [1 - \chi_n^2(R^2)]/m \quad \rightarrow \quad i = 1, 2, \cdots, m \tag{5}$$

where m is the total number of sections into which domain Ω will be split. In the more usual case where the limit state is a hypersurface rather than a hypersphere (Fig. 2), the hypersurface is approximated by a series of hyperspherical segments, each with its central point Q_i located at the point of intersection between $\mathbf{e}_i$ with the current limit state function (Nie and Ellingwood, 2000).

$$P_{fi} = [1 - \chi_n^2(R_i^2)]/m \tag{6}$$

Considering that the failure probability of any subdomain Ω_{iF} is approximated by Eq. (6), the total failure probability P_f, which corresponds to the summation of all subdomains, and the reliability index β, are respectively:

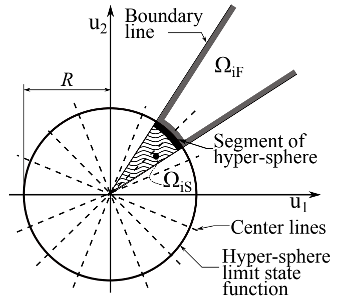
$$P_f = \sum_{i=1}^{m} P_{fi} = \sum_{i=1}^{m} [1 - \chi_n^2(R_i^2)]/m \tag{7}$$

$$\beta = \Phi^{-1}(1 - P_f) \tag{8}$$

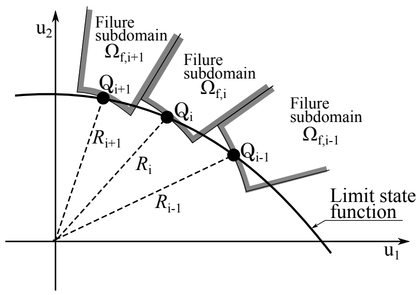
where R_i is the radius of the hypersphere segment of subdomain i.

3.4 Contribution of Each Subdomain P_{fi} to the Total Failure Probability

The failure probability of a general system $P_{f_{sys\ g}}$, concerns the calculation of the failure probability when the structure has more than one failure mode (multimode failure). Such problems arise when multiple performance functions g^k,



(a) Radial subdamains



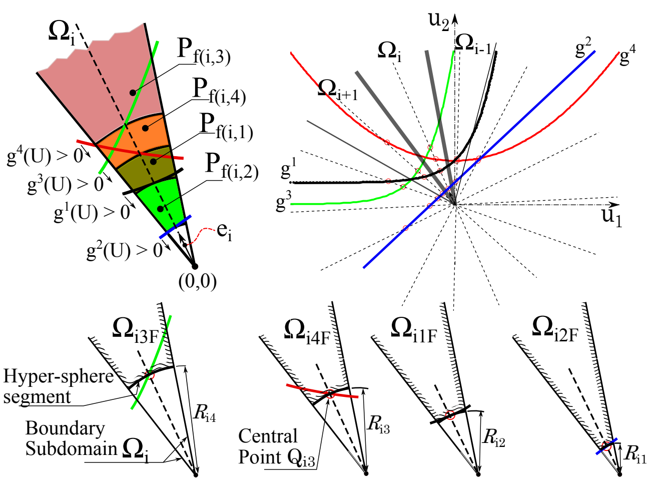
(b) Approximate nonlinear limit state function

Figure 2. Failure domain and segments for limit states circular and nonlinear: Adapted Katsuki and Frangopol, 1994

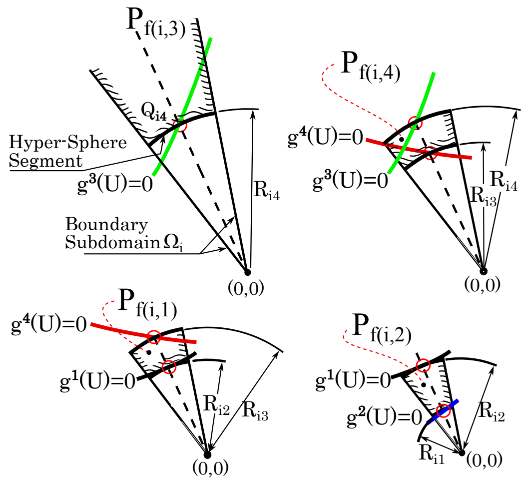
$k = 1, 2, \cdots, k$, are imposed. This case is illustrated in Fig. 3 for four limit state functions in a two-dimensional standard normal space (u_1, u_2). The failure probability of a general system is given by:

$$P_{f_{sys\ g}} = \sum_{i=1}^{m} [1 - \chi_n^2(R^2_{(i,n_{felC})})]/m \quad \rightarrow \quad n_{felC} = 1, 2, \cdots, N_{felC} \tag{9}$$

where n_{felC} is the limit state function which corresponds to the critical element of the system and N_{felC} is the total number of elements that cause the system to collapse.



(a) Approximate multiple limit states by hyperspherical segments



(b) Failure probability of any subdomain Ω_{iF}

Figure 3. Multiple limit state functions and its approximations with spherical segments

As shown in Fig. 3, there exist two extreme cases of failure. The two extreme cases of series and parallel systems are associated with the minimum and maximum distances of the current limit state functions, i.e:

$$P_{f_{sys\ s}(i)} = [1 - \chi_n^2((min(R_{(i,n_{fel})}))^2)]/m; \qquad P_{f_{sys\ p}(i)} = [1 - \chi_n^2((max(R_{(i,n_{fel})}))^2)]/m \tag{10}$$

where $P_{f_{sys\ s}(i)}$ and $P_{f_{sys\ s}(i)}$ are the total probabilities of failure associated with series and parallel systems, respectively, $R_{(i,n_{fel})}$ is the distance between the origin and the current limit state function $g^{n_{fel}}$ along the $\mathbf{e}_{(.)}$, and n_{fel} is an nth limit state function.

In the proposed method, the evaluation of structural failure probability defined by Eq. (9) consists in splitting the domain in the standard normal space Ω, into subdomain meshes Ω_i, such that the volume of each $\Omega_{i,n_{fel}}$ corresponds to the P_f of the current limit state function $g^{n_{fel}}$ along the centerline $\mathbf{e}_i$. Therefore, the contribution of each subdomain Ω_i to the total probability of failure by a limit state function in a two-dimensional standard normal space is searchd as follows:

- one calculates the volume below the joint probability density function (PDF), outside the limit state function (that consists in approximating of the current hypersurface by a hypersphere segment), between the bounds of the sector corresponding to the direction $\mathbf{e}_i$ (Fig. 3a). This procedure is carried out from the limit state function closest to the origin to the most distant one;

- since the P_f is determined solely by the distance from the origin $|\mathbf{U}|$, the limit state function closer to the origin is likely to contribute more to the system failure probability; therefore, the volume of the limit state function to the origin, along the direction $\mathbf{e}_i$, is subtracted from the volume of the second closest limit state function to the origin. This sequence is applied to all limit state functions along $\mathbf{e}_i$ (Fig. 3b).

Table. 1 shows the formulation used to estimate the contribution of each subdomain mesh along the centerline $\mathbf{e}_i$ in Fig. 3.

Table 1. Probability of failure to each subdomain mesh.

Classification Systems	Subdomain Mesh Ω_i	Formulation
Parallel	$P_{f_{sys\ p}(i3)} = \Omega_{i3F}$	$P_{f(i3)} = [1 - \chi_n^2(R_{i4}^2)]/m$
Series	$P_{f_{sys\ s}(i2)} = \Omega_{i2F}$	$P_{f(i2)} = [1 - \chi_n^2(R_{i1}^2)]/m$
Hybrid or Parallel	$P_{f_{sys\ g}(i3)} = \Omega_{i3F}$	$P_{f(i3)} = [1 - \chi_n^2(R_{i4}^2)]/m$
Hybrid	$P_{f_{sys\ g}(i4)} = \Omega_{i4F} - \Omega_{i3F}$	$P_{f(i4)} = \{[1 - \chi_n^2(R_{i3}^2)]/m\} - \{[1 - \chi_n^2(R_{i4}^2)]/m\}$
Hybrid	$P_{f_{sys\ g}(i1)} = \Omega_{i1F} - \Omega_{i4F}$	$P_{f(i1)} = \{[1 - \chi_n^2(R_{i2}^2)]/m\} - \{[1 - \chi_n^2(R_{i3}^2)]/m\}$
Hybrid	$P_{f_{sys\ g}(i2)} = \Omega_{i2F} - \Omega_{i1F}$	$P_{f(i2)} = \{[1 - \chi_n^2(R_{i1}^2)]/m\} - \{[1 - \chi_n^2(R_{i2}^2)]/m\}$

3.5 Search Technique for Multimode Failure

The technique proposed in the previous section identifies the portions of P_f in the decreasing order of their likelihood. Since the proposed searching method aims to capture multiple failure modes, the remaining tasks consist in performing a combination of failure sequences in order to identify dominant failure modes, also in the decreasing order of their likelihood. The failure modes causing the system failure are recorded and their occurrence frequencies are counted. Therefore, the orders of the failure modes correspond to the significance of the contribution to the system reliability of the truss structure. Table. 2 shows the sequential failure modes obtained by the proposed method in the decreasing order of their occurrence.

Table 2. Failure modes of subdomain Ω_i.

Sequential Failure Modes	System Failure Event	Probabilities of the Identified Failure Modes
2	$E_{sys} = (E_2)$	$P_{f(i2)} = [1 - \chi_n^2(R_{i1}^2)]/m$
$2 \to 1$	$E_{sys} = (E_2 \cup E_1)$	$P_{f(i2)} = \{[1 - \chi_n^2(R_{i1}^2)]/m\} - \{[1 - \chi_n^2(R_{i2}^2)]/m\}$
$2 \to 1 \to 4$	$E_{sys} = (E_2 \cup E_1 \cup E_4)$	$P_{f(i4)} = \{[1 - \chi_n^2(R_{i3}^2)]/m\} - \{[1 - \chi_n^2(R_{i4}^2)]/m\}$
$2 \to 1 \to 4 \to 3$	$E_{sys} = (E_2 \cup E_1 \cup E_4 \cup E_3)$	$P_{f(i3)} = [1 - \chi_n^2(R_{i4}^2)]/m$

4. IMPLICIT LIMIT STATE FUNCTION AND SOURCES OF NONLINEARITY

In many cases of practical importance, particularly for complicated structures, the performance function g($\mathbf{X}$) is generally not available as an explicit, closed-form function of the input variables (Haldar, 2000). Therefore, the response has to be computed through a numerical procedure such as finite element analysis. The deterministic finite element method (FEM) consists of an iterative procedure to capture the nonlinear behavior of the structure (Haldar, 2000). The efficiency of the deterministic FEM is important for the success of the probabilistic method.

Most existing studies found in the literature, on the formulation of the numerical method incorporated into the probabilistic system, utilize the deterministic FEM model, assuming linear behavior for the structure. In this paper, the deterministic finite element method is developed considering sources of nonlinearity in a typical truss structure. Among the several sources of nonlinearity, the following stand out: (a) physical (or material) nonlinearity and (b) geometrical nonlinearity. These nonlinearities will cause the configuration of a structure to deviate quite noticeably from its undeformed configuration. In such cases, linear analysis is obviously inappropriate, and nonlinear structural analysis should be used (Haldar, 2000).

In physical nonlinearity, the constitutive relation describing the material is itself nonlinear and the structural response associated with physical phenomena such as plasticity or strain softening must be captured. Physical nonlinearity is due to changes in geometry, arising from large strains and/or rotations, which enter the formulation from a nonlinear strain displacement relationship, and may occur even if the constitutive relation is linear (Leon *et al.*, 2011).

Nonlinear problems arising from geometric or material nonlinearity feature critical points along the solution path. Critical points or stability points are points on the solution path where the structure loses stability or where bifurcation occurs. Load limit points occur when a local maximum or minimum load is reached on the load versus displacement curve. A horizontal tangent is present at load limit points. Displacement limit points occur at vertical tangents on the solution curve. Methods capable of run through displacement limit points are said to capture snap-back behavior. In this context, the unstable region following a load limit point corresponds to a physical instability in the structural system, such as buckling. Therefore, if snap-back behavior was not captured the structure would appear to have a sharp drop in the load at point and the nature of the unloading might be lost (Leon *et al.*, 2011).

Usually, the solution to non-linear problems is obtained by the use of incremental and iterative techniques, which should be able to follow the equilibrium path beyond critical points; therefore, most of the methods will trace the equilibrium path point by point, through an iterative scenario. Most of these techniques are a natural extension of the well-known Newton-Raphson iterative method, such as: load control, displacement control, arc-length control, and generalized displacement control method (GDCM). The GDCM was initially proposed, Yang and Shieh, 1990, as an alternative to the existing methods for solving non-linear problems with multiple limit and snap-back points (Cardoso and Fonseca, 2006). In this work, we use the GDCM to solve a complex non-linear problem. For further discussion on non-linear systems, the reader is referred to Leon *et al.*, 2011; Yang and Kuo, 1994; Gu, 2004; Chen, 2008 and Bathe, 1996.

5. NUMERICAL EXAMPLES

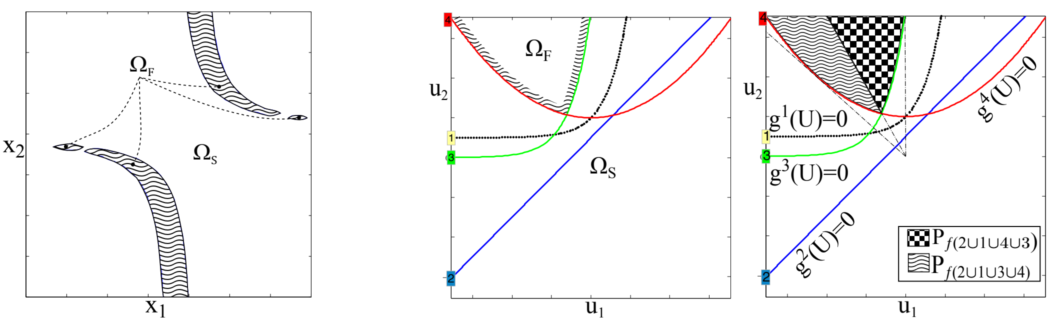
The proposed methodology is demonstrated first through numerical examples that are often used as benchmark structures in the literature: two explicit functions and one statically undertermined 6-bar truss. In order to examine the accuracy and computational efficiency of the proposed method, the results are compared with the Monte Carlo simulation and with the best results in literature.

Problem 1 is the same used by Wang and Shan, 2006. This problem has only one failure mode. The limit state function for this example is $g(\mathrm{x}_1, \mathrm{x}_2) = 28.0 - f(\mathrm{x}_1, \mathrm{x}_2)$. Variables x_1,x_2 are used in the function $f(\mathrm{x}_1, \mathrm{x}_2)$ for consistency instead of β_1 and β_2 (see Wang and Shan, 2006). This problem has multiple failure regions, as shown in Fig. 4a. β_1 and β_2 are random variables with a distribution $\beta_1 \sim N(1,\ 0.025)$ and $\beta_2 \sim N(1,\ 0.025)$. The reliability index and failure probabilities are presented in Tab. 3, compared with those of the original problem obtained by Wang and Shan, 2006 and by the Monte Carlo simulation. It is shown that our failure probability is smaller compared with the solution of Wang and Shan, 2006, leading to a smaller failure probability of the system.

Table 3. Test result comparison.

Limit State Function	Proposed Method		MCS		Wang and Shan, 2006	
	β	P_f	β	P_f	β	P_f
$g(x_1, x_2) = 28.0 - f(x_1, x_2)$	2.3180	1.022×10^{-2}	-	1.022×10^{-2}	-	1.09×10^{-2}

Problem 2 is the same example used by Sorensen, 2004, which consist of a parallel system of four failure elements. After the transformation of the stochastic variables x_1 and x_2 into the standard normal space of variables u_1 and u_2, the four failure elements are described by the following failure functions: $g_1(\mathbf{U}) = \exp(\mathrm{u}_1) - \mathrm{u}_2 + 1$; $g_2(\mathbf{U}) = \mathrm{u}_1 - \mathrm{u}_2 + 1$; $g_3(\mathbf{U}) = \exp(\mathrm{u}_1 + 2) - 2$; $g_4(\mathbf{U}) = 0.1\mathrm{u}_1^2 - \mathrm{u}_2 + 2$. The failure functions, are shown in Fig. 4b.



(a) Nonlinear limit state function (b) Failure domain for a parallel system

Figure 4. Numerical examples: explicit functions

For illustration purposes, Fig. 4b shows the domain of a failure mode in the two-dimensional standard normal space. The lines show the limit state surfaces of component events while the space enclosed by limit state surfaces represent the failure mode. Table 4 shows the sequences of components failures (column 1) and reliability index (column 2) of the proposed method. The "exact" solution β = 3.56399 (by chi-square distribution) or β = 2.92092 (by normal distribution) was obtained by the Monte Carlo simulation using one million simulation runs, when the coefficient of variation (c.o.v) reaches 0.02. The reliability index calculated by Sorensen, 2004, is 2.92. Note that the proposed procedure converges very quickly to a probability estimate close to that obtained by the Monte Carlo analysis and Sorensen, 2004.

Table 4. Reliability index and failure probability of failure modes and overall system.

Ultimate Failure Modes	Proposed Method	
	β	Failure Probabilities
$2 \to 1 \to 4 \to 3$	3.7519	8.7749×10^{-4}
$2 \to 1 \to 3 \to 4$	3.7704	8.1843×10^{-4}
System	3.5720	1.6959×10^{-3}
Serviceability Failure Modes		
2	1.8117	1.9376×10^{-1}
$2 \to 4$	3.2057	5.8681×10^{-3}
$2 \to 4 \to 1$	3.4444	2.6537×10^{-3}
$2 \to 3$	3.4638	2.4811×10^{-3}
$2 \to 1$	2.8255	1.8468×10^{-2}
$2 \to 1 \to 3$	3.2219	5.5701×10^{-3}
$2 \to 3 \to 1$	3.5996	1.5362×10^{-3}
$2 \to 1 \to 4$	3.1294	7.4728×10^{-3}
4	3.8146	6.9231×10^{-4}
$4 \to 2$	4.0978	2.2576×10^{-4}

Problem 3 is a practical engineering example. It consists of six-member truss structure as shown in Fig. 5. The data on loads and materials are listed in Tab. 5. All the random variables are assumed to follow the independent normal distribution except for load L5, which is assumed to follow a lognormal distribution. The truss is a statically indeterminate structure in which the failure of any two members results in structural collapse. Kim *et al.*, 2013, Shao and Murotsu, 1999, using the same example, in which all the elements are assumed to have ideal elasto-plastic behavior. However, in this work, nonlinearity is considered by adopting an elastic plasticity model with linear hardening. This example will be used to demonstrate the ability of the nonlinear solution algorithms to capture both load and displacement limit points, as well as sudden changes of direction on the equilibrium paths.

The failure modes identified by the proposed method, together with the value obtained for the reliability index, are presented in Tab. 6. In the fist stage, member 6 turns out to be the weakest one for the given loading conditions. This result confirms that the proposed method can successfully identify the same failure modes as those by Kim *et al.*, 2013. Its numerical efficiency is investigated to examine the number of simulations required to identify the ultimate failure mode (structural collapse). The number of simulations required by the proposed method is only one-thousandth of that required by MCS and one-hundredth of that required by the method proposed by Kim *et al.*, 2013. These results confirm that the proposed method greatly improves the computational efficiency both in searching failure modes and in evaluating the system failure probability. The accuracy of the proposed method in computing the probabilities is also demosntrated in Tab. 6 by the reliability index of the four ultimate failure modes together with the serviceability failure modes. The results show that the reliability index calculated by the proposed method are fairly close to those obtained by the MCS method.

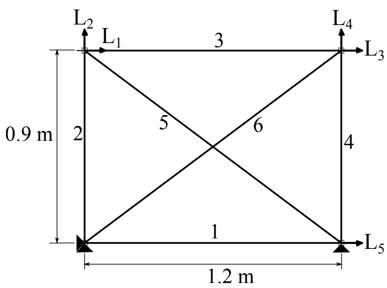


Figure 5. Six-bar truss

Table 5. Statistical parameters and distribution types.

Random Variables	Distribution	Mean	c.o.v
L_1	Normal	50 kN	0.1
L_2	Normal	30 kN	0.1
L_3	Normal	20 kN	0.1
L_4	Normal	30 kN	0.1
L_5	Lognormal	20 kN	0.1
σ_{yi}	Normal	276 MPa	0.05

6. CONCLUSIONS

An iterative technique for finding the failure probabilities associated with the intersections of nonlinear limit state functions was presented. The method could identify failure modes of a structural system and compute the probabilities of the identified and of the overall system failure event. This technique is based on the conventional HDM procedures which approximate the limit state surface by a series of spherical segments in order to provide accurate estimates of

Table 6. Reliability index and number for simulations of failure modes and overall failure of the system.

Proposed Method		Kim *et al.*, 2013		MCS
Ultimate Failure Modes	β	Ultimate Failure Modes	β	β
$6 \to 2$	3.1142	$6 \to 2$	3.0735	3.0700
$2 \to 6$	4.5334	$2 \to 6$	3.4300	-
$6 \to 1$	3.5110	$6 \to 1$	3.4346	3.4980
$6 \to 5$	6.3732	-	-	-
System	3.0189		2.9775	3.0330
Serviceability Failure Modes				
6	1.2902	-	-	
2	3.5533	-	-	
1	5.2863	-	-	
Number of Simulations	2983	28400		1.0×10^6

failure probabilities of components or systems. The accuracy of HDM depends on its ability to efficiently generate a set of directions along which the probability increments are estimated. This paper has presented a procedure to generate these "evenly distributed" directions. Numerical applications showed the accuracy of the methodology in the evaluation of the failure probability of problems represented by high nonlinear limit state functions with multiple failure regions. Furthermore, the superior performance of proposed method is demonstrated in comparison with the example presented by Kim *et al.*, 2013 and independent evaluations by Monte Carlo simulation.

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