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THE USE OF A VIRTUAL PENDULUM AS PASSIVE ENERGY DISSIPATION SYSTEM FOR SEISMIC PROTECTION OF BUILDINGS

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Abstract. *The passive control of structures using a pendulum absorber has been extensively studied in the technical literature and used in tall buildings such as the Taipei 101 in Taiwan. As the frequency of the pendulum depends only on its length and the acceleration of gravity, to tune the frequency of the pendulum with that of the structure, the pendulum length is the only design variable. However, in many cases, the required length and the space required for its installation are not compatible with the project. In these cases one can replace the classical pendulum by a virtual pendulum which consists of a mass moving over a curved surface, allowing thus for a greater flexibility in the absorber design, since the length of the pendulum becomes irrelevant and the shape of the curved surface can be optimized. Several designs are possible and the device can be used as a tuned mass damper or a base isolation system. The aim of this work is to develop a mathematical model for a building with a virtual pendulum and to study the influence of this device on the nonlinear oscillations and stability of the main system under a base excitation. Through simulations under harmonic and recorded ground motions of increasing intensity, the performance of the virtual pendulum used as a tuned mass damper or base isolation is evaluated on an SDOF structure in order to illustrate their respective advantages as well as the drawbacks inherent to their non-linear behavior. Several values of the relevant parameters are assumed in the analyses.*

Keywords: *Seismic protection, virtual pendulum, tuned mass damper, base isolation, nonlinear vibrations.*

1. INTRODUCTION

As structures become lighter and more slender, the fundamental frequency of the structure decreases and approaches the values of the excitation frequency associated with environmental loads, such as wind, waves or earthquakes. With this the structures may have undesirable levels of vibration, causing discomfort to users, damage to structural elements and, in extreme cases, the ruin of the structure. This is quite common in high seismicity regions. The seismic energy and its frequency content are very dangerous for buildings regardless of their height. Passive vibration control systems have been studied analytically and experimentally by different researchers and used in various kinds of structures in order to minimize the effects of excitation on the structure. In the case of earthquakes, the passive vibration control can be obtained using attenuators generating control forces on the structure which oppose excitation, or by base isolation mechanisms (Soong, et.al.1997).

One mechanism that can be used both as a tuned mass damper and base isolation is the so called virtual pendulum, which consists of a mass that moves over a curved surface, causing forces opposed to movement, in the case of the tuned mass damper, or allows the displacement of the structure as a whole, in the case base isolation, reducing stresses and displacements within the structure.

Several researches are found in the literature on the use of pendulum absorbers. Gonçalves and Orlando (2013) studied a hybrid control system to mitigate the vibrations of tall towers. The hybrid control is based on the simultaneous use of a pendulum absorber with an external force applied at the tower-pendulum connection. Oliveira (2012) studied the efficiency of a tuned pendulum on the vibrations of a ten-story building modeled as a shear frame. As the frequency of the pendulum depends only on its length and the acceleration of gravity, to tune the frequency of the pendulum with that of the structure, the pendulum length is the only design variable. However, in many cases, the required length and the space required for its installation are not compatible with the project. In these cases one can replace the classical pendulum by a virtual pendulum which consists of a mass moving over a curved surface, allowing thus for a greater flexibility in the absorber design, since the length of the pendulum becomes irrelevant and the shape of the curved surface can be optimized, as shown by Pinheiro (1997).

Several similar devices for vibration control have been studied in recent years. Matta and De Stefano, A. (2009) proposes a robust design methodology of a mass-uncertain rolling-pendulum TMD for the seismic protection of buildings. Through simulations under harmonic and recorded ground motions of increasing intensity, the performance of circular and cycloidal rolling-pendulum TMDs is evaluated. Legeza (2010) studied the efficiency of a vibro-protection system with an isochronous roller damper under the action of an external harmonic excitation. The working surface of the damper consists of a brachistochrone. The dynamic equations of common no-slip motion of the damper on a hinged roller and of the bearing body are formulated and the roller damper tuning parameters are determined. Chen and Georgakis (2013) used a tuned rolling-ball damper characterized by single or multiple steel balls rolling in a spherical container and mounted on the top of a wind turbine to reduce the wind-induced vibration. A 1/20 scale shaking table model was developed to evaluate the control effectiveness of the damper. The test results indicated that the rolling-ball dampers could effectively suppress the wind-induced vibration of wind turbines. Recently, Cano (2013) developed a numerical model for the seismic analysis of buildings with base isolation systems composed of elastomer supports or moving on roller supports. The results were validated by experimental tests on a physical scale model to a four-story structure, subjected to a base excitation.

The aim of this work is to develop a mathematical model for a building with a virtual pendulum and to study the influence of this device on the nonlinear oscillations and stability of the main system under a base excitation. Through simulations under harmonic and recorded ground motions of increasing intensity, the performance of the virtual pendulum used as a tuned mass damper or base isolation is evaluated in order to illustrate the marked reduction caused by both systems on the displacements and accelerations of the main structure. Several values of the relevant parameters are assumed in the analyses.

2. PROBLEM FORMULATION

The main structure is modeled as a SDOF system with mass *Ms*, column stiffness *Ks* and damping constant *Cs*. The lateral displacement of the frame is denoted by *Xs*. The virtual pendulum absorber is composed by an additional mass *Ma* with generalized horizontal coordinate *Xa* and the respective vertical displacement *Za* (see Fig. 2) which moves over a circular surface of radius *R*, as shown in Figure 1.a. The mass and inertia forces of the rollers are considered negligible. The base isolation system is illustrated in Fig. 1.b. In both cases the structure is subjected to a time-dependent base displacement *Xg*.

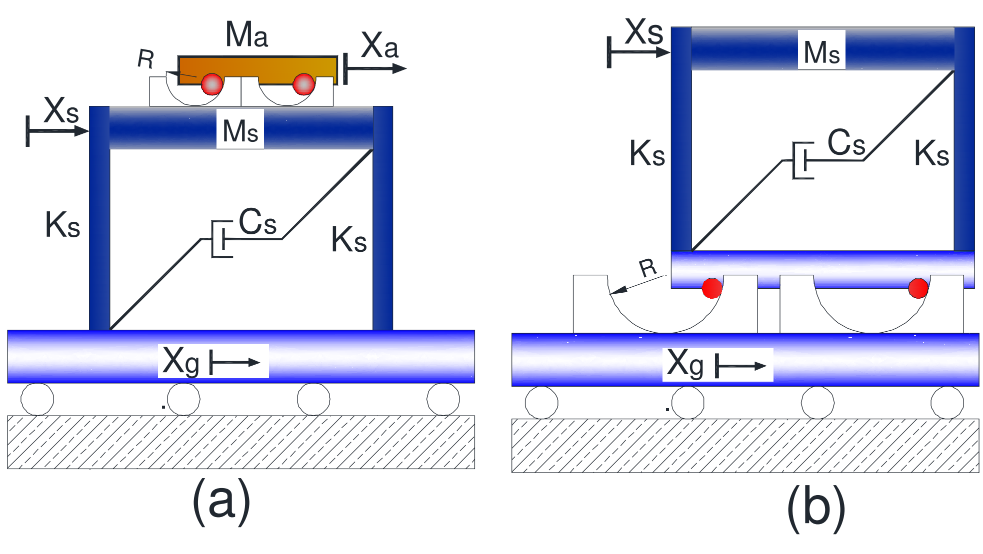


Figure 1 – Virtual pendulum as a tuned mass damper (a) and base isolation (b).

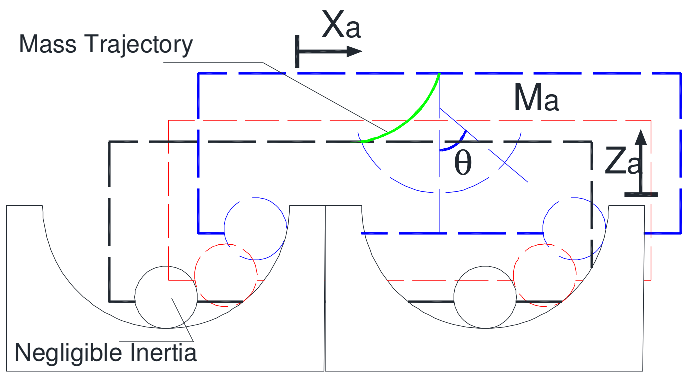


Figure 2 – Displacements of mass *Ma* on the curved surface

2.1 Mathematical Formulation

The differential equations of motion of the vibration control systems with virtual pendulum are obtained by applying Hamilton's principle to the two alternative systems shown in Figure 1.

2.2 Virtual pendulum as a tuned mass damper (TMD)

The kinetic energy of the shear-building, T_s, and of the pendulum, T_a, are given, respectively, in terms of generalized coordinates by:

$$T_S = \frac{1}{2} M_s \left(\dot{X}_g + \dot{X}_s \right)^2 \tag{1}$$

$$T_a = \frac{1}{2} M_a \left(\left(\dot{X}_g + \dot{X}_s + \dot{X}_a \right)^2 + \dot{Z}_a^2 \right) \tag{2}$$

where: X_s= horizontal displacement of the structure, X_g= horizontal base displacement, X_a= horizontal displacement of the control mass, Z_a= vertical displacement of the control mass, M_s= mass of the structure and M_a= virtual pendulum mass.

The displacements of the additional mass, X_a and Z_a, are given in terms of the rotation angle θ as (see Figure 2):

$$X_a = R \sin(\theta) \tag{3}$$

$$Z_a = R(1 - \cos(\theta)) \tag{4}$$

where R is the radius of curvature of the circular surface.

The internal strain energy of the structure is given by:

$$U_s = \frac{1}{2} K_s X_s^2 \tag{5}$$

where K_s is the structure stiffness.

The potential energy is given by:

$$V_a = Ma \cdot g \cdot Z_a \tag{6}$$

where g is the acceleration of gravity.

The variation of the internal work of the damping forces of the structure and the virtual pendulum and additional non-conservative forces is given by:

$$\begin{aligned} \delta W &= -C_s \dot{X}_s \delta(X_s) - C_a \left(X_{a_\theta}\right)^2 \dot{\theta}\delta(\theta) \\ \delta W &= -Q_{X_s} \delta(X_s) - Q_\theta \delta(\theta) \end{aligned} \tag{7}$$

where C_s is the damping coefficient of the structure, C_a is the damping coefficient of the pendulum and Q_{xs} and Q_θ are additional non-conservative forces.

By applying Hamilton's principle, the following system of Euler-Lagrange equations of motion are obtained:

$$\frac{d}{dt}\left(\frac{\delta T}{\delta \dot{q}_i}\right) - \frac{\delta T}{\delta q_i} + \frac{\delta \Pi}{\delta q_i} - Q_i = 0 \tag{8}$$

where $T = T_S + T_a$ is the total kinetic energy, $\Pi = U_s + V_a$ is the total potential energy and q_i is the i-th generalized coordinate.

The following nondimensional parameters are used in the analysis:

$$\begin{aligned} &\overline{X}_s = \frac{X_s}{R}; \quad \tau = \omega_s t; \rightarrow \frac{d\overline{X}_s}{dt} = \omega_s \frac{d\overline{X}_s}{d\tau}; \frac{d^2\overline{X}_s}{dt^2} = \omega_s^2 \frac{d^2\overline{X}_s}{d\tau^2}; \quad \omega_s^2 = \frac{K_s}{M_s}; \quad \overline{A} = \frac{A}{R}; \\ &\delta = \frac{M_a}{M_s}; \ \frac{C_s}{M_s} = 2\xi_s\omega_s; \ \omega_p^2 = \frac{g}{R}; \ C_a = \Gamma C_s; \ \frac{\Omega}{\omega_S} = \beta \ \ \Delta = \frac{\omega p}{\omega_S}, \ \eta = \frac{r}{R}, \ \overline{I}_{cm} = \frac{I_{cm}}{r^2 m_s} \end{aligned} \tag{9}$$

where ω_p is the pendulum characteristic frequency of length R and ω_S is the frequency of the structure:

From Eqs. (1) to (8), the following system of nonlinear equations of motion in terms of the nondimensional parameters (9) is obtained:

$$(\delta+1)\left(\overline{X}_{s_{,\tau,\tau}} + \overline{X}_{g_{,\tau,\tau}}\right) + \delta\cos(\theta)\theta_{,\tau,\tau} + 2\xi_s \overline{X}_{s_{,\tau}} - \delta\sin(\theta)\left(\theta_{,\tau}\right)^2 + \overline{X}_s = 0 \tag{10}$$

$$\delta\cos(\theta)\left(\overline{X}_{s_{,\tau,\tau}} + \overline{X}_{g_{,\tau,\tau}}\right) + \delta\theta_{,\tau,\tau} + 2\Gamma\xi_s\cos(\theta)^2\theta_{,\tau} + \Delta^2\delta\sin(\theta) = 0 \tag{11}$$

The excitation consists of a base acceleration produced by a harmonic displacement with excitation frequency Ω and magnitude A:

$$\ddot{\overline{X}}_g = \beta^2 \overline{A}\sin(\beta\tau) \tag{12}$$

or a seismic excitation.

The two system frequencies are given in terms of the previously defined dimensionless parameters as:

$$\overline{\omega}_i = \frac{1}{2}\sqrt{2\delta\Delta^2 + 2\Delta^2 + 2 \mp 2\sqrt{\delta^2\Delta^4 + 2\delta\Delta^4 + \Delta^4 + 2\delta\Delta^2 - 2\Delta^2 + 1}} \tag{13}$$

The variation of the two natural frequencies as a function of the frequency ratio $\Delta = \omega_p / \omega_s$ is depicted in Fig. 2 for two values of the mass ratio $\delta = M_a / M_s$.

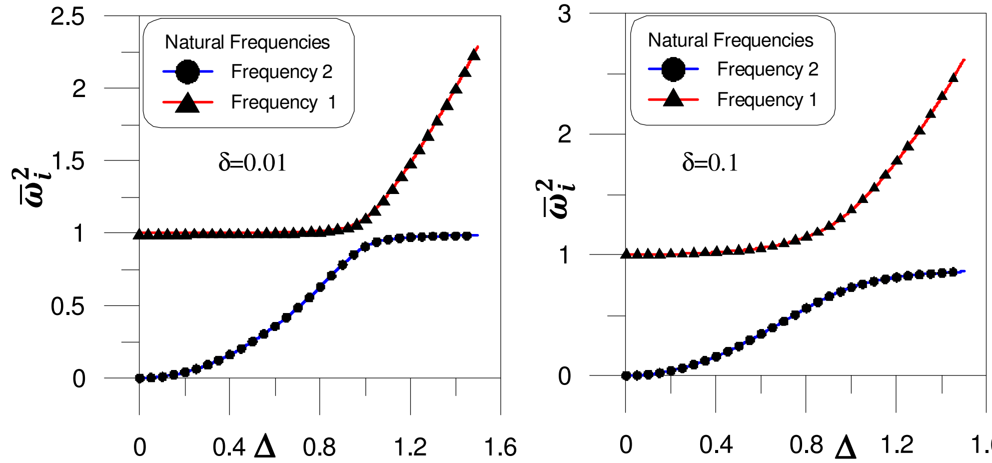


Figure 3 – TMD: variation of the natural frequencies as a function of the nondimensional parameters δ and Δ.

2.3 Virtual pendulum as a base isolation system

Consider now the base isolation system shown in Fig. 1.b. Now the structure moves on a series of circular surface of radius R. This system's main characteristic is the fact that when it is destabilized, it automatically returns to the starting equilibrium position due to the stabilizing effect of the vertical component of the structure's own weight. Considering that the structure, subjected to the base displacement X_g, has an horizontal and vertical displacement components $X_s(\theta)$ and $Z_s(\theta)$, the energy contributions can be written as:

$$T_S = \frac{1}{2} M_S \left(\left(\dot{X}_g^2 + \dot{X}_S^2 \right)^2 + \dot{Z}_S^2 \right) \tag{14}$$

$$U_S = \frac{1}{2} K \cdot X_S^2 \tag{15}$$

$$V_S = M_S Z_S g \tag{16}$$

$$\Pi = U_S + V_S \tag{17}$$

After applying Hamilton's principle and using the nondimensional parameters (9), the following nonlinear equation of motion is obtained:

$$2\cos\left(\theta(\tau)\right)^2 \xi_s \frac{d\theta(\tau)}{d\tau} + \frac{d^2\theta(\tau)}{d\tau^2} + \Delta^2 \sin\left(\theta(\tau)\right) + \sin\left(\theta(\tau)\right)\cos\left(\theta(\tau)\right) + \frac{\cos\left(\theta(\tau)\right)}{\omega_S^2} \frac{d^2 \bar{X}_g}{d\tau^2} = 0 \tag{18}$$

The natural frequency of the isolated structure is given by:

$$\omega = \sqrt{\Delta^2 + 1} \tag{19}$$

3. NUMERICAL ANALYSIS

3.1 Harmonic excitation

The basic parameters of the structure used in the numerical analysis are summarized in Table 1 (Benevile, 2002).

Table 1 - Basic parameters values of the structure. (Benevile, 2002)

Mass	Ms	506617.43	Kg
Stiffness	Ks	5.1045E+06	N/m
Frequency	ωs	3.17	Rad/s
Acceleration of gravity	g	10	m/s2
Excitation frequency	Ω	Variable	Rad/s
Excitation magnitude	A	0.01	m
Damping coefficient	ξ	Variable	

3.2 **Virtual pendulum as a tuned mass damper**.

Figure 4 shows the bifurcation diagrams obtained by the brute force method, where the maximum lateral displacement of the structure and controlling mass rotation is plotted as a function of the forcing frequency ratio $\beta = \Omega / \omega_s$ (resonance curves) for three diferent values of the mass ratio $\delta = M_a / M_s$. The controlling mass varies from 1% to 5% of the mass of the structure. The results are obtained considering a virtual pendulum length *R*=1.2m and a frequency ratio $\Delta = \omega_p / \omega_s = 0.9$. As the controlling mass increases the effectiveness of the control increases reducing the displacements of the structure and of the TMD. The maximum displacement of the uncontrolled structure is 0.202.

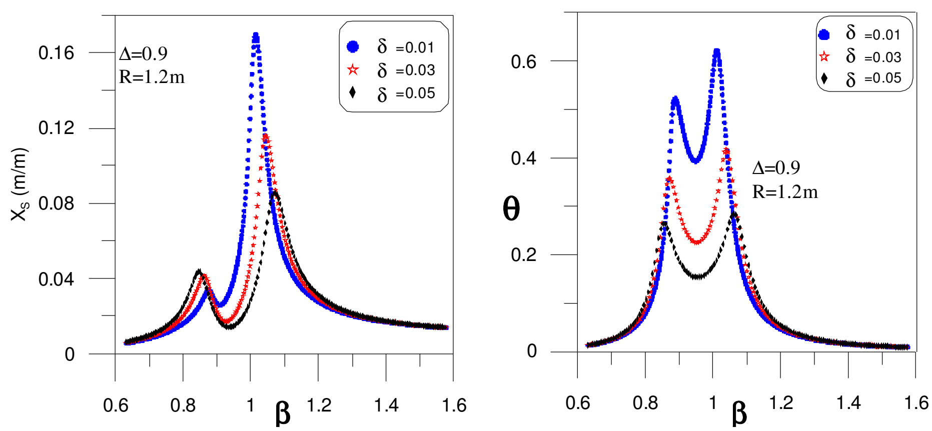


Figure 4 Bifurcation diagrams (a) structure, (b) virtual pendulum. δ=0.01, 0.03, 0.05, Δ=0.9, ξ=2%.

Figure 5 shows the resonance curves for selected values of $\Delta = \omega_p / \omega_s$, considering $\delta = M_a / M_s = 0.01$. Again the beneficial effect of the TMD on the structure's displacements is evident. The best tuning value is $\Delta = 0.9$.

3.3 **Virtual pendulum as a base isolation system**.

Figure 6 shows the resonance curves for selected values of $\Delta = \omega_p / \omega_s$, considering the base isolation system, while Figure 7 shows as comparison between the two vibration control systems for different sets of system parameters. It is evident that the base isolation system is more effective in reducing the structure's displacements than the TMD.

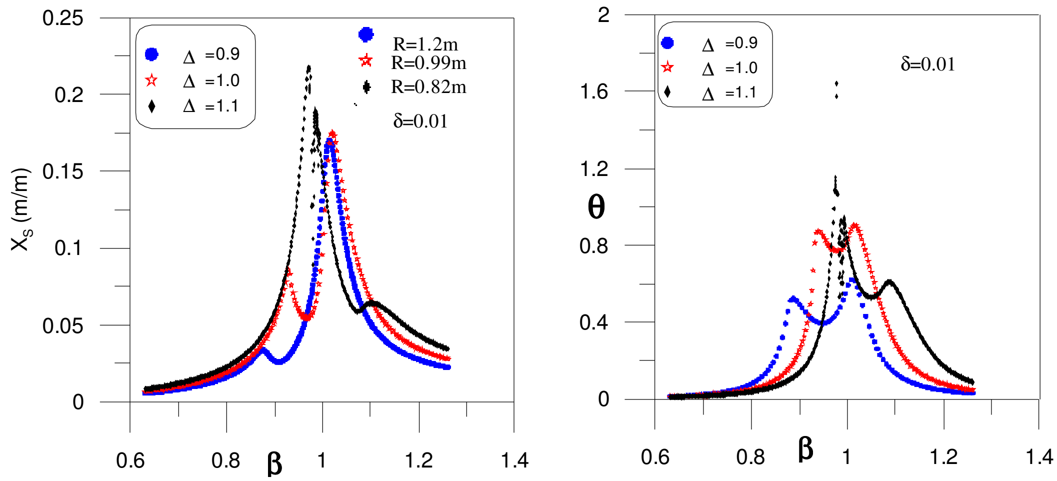


Figure 5 Bifurcation diagrams (a) structure, (b) virtual pendulum. δ=0.01, Δ=0.9, 1.0, 1.1, ξ=2%

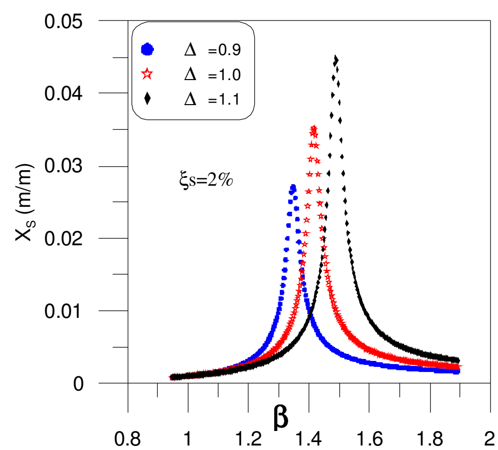


Figure 6 - Bifurcation diagrams of the isolated structure. Δ=0.9, 1.0, 1.1

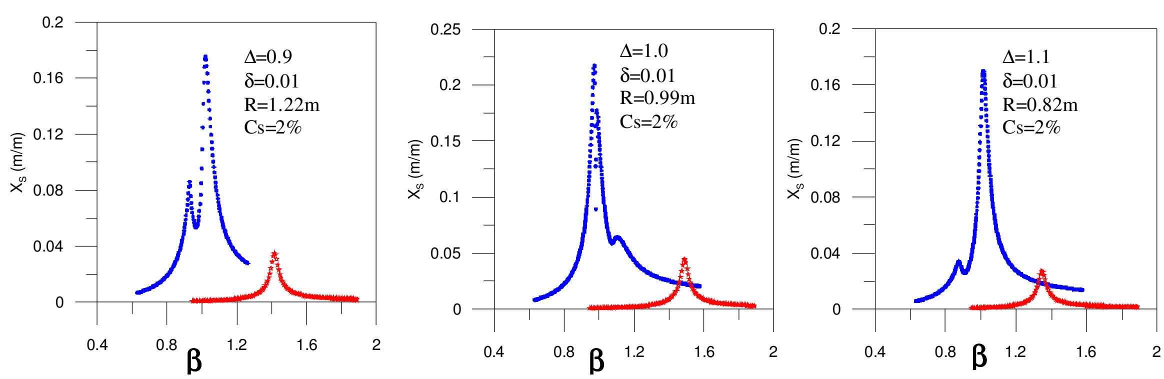
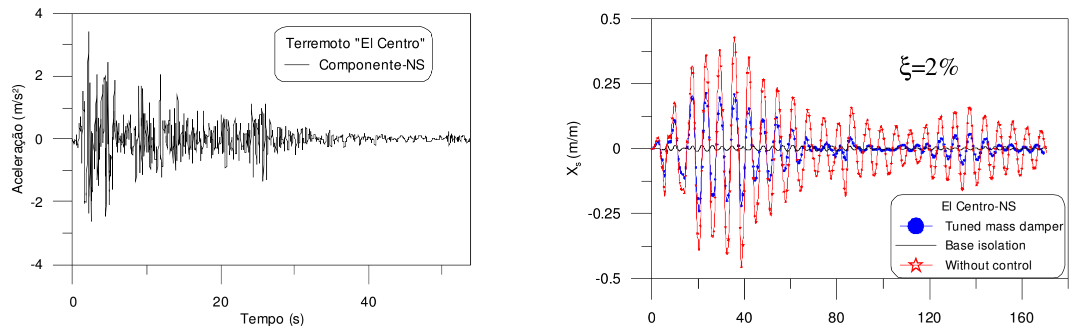


Figure 7 – Comparison of the effectiveness of the tuned mass damper (blue) and the base isolation system (red) for different sets of the system parameters.

3.4 Seismic Excitation

Figure 8 shows the time response of the structure subjected to the well-known base acceleration of the El Centro earthquake. Again, both control systems show excellent performance decreasing markedly the structural vibration amplitude and, consequently, the internal forces. In particular, the base isolation system decreases in about 97% the vibration amplitudes.



(a) Base acceleration (b) Time response

Figure 8 – Time response of the controlled and uncontrolled system under seismic excitation. El Centro.

4. CONCLUSIONS

Two mechanisms for seismic protection of buildings were studied and compared in this study using in both cases a virtual pendulum: a tuned mass damper on the top of the building and a base isolation system. The virtual pendulum consists of a mass that moves on a circular surface, having more flexibility than the classical pendulum. The dimensionless nonlinear equations of motion are derived for each model and used in parametric analysis. The results show that both control mechanisms are quite effective when the structure is subjected to a base harmonic excitation; leading to a significant reduction in the amplitudes of vibration and accelerations of the main structure. The base isolation system led to the best reductions for harmonic and seismic base excitation. However, the system to be used depends on the characteristics of design and ease of installation and maintenance of the seismic protection mechanism in the structure. This is a research project in progress. Future work will include a parametric analysis to determine optimal parameters and to assess the effectiveness of the control in the presence of various types of loading.

5. REFERENCES

Beneveli, S.M.A., 2002, Controle Híbrido para Atenuação de Vibrações em Edifícios, Suzana Moreira Avila Beneveli, Tese de Doutorado, DEC, PUC-Rio, Brasil.

Cano, N.A.O., 2013, Avaliação de Sistemas de Isolamento para o Controle de Vibrações de Edifícios Submetidos a Excitação de Base., Tese de Doutorado, COPPE, Universidade Federal do Rio de Janeiro- Brasil.

Chen, J., & Georgakis, C. T., 2013. Tuned rolling-ball dampers for vibration control in wind turbines. *Journal of Sound and Vibration*, *332*(21), 5271-5282.

Legaza, V.P., 2010, Efficiency of a Vibroprotection System with an Isochronous Roller Damper, *Mechanics of Solids vol.48 Nro.2-2013*

Matta, E., & De Stefano, A. (2009). Robust design of mass-uncertain rolling-pendulum TMDs for the seismic protection of buildings. *Mechanical Systems and Signal Processing*, *23*(1), 127-147.

Oliveira, F.S. 2012, Critérios de projeto para amortecedor tipo pêndulo para controle de vibrações em edifícios altos, Dissertação de M.Sc., Universidade de Brasília, Brasília, DF, Brasil.

Orlando, D., Gonçalves, P.B., "Hybrid nonlinear control of a tall tower with a pendulum absorber", *Structural Engineering and Mechanics*, Vol. 46, No. 2 153-177, **2013** DOI: 10.12989/sem.2013.46.2.153

Pinheiro, M.A.S., 1997, Absorsor Pendular Não-Linear para Redução de Vibrações em Torres Esbeltas. Dissertação de Mestrado – COPPE/UFRJ, Rio de Janeiro.

Soong, T.T. & Dargush, G.F.,1997, Passive energy dissipation systems in structural engineering, Chichester, John Wiley & Sons.