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NUMERICAL SIMULATION OF NON-DARCY OIL FLOW IN NATURALLY FRACTURED RESERVOIRS: HORIZONTAL WELL

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Abstract. *Numerical simulation of oil flow is applied in order to obtain pressure drop forecasts in hydrocarbon reservoirs under different production strategies. Due to differences in properties such as porosity and permeability between the porous medium and fractures in a heterogeneous reservoir, there is a growing interest to study flow regimes in fractured reservoirs. In Naturally Fractured Reservoirs (NFRs), for example, contrasts in permeability and in porosity are due to geological factors. In this work, a numerical simulator has been developed in order to study the non-Darcy oil flow in Naturally Fractured Reservoirs producing through a horizontal well. The physical-mathematical model considers a three-dimensional and isothermal single-phase flow. Non-Darcy effects are incorporated using the Barree and Conway's model, taking into account the inertial effects of the flow. The governing nonlinear partial differential equations are discretized using the finite difference method and a time implicit formulation. The numerical code also allows the use of grid refinement, in order to represent fractures and the horizontal well, providing a better capture of the physical phenomena. A factorization preconditioning technique has been chosen for the numerical solution of the system of algebraic equations, due to its ability to solve matrix equations for problems where the heterogeneity is an important feature. Several production scenarios for hydrocarbon primary recovery have been considered. As expected, it has been concluded that the spatial variation of the properties of the porous medium, due to the heterogeneity of the reservoir, and the flow regime have significant influence in determining pressure and velocity fields, with a consequent impact on the wellbore pressure.*

Keywords: *Barree and Conway's model, finite-difference method, oil flow, naturally fractured reservoir, horizontal well.*

1. INTRODUCTION

In the past few decades, numerical reservoir simulation has increased its importance in the oil and gas industry. Numerical experiments have been considered in order to study, for example, flow in heterogeneous porous media, reservoir management and reservoir characterization. Numerical simulations have been used to optimize oil and gas well production, in order to maximize hydrocarbon recovery. In this context, there is a significant number of examples of relevant problems in which the simulation of heterogeneous reservoirs has been applied successfully (Tavares *et al.*, 2006; Mendes *et al.*, 2012; He and Durlofsky, 2013; Mikelić and Wheeeler, 2013). The heterogeneities present in the reservoir can be detected, for example, from wellbore gauges that provide data during the production/injection of fluids.

In fact, physical-mathematical modeling of heterogeneous porous media has attracted researchers from different areas for a long time. The flow in Naturally Fractured Reservoirs (NFRs) is a situation of practical interest (Nelson, 2001; Biryukov and Kuchuk, 2012). In these rock formations, there are many regions of high contrast in permeability ($\mathbf{k}$) and in porosity (ϕ) caused by geological factors (Fig. 1). A fracture is a naturally occurring macroscopic discontinuity in the reservoir rock due to deformation or physical diagenesis (chemical and physical changes after deposition) (Tiab and Donaldson, 2004). Depending on the scale and on the spatial distribution of heterogeneities, cavities, called vugs, may also be formed (Nair *et al.*, 2008; Guo *et al.*, 2012). High permeability channels, natural fractures and vugs in carbonates, for example, arise from the rock dissolution processes.

Typically, flow in porous media is modeled using the well-known Darcy's law, that predicts a linear relationship between the superficial velocity and the pressure gradient. However, in some cases, inertial flow effects can lead to deviations from the predicted values obtained with the original Darcy's law. High flow velocities in the presence of fractures and/or near the wells are situations in which inertial effects should not be neglected. The Forchheimer's equation, for example, models the inertial effects by adding a non-linear term to the Darcy's law (Tavares *et al.*, 2006). Barree and Conway (2004) have also proposed a model to incorporate the inertial effect on fluid transport in a porous medium, the so-called Barree and Conway's model. For example, Tavares *et al.* (2006) have used the Forchheimer equation for modeling flow in a fractured reservoir, while Al-Otaibi and Wu (2010) have applied the Barree and Conway's model.

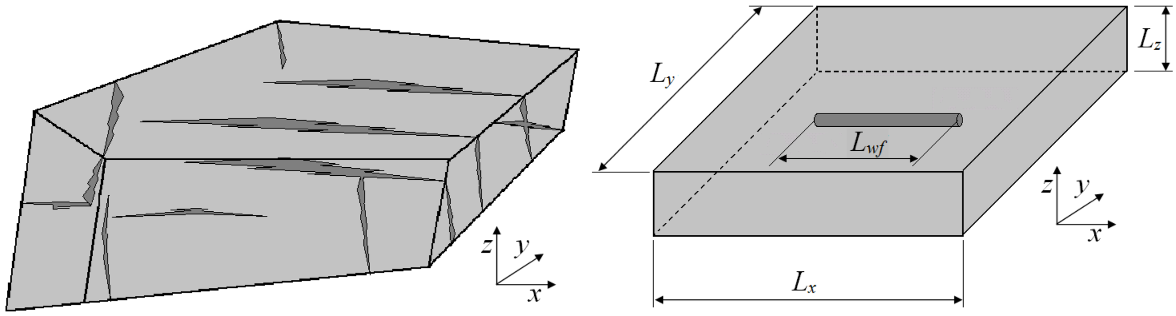


Figure 1. On the left, schematic representation of a NFR: fractures in dark gray color and matrix in light gray color. On the right, schematic representation of a reservoir and a horizontal well, of length L_{wf}, parallel to x-direction.

Aiming to enable or increase the production of certain reservoirs, horizontal wells (Fig. 1) have been utilized to augment the contact area between reservoir and well (Akgun, 2004). They are indicated for low-thickness reservoirs, low-permeability formations and naturally fractured reservoirs. Compared to vertical wells, productivity forecasts are more subject to uncertainty in the case of horizontal wells. This is due to the fact that flow regimes in a reservoir with a horizontal well are not so clearly identified as those for a vertical well (Chaudhry, 2004). This partially results from the fact that the nature of the flow is inherently three-dimensional (the radial symmetry observed in vertical wells does not occur in the same way for horizontal wells). Because of its importance for reservoir engineering, much research has been done on reservoir simulation with horizontal wells (Al-Mohannadi *et al.*, 2007; Dumkwu *et al.*, 2012; Guo *et al.*, 2012), including naturally fractured reservoirs (Shahbazi *et al.*, 2015) and non-Darcy flow (Al-Mohannadi, 2004).

In this work, we study the single-phase flow of slightly compressible oil, in a naturally fractured reservoir, taking into account inertial effects using the Barree and Conway's model and considering production through a horizontal well. Here, fractures do not form a continuous conductive network, so that only a limited number of fractures can communicate hydraulically with each other. Fractures and the matrix contribute to the effective conductivity of the reservoir, but the total capacity of fluid storage is directly related to the matrix porosity (Biryukov and Kuchuk, 2012). In order to address this problem effectively, a three-dimensional reservoir simulator, suitable for applications in the field scale has been developed using the Cartesian coordinate system and the finite difference method. The resulting non-linear governing partial differential equation, written in terms of the pressure, is solved numerically after discretization and linearization using an iterative preconditioned conjugate gradient algorithm. The main objective of this work is to investigate the combined effects of non-Darcy flow and fractures, considering different oil production scenarios, on horizontal well pressure behavior.

2. PHYSICAL MODELING

In single-phase flow in porous media, a partial differential equation for the pressure unknown can be obtained from the governing equations, namely conservation of mass, momentum balance and an equation of state. Here, besides these equations, the following assumptions are considered: i) the rock formation is heterogeneous; ii) the permeability varies in space but not in time; iii) the rock compressibility is small and constant; iv) the fluid is Newtonian; v) the reservoir pressure is above the bubble point pressure; vi) the fluid has a constant composition; and vii) the flow is isothermal.

The mass conservation equation can be written in an alternative form if we introduce the formation-value-factor $B(p,T) = V(p,T)/V_{sc}$ (Ertekin *et al.*, 2001), where the subscript sc indicates standard conditions, and p and T are the pressure and the temperature. Therefore, this factor defines the ratio between the fluid volume measured at reservoir conditions and the fluid volume measured at standard conditions. Thus, using mass conservation, we write

$$\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) + \nabla\cdot\left(\frac{\mathbf{v}}{B}\right) - \frac{q_m}{V_b\rho_{sc}} = 0, \tag{1}$$

where ϕ is the porosity, $\mathbf{v}$ is the superficial fluid velocity, q_m is the source term, V_b is the bulk volume and ρ_{sc} represents the density in standard conditions.

Barree and Conway (2004) proposed a way to incorporate the inertial effects on flow modeling in porous media, in which the permeability $\mathbf{k}$ is replaced by an apparent permeability $\mathbf{k}_{app} = \mathbf{k}_{BC}\mathbf{k}$, depending on the flow conditions and defined as

$$\mathbf{k}_{BC} = \left[\mathbf{k}_{mr} + \frac{\mathbf{1} - \mathbf{k}_{mr}}{1 + \left[\left(\rho|\mathbf{v}|\right)/\left(\mu\boldsymbol{\tau}_c\right)\right]}\right], \tag{2}$$

where $\mathbf{k}_{mr} = \mathbf{k}_{min}\mathbf{k}^{-1}$, and $\mathbf{k}_{min}$ is a minimum permeability value for high flow rates and $\boldsymbol{\tau}_c$ has the dimension of the inverse of a characteristic length (Barree and Conway, 2004).

According to Lai *et al.* (2009), Barree and Conway's model has some advantages compared to other equations which govern momentum balance in porous media, as example, the traditional Darcy's law

$$\mathbf{v} = -\left(\mathbf{k}/\mu\right)\left[\nabla p - \rho g \nabla D\right], \tag{3}$$

applied when inertial effects (among others) are absent, and the Forchheimer's equation

$$\mathbf{v} = -\left(\mathbf{k}/\mu\right)\left[\nabla p - \rho g \nabla D - \rho \boldsymbol{\beta} \mathbf{v}\left|\mathbf{v}\right|\right], \tag{4}$$

that is an expression for momentum balance incorporating inertial effects, in which $\boldsymbol{\beta}$ is a coefficient, described by

$$\boldsymbol{\beta} = c_1 \mathbf{k}^{c_2} \phi^{c_3} \tau_p^{c_4}, \tag{5}$$

where τ_p represents the porous media tortuosity, while c_1, c_2, c_3 and c_4 are constants for a given rock formation.

Using Barree and Conway's model, a single equation can describe a entire range of flow. This is an advantage in context of code implementation on the numerical simulation. Moreover, transition regions between flow regimes are better captured. Lai *et al.* (2009) also argued that Barree and Conway's model provides a plateau associated to high flow rates (described in the literature) and honors Darcy's law and Forchheimer's equation.

It is worth noting that in some applications involving well testing analysis (mainly for gas flow), the term $\boldsymbol{\beta}$ is used in order to compute the so called non-Darcy flow coefficient, D, then considered in the computation of the total skin s',

$$s' = s_M + Dq, \tag{6}$$

where s_M is the skin related to the formation damage, q is a flow rate and Dq represents a skin effect due to non-Darcy flow (Lee *et al.*, 2003). According to Al-Otaibi and Wu (2010), this simplified approach has limitations, in way that it is very relevant the research on non-Darcy flow modeling.

2.1 Hydraulic diffusivity equation

In order to obtain an equation in terms of the unknown pressure, Barree and Conway's model allows us to rewrite the Darcy's law using the apparent permeability tensor ($\mathbf{k}_{app}$)

$$\mathbf{v} = -\left(\mathbf{k}_{app}/\mu\right)\left[\nabla p - \rho g \nabla D\right], \tag{7}$$

where $\mathbf{k_{app}}$ is the tensor of apparent absolute permeability, μ is the fluid viscosity, p is the pressure, D is the depth and g is the gravity acceleration magnitude and $\mathbf{k}_{app}$ is equal to: i) $\mathbf{k}$ for the classical Darcy's law and ii) $\mathbf{k}_{BC}\mathbf{k}$ for the Barree and Conway's model.

Substituting Eq. (7) into Eq. (1), multiplying by V_b and introducing $q_m = q_{sc}\rho_{sc}$, we obtain

$$\Gamma \frac{\partial p}{\partial t} - V_b \nabla \cdot \left[\frac{\mathbf{k}_{app}}{B\mu}\nabla p\right] + J_w(p - p_{wf}) + \Lambda = 0, \tag{8}$$

where (Ertekin *et al.*, 2001)

$$\Gamma = V_b\left\{\left[\left(\phi^0 c_\phi\right)/B\right] + \phi\left(\partial B^{-1}/\partial p\right)\right\} \tag{9}$$

and ϕ^0 and p^0 are respectively the porosity and the pressure of reference, for

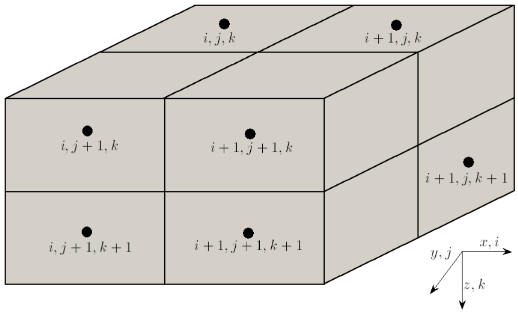
$$\phi = \phi^0\left[1 + c_\phi(p - p^0)\right], \tag{10}$$

c_ϕ is the rock compressibility and $\Lambda = V_b G$ includes the gravitational effects. The source term q_{sc} is used to introduce the well pressure, p_{wf}, by the relation $q_{sc} = -J_w(p - p_{wf})$, where J_w is referred to as the Productivity Index (PI) (Dumkwu *et al.*, 2012). Equation (8) is written in the form of the hydraulic diffusivity equation. It is a nonlinear partial differential equation and the dependent variable is the oil pressure.

For an isothermal flow and pressures above the bubble point pressure, we can also use the following approximations (Ertekin *et al.*, 2001)

$$B = B^0/\left[1 + c_o(p - p^0)\right], \qquad \rho = \rho^0\left[1 + c_o(p - p^0)\right] \qquad \text{and} \qquad \mu = \mu^0/\left[1 + c_\mu(p - p^0)\right], \tag{11}$$

where c_o represents the oil compressibility factor, superscript 0 indicates measurement at the reference pressure and c_μ is the relative rate of viscosity change with pressure.



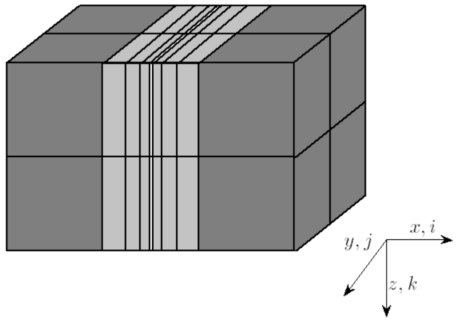


Figure 2. On the left, discretization by finite difference using Cartesian coordinates and centered blocks. The integer subscripts indicate the centers of the cells. On the right, region of local grid refinement (in light gray color) to better represent fractures.

2.2 Discretization of the hydraulic diffusion equation

In the discretization of Eq. (8) we use the finite difference method (Ertekin *et al.*, 2001). A schematic representation of a discretized three-dimensional domain can be seen in Fig. 2 for the Cartesian coordinate system. Subsequently, the integer subscripts i, j and k indicate the centers of the cells along the x-, y- and z-directions, respectively. The fractional subscripts, such as $i \pm 1/2, j, k$, indicate the boundaries of the cells.

In this study, we assume that the permeability tensor is diagonal. Considering centered blocks and an approach based on centered differences and backward time difference approximations (Ertekin *et al.*, 2001), Eq. (8) can be rewritten as

$$\Gamma_{i,j,k}^{n+1}/\Delta t\left(p_{i,j,k}^{n+1}-p_{i,j,k}^{n}\right)+\sum_{l\in\psi_m}\left[T_{l,m}^{n+1}\left(p_l^{n+1}-p_m^{n+1}\right)\right]-J_{w,i,j,k}^{n+1}\left(p_{wf,i,j,k}^{n+1}-p_{i,j,k}^{n+1}\right)=-\Lambda_{i,j,k}^{n+1} \tag{12}$$

where $m=(i,j,k)$, $\psi_m=\{(i-1,j,k),(i+1,j,k),(i,j-1,k),(i,j+1,k),(i,j,k-1),(i,j,k+1)\}$, knowing that $V_b=L_xL_yL_z$, $A_x=L_yL_z$, $A_y=L_xL_y$ and $A_z=L_xL_y$ and introducing the transmissibility in the x-direction as being given by

$$T_{x,i\pm\frac{1}{2},j,k}^{n+1}=\left[\left(k_{app,xx}A_x\right)/\left(B\mu\Delta x\right)\right]_{i\pm\frac{1}{2},j,k}^{n+1} \tag{13}$$

and the values of the transmissibilities in the $i \pm 1/2, j, k$ positions are calculated, by a harmonic average, from their known values in i, j, k and $i+1, j, k$. The same procedure is adopted on the other two spatial directions. Thus, non-Darcy flow effect is present locally for each cell when the apparent permeability is updated and, then, on the computation of transmissibilities. A grid refinement is used so as to better represent the fractures in the field scale (Fig. 2). More detailed information can be obtained in Souza and Souto (2014). A refinement procedure is also applied in order to better capture flow around the horizontal well (De Souza, 2013).

2.3 Numerical Determination of Pressure

The discretized form (12), applied to each cell of the mesh, leads to a nonlinear system of algebraic equations written in terms of the pressure. In order to be able to use specific techniques for solving linear systems of algebraic equations, we have to linearize the transmissibilities, Γ, J_w and Λ. For a simple iteration we get for the terms of transmissibility along the x-direction (Ertekin *et al.*, 2001),

$$T_{x,i\pm\frac{1}{2},j,k}^{n+1}\cong T_{x,i\pm\frac{1}{2},j,k}^{n+1,v}=\left[\left(k_{app,x}A_x/\left(B\mu\Delta x\right)\right]_{i\pm\frac{1}{2},j,k}^{n+1,v}. \tag{14}$$

Therefore, Eq. (12) can be rewritten as

$$\frac{\Gamma_{i,j,k}^{n+1,v}}{\Delta t}\left(p_{i,j,k}^{n+1,v+1}-p_{i,j,k}^{n}\right)+\sum_{l\in\psi_m}\left[T_{l,m}^{n+1,v}\left(p_l^{n+1,v+1}-p_m^{n+1,v+1}\right)\right]+J_{w,i,j,k}^{n+1,v}\left(p_{wf,i,j,k}^{n+1,v+1}-p_{i,j,k}^{n+1,v+1}\right)=-\Lambda_{i,j,k}^{n+1,v} \tag{15}$$

where superscripts v (known values) and $v+1$ (unknown values) stand for the iteration levels. The same procedure is also valid for the terms $\Gamma_{i,j,k}$, $J_{w,i,j,k}$ and $\Lambda_{i,j,k}$.

For the linearized well-reservoir coupling, it is necessary a specific equation for determining the flow rate in the production well so as to calculate the index J_w. In this work, we consider vertical and horizontal wells, neglecting friction and inertia effects within the wells. The total flow rate of the well, Q_{sc}, must be equal to the flux leaving all boundaries of the cells connected to the production well, thus,

$$Q_{sc} = -\sum_{k} J_{w,i,j,k}^{n+1,v} \left[p_{i,j,k}^{n+1,v+1} - (p_{wf})_{i,j,k}^{n+1,v+1} \right] \tag{16}$$

for a well trespassing k layers (horizontal for a vertical well and vertical for a horizontal well). Adopting one of the layers as the reference ($k = W$) and considering the gravitational effects (only for vertical wells) the well pressure can be expressed as

$$(p_{wf})_{i,j,W}^{n+1,v+1} = \frac{\sum_{k} J_{w,i,j,k}^{n+1,v} (p_{wf})_{i,j,k}^{n+1,v+1}}{\sum_{k} J_{w,i,j,k}^{n+1,v}} + \frac{\sum_{k} J_{w,i,j,k}^{n+1,v} \left[-(\rho g)_{i,j,k+1/2}^{n+1,v} (D_{i,j,k} - D_{i,j,W}) \right] + Q_{sc}}{\sum_{k} J_{w,i,j,k}^{n+1,v}} \tag{17}$$

and

$$(p_{wf})_{i,j,k}^{n+1,v+1} = (p_{wf})_{i,j,W}^{n+1,v+1} + (\rho g)_{i,j,k+1/2}^{n+1,v} (D_{i,j,k} - D_{i,j,W}), \tag{18}$$

where $\rho_{i,j,k+1/2}^{n+1,v}$ is the arithmetic average between $\rho_{i,j,k}^{n+1,v}$ and $\rho_{i,j,W}^{n+1,v}$.

In the case of a horizontal well (parallel to x-direction) and assumptions adopted for the fluid flow, the wellbore here is assumed to have infinite conductivity and the productivity index J_w is given by (Al-Mohannadi, 2004),

$$J_{w,i,j,k}^{n+1,v} = \left\{ \frac{2\pi\sqrt{k_{app,z}k_{app,y}}}{\mu B} \frac{\Delta x}{\left[1 - (r_w/r_{eq})^2\right] \ln (r_{eq}/r_w)} \right\}_{i,j,k} \quad \text{and} \quad r_{eq,i,j,k}^{n+1,v} = \sqrt{\frac{(\Delta z \Delta y)_{i,j,k}}{\pi}} \exp(-0.5), \tag{19}$$

where r_{eq} is the equivalent radius (Peaceman, 1983) and r_w the wellbore radius. In order to take into account the inertial effects of a non-Darcy flow, the apparent permeabilities are used in the calculation of J_w and r_q in Eq. (19), according to Al-Mohannadi (2004).

Equations (15) to (17) form a system of equations for reservoir and well pressures. The resulting linearized system of equations is solved using an approximate factorization approach. This technique is able to deal with practical problems in the numerical simulation of heterogeneous reservoir, whose properties can impact the coefficient matrix (matrix conditioning) mainly due to the large variability of the permeability field. The Preconditioned Conjugate Gradient method is employed to solve the system of algebraic equations using a Jacobi preconditioner (Ertekin *et al.*, 2001; Saad, 2003). A nested iteration process (Ertekin *et al.*, 2001) is used in the iteration of the inner loop: the method of Conjugate Gradient is applied to determine the pressure and, during the iteration of the outer loop, the transmissibilities are updated.

3. NUMERICAL RESULTS

The methodology previously discussed was used in numerical simulations for constant production rate and horizontal well. Table 1 summarizes default data for simulations, considering first a reservoir without fractures. P_i is the pressure at the reservoir top, Q_{sc} is the wellbore production rate at standard m^3/day and n_x, n_y and n_z are the numbers of cells in x-, y- and z-directions, respectively. In all cases, μ is a constant. Tables 2 and 3 show specific data for studies involving horizontal well characteristics and heterogeneities. Figure 3 depicts arrangement of fractures considered for a NFR. When Barree and Conway model was applied, default data are $k_{mr} = 0.5$ and $\tau_c = 3.28 \times 10^{-1}$ m^{-1}.

Table 1. Default data used for simulations.

Parameter	Value	Parameter	Value	Parameter	Value
μ (Pa · s)	0.001	$P_i = p_0$ (MPa)	31.03	L_{wf} (m)	609.6
$k_x = k_y$ (m^2)	19.74×10^{-15}	n_x	48	$L_x = L_y$ (m)	1828.8
k_z (m^2)	4.94×10^{-15}	n_z	25	L_z (m)	67.06
c_ϕ (Pa^{-1})	1.45×10^{-9}	n_y	49	$\phi_0 = \phi_{init}$	0.2
c_o (Pa^{-1})	0.58×10^{-9}	r_w (m)	0.0762	ρ_b^0 (kg/m^3)	839.4
Q_{sc} (std m^3/day)	-114.47	B_o^0 (m^3/std m^3)	1.0	t_{max} (days)	720.0

Figure 4 includes a plot of $p_{wf} \times \log t$ for different numerical grids (Table 4). These data were based on those used by Al-Mohannadi (2004) for a homogeneous and anisotropic reservoir. One can observe characteristic slopes obtained and there is evidence of numerical convergence as the grid is refined in a large period of simulation. It is worth noting that

Table 2. Changes in default data for matrix region in porous media.

Parameter	Value	Parameter	Value	Parameter	Value
$k_x = k_y$ (m^2)	1.974×10^{-15}	L_{wf} (m)	304.8	$\phi_0 = \phi_{init}$	0.1
k_z (m^2)	0.494×10^{-15}	Q_{sc} (std m^3/day)	-79.49	L_z (m)	152.4

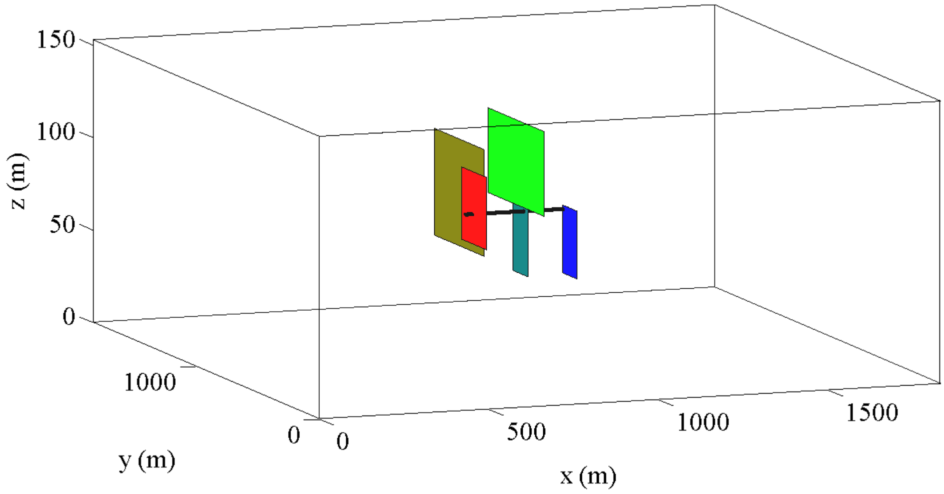


Figure 3. Horizontal well and arrangement of fractures considered, for the reservoir named NFR 1. Along with $x-$direction, from the left to the right, fractures are numbered from 1 to 5.

Table 3. Default data for fractures in porous media, considering Fig. 3.

Fracture's number	k_f (10^{-12} m^2)	ϕ_f	h_f (10^{-3} m)	A_{yz} (m^2)
1	14.80	0.60	1.22	7944.0
2	11.84	0.45	0.76	23370.7
3	24.66	0.65	1.52	20760.9
4	19.74	0.70	1.83	5958.2
5	29.68	0.55	2.29	4388.6

external boundary effects on wellbore pressure were detected (for the same time) for all grids. It is possible to observe the existence of a plateau for all curves in short times and it is dependent on the grid size. The smaller plateau is associated with the finer mesh. This is a numerical artifact well known in the literature, and it is due to the model of well-reservoir coupling technique (Peaceman, 1983). Although numerical results at early times can be improved by some specific techniques (Dumkwu *et al.*, 2012; Al-Mohannadi *et al.*, 2007), focus of this work is not on very early times in porous media. Thus, considering the results after 10^{-2} days of oil production, $n_x = 48$, $n_y = 49$ and $n_z = 25$ were chosen to be used in the numerical simulations.

Table 4. Data for grid refinement. Δy_{wf} and Δz_{wf} are dimensions associated to the block that hosting the well.

Grid	n_x	n_y	n_z	$\Delta y_{wf} = \Delta z_{wf}$ (m)
1	12	13	7	12.192
2	24	25	13	6.096
3	48	49	25	3.048
4	96	97	49	1.524

Still referring to Fig. 4, there is a comparison among a set of horizontal wells of different lengths and a fully penetrating vertical well, for the same reservoir. Simulations were run for a smaller time, because for vertical well the pressure drop is excessive. One can note that for greater horizontal well lengths there are smaller pressure drops, as expected. For the same length, horizontal well conducted to a smaller pressure drop than the vertical well.

Hereafter, results are presented for Δp_{wf} and $d\Delta p_{wf}/d\ln t$, for P_i used as reference pressure ($\Delta p_{wf} = P_i - p_{wf}$). Figure 5 shows comparisons involving Darcy's law and non-Darcy flow modeled using the Barree and Conway model. Firstly, there is a study comparing a homogeneous reservoir and the NFR 1 (the homogeneous reservoir plus arrangement of fractures). It is possible to observe that non-Darcy flow presented bigger pressure drop than Darcy flow. Comparing

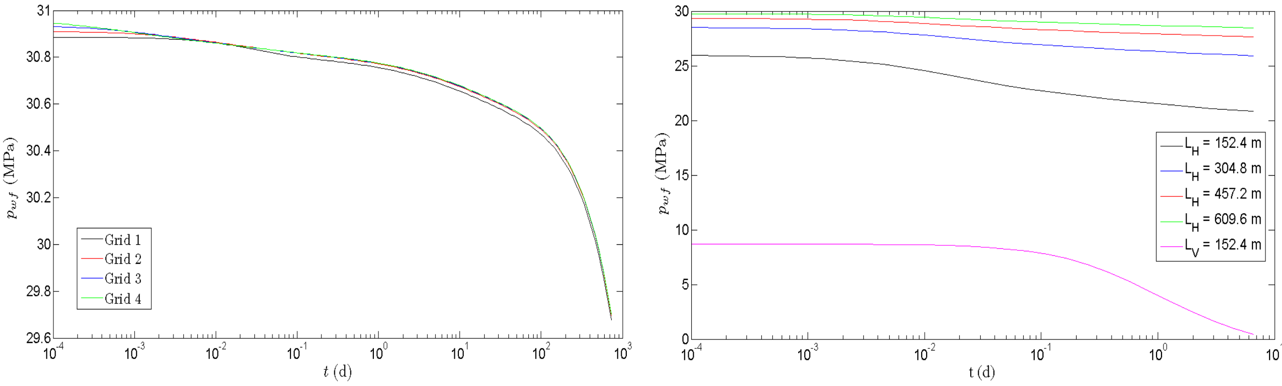


Figure 4. On the left, results from different grids using data from Tables 1 and 4 (for Darcy flow). On the right, comparison of lengths for vertical (L_V) and horizontal (L_H) wells, using data from Tables 1 and 2 (for non-Darcy flow).

reservoirs, NFR 1 showed smaller pressure drop than the homogeneous reservoir. These behaviors are expected and proved the influence of non-Darcy flow and NFR on the flow. Slopes at very early times, at the region related to the numerical artifact associated to well-reservoir coupling technique, were influenced by the presence of fractures. Figure 5 also present comparisons involving different production rates for Darcy and non-Darcy flows, all results for the NFR 1.

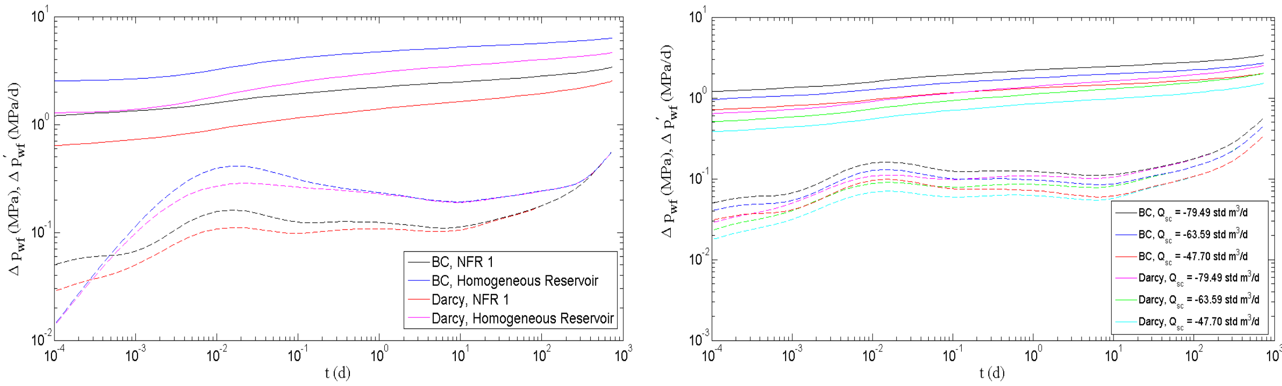


Figure 5. On the left, comparison among Darcy's law and Barree and Conway's model (BC) in a homogeneous reservoir and in a NFR. On the right, variation of production rates in a NFR.

Important parameters for NFRs were also studied using the Barree and Conway's model and the corresponding results are depicted in Fig. 6. Greater k_f values lead to lower pressure drops because hydraulic conductivity increases as a function of this parameter. For fractures porosity variations, there are small effects in very early times and in late times (less pronounced yet). The lowest values are related to a reduced fluid storage and to an earlier perception of the reservoir boundaries (porous media hydraulic conductivity increases). Different values of k_{mr} and τ_c were also tested for Barree and Conway's model and the NFR 1and results are presented in Fig. 7. As expected, inertial effects are more significant for low values of k_{mr} and τ_c. Results for Darcy's law are included for comparison.

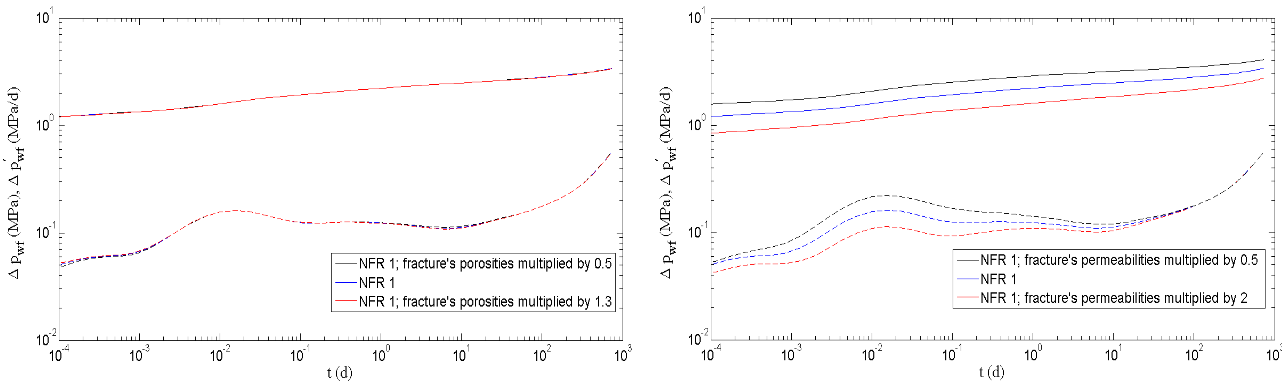


Figure 6. On the left, variation of fracture's porosity in a NFR. On the right, comparison among different fracture's permeabilities in a NFR.

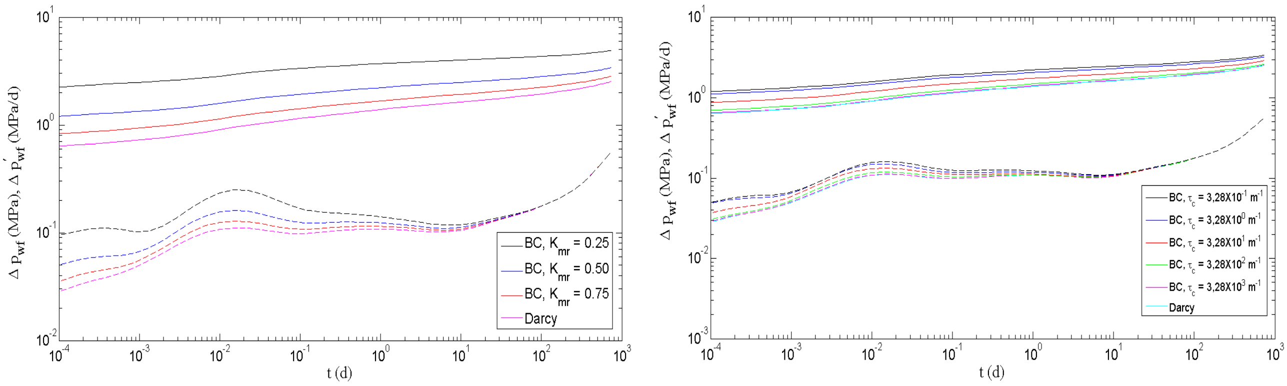


Figure 7. On the left, comparison involving different k_{mr} for Barree and Conway model, for a NFR. On the right, tests involving different τ_c for Barree and Conway model, for a NFR.

4. CONCLUSIONS

In this paper we discussed the application of the Barree and Conway's model to numerical reservoir simulation of oil flow in a naturally fractured reservoir (modeled in Cartesian coordinates) considering only discretely fractured reservoirs and horizontal well. Finite-difference method and an implicit formulation were applied in order to obtain numerical solution for the wellbore pressure for a constant production rate. Comparing solutions from Darcy's law and Barree and Conway's model it was possible to note significant differences between the pressure drop curves and to give us insights as to how parameters like k_{mr} and τ_c influence the wellbore pressure. The methodology employed was able to detect the combined effects of inertial flow and heterogeneities. Indeed, two effects should be highlighted when we consider numerical simulation of naturally fractured reservoirs with non-Darcy flow: (i) fractures reduce wellbore pressure drop while (ii) inertial effects increase wellbore pressure drop. Thus, an accurate modeling of flow in hydrocarbons reservoirs is fundamental to properly quantify both effects, and this is very important to reservoir management. Also, it is worth noting the role of the horizontal well: (i) on the pressure drop reduction compared to vertical well and (ii) in a combined application to natural fractures. Results showed that non-Darcy can be important also for horizontal well-reservoir systems.

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