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ROBUST CONTROL DESIGN BY NUMERICAL OPTIMIZATION APPLIED TO HYDROELECTRIC TURBINES UNDER PARAMETRIC UNCERTAINTIES

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Abstract. *This work proposes the use of numerical optimization technique based on Direct Transcription as a method for the robust design of the control system for hydroelectric turbines. The controller design for this type of application involves physical and operational constraints. The usual control design techniques typically do not explicitly incorporate constraints in the formulation of the problem resulting in a trial and error design. By means of optimization is possible to formulate a controller design problem that explicitly takes into account the constraints and issues such as the modeling errors. For this purpose normally one may use the Shooting optimization technique that is based on the integration of differential equations of the model but it may have convergence difficulties due to its typical sensitivity to initial estimates of the parameters. This paper examines the technique of Direct Transcription as an alternative for the optimization-based designs where the system dynamics are included as constraints to the optimization problem. This causes the errors due to the quality of the initial estimate to be diluted over the discretization nodes, usually implying smaller convergence issues. Although the method might be extended to nonlinear systems, in this application it was used a linear deviation model suggested in the literature for the configuration of a single generator unit connected to an isolated system. Although the proposed method can be extended to other kinds of controllers and because it is usual in practical applications, the PID controller was employed here. The constraints considered are the maximum displacement and speed of the actuator and other operating limits such as the maximum inverse response amplitude of mechanical power in a maneuver for changing the operating condition. The problem of robustness was formulated for parametric uncertainties of the model. The numerical results suggest that the Direct Transcription technique is well suited to the robust control design of hydroelectric turbine system exhibiting better results than the Shooting technique.*

Keywords: *Numerical Optimization, Direct Transcription, Hydraulic Turbines Governors.*

1. INTRODUCTION

Hydropower plants are renewable forms of energy generation. According to ONS, in Brazil, in 2013 the electricity generated by hydropower accounted for about 79% of the energy matrix. Optimizing the control means increasing its generation capacity, and contribute to fewer emissions resulting from fossil fuels into the environment, by thermoelectric plants. The control improvement is in addition to better performance and protection of facilities and reduced maintenance costs.

This paper proposes an alternative to the solution of an existing control problem in many installations of hydroelectric plants; the optimization of the parameters of PID controller based on the so-called direct transcription control strategy. The goal is to implement this strategy taking advantage of features present in existing installations; they are common resources to various manufacturers, both in new installations and in installations in operation for some time. To apply the technique of direct transcription, a simple model was adopted but with uncertain parameters.

Fig. 1 shows a simplified schematic of a hydroelectric power plant adopted here. It has no intake tunnel, with a single penstock, providing mechanical power to a single isolated generating unit feeding a single load.

The water contained in a reservoir is conducted through the penstock and transfers its energy to a Francis type turbine.The wicket gates are powered by an electro-hydraulic actuator that controls the opening and consequently the turbine rotation speed. The same shaft is shared by the turbine and a generator, which transforms mechanical power into electrical power that is supplied to a given load. After passing through the turbine water is discharged to the downstream level through a draft tube.The control action is exerted on electro-hydraulic actuator.

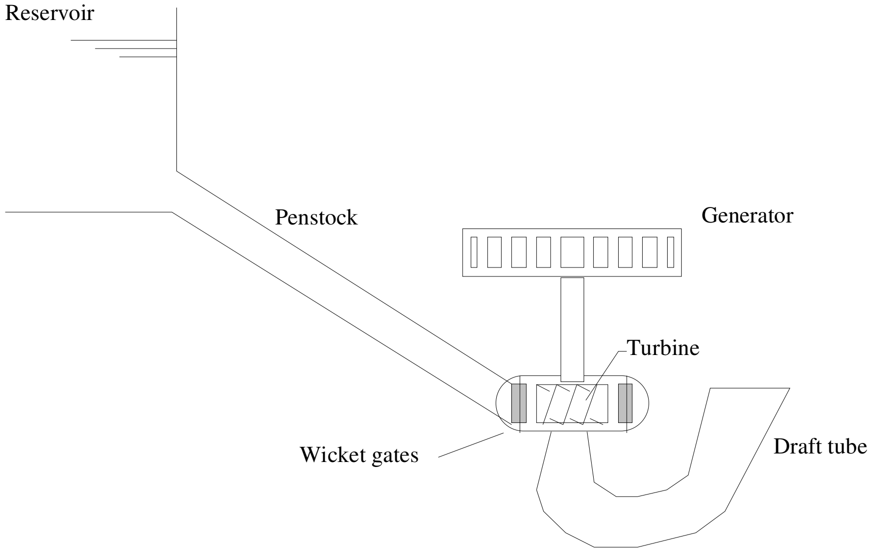

Figure 1. Simplified schematic of a hydroelectric power plant

2. MATHEMATICAL MODEL OF A TYPICAL HYDRO POWER PLANT

A hydropower plant is a complex system involving hydrodynamics and mechanic-electric dynamics all coupled and nonlinear. A mathematical model of a hydropower plant has many variables, depending on the individual physical characteristics of each installation. Despite its complexity, starting from a simple linear model and operational contraints is enough to keep important features of the process. The mathematic model can be splited into four parts, connecting to each other: Penstock and turbine, electro-hydraulic actuator, generator and controller.

2.1 Turbine and penstock

In the following, we assume that:

- The penstock pipe is inelastic
- The water is incompressible
- The hydraulic resistance is negligible
- The velocity of the water varies directly with the gate opening and with the square root of the net head
- The turbine output power is proportional to the product of head and volume flow.

The turbine output power is proportional to the product of head and volume flow. According to Kundur (1993) and IEEE (1992), the turbine and penstock characteristics are determined by equations of velocity of the water in the penstock, turbine mechanical power and acceleration of water column. For small displacement about an operating point, in the frequency domain, the model become

$$\frac{P_m}{g} = \frac{2-2T_W s}{2+T_W s} \tag{1}$$

The equation (1) represents the classical transfer function of a hydraulic turbine. It reveals how the turbine power output behaves in response to a change in gate opening for an ideal lossless turbine. P_m is the turbine mechanical power variation, g is the deviation of the gate position and $T_W = \frac{LQ_0}{a_g A H_0}$, the water starting time. L is the length and A the cross section area of the penstock, Q_0 is the rated flow rate and H_0 is the water head of hydraulic turbine, and a_g is the gravity acceleration.

2.2 Electro-hydraulic actuator

Based on the work of Qian, Yi and Liu (2011), the actuator composed of the action set as a pilot valve and servo can be expressed as a first order transfer function, where T_G is the time constant of the actuator. Thus, the transfer function of the electro-hydraulic actuator is:

$$\frac{g}{u} = \frac{1}{1+T_G s} \tag{2}$$

The equation (2) models how the gate opening changes in response to control action u.

2.3 Generator unit and load

If the generator unit operates isolated, the the transfer function in the equation (3) models how is the deviation of the angular velocity ω in response to the deviation of the net power (mechanical – electric) demanded by the load.

$$\frac{\omega}{P_m - P_e} = \frac{1}{K_D + T_M s} \tag{3}$$

K_D is the auto load control constant and T_M is the starting mechanical time constant of the generator.

2.4 Controller

The controller is represented by means of the $G(s)$ transfer function, and is considered here a PID controller:

$$\frac{e}{\omega_r - \omega} = k_P + \frac{k_i}{s} + \frac{k_d s}{T_d s + 1} = G\left(s\right) \tag{4}$$

k_p, k_i and k_d gains are called: proportional, integral and derivative, respectively. The T_d time constant is associated with derivative action for filtering purposes to reduce noise amplification; e is the error between the reference speed and ω_r the actual speed ω.

2.5 Block diagram

The block diagram in Figure 2 represents the complete mathematical model.

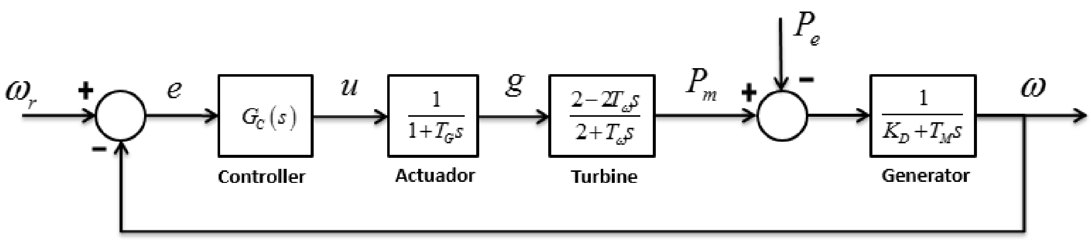

Figure 2. Block diagram of the complete mathematical model

3. OPTIMAL CONTROL PROBLEM FORMULATION

The $G(s)$ controller design can be done by means of a nonlinear optimization problem in which the system dynamic appear as a constraint. For the solution of this problem it was used the Direct Transcription technique which requires the discretization of time. The set of state variables and control at all times are part of the set of free parameters P of the optimization problem. That is, the optimal control problem is converted into a parametric optimization problem.

In this paper it is used the PROPT(Optimal Control Software) which is a compiler that automates the conversion of an optimal control problem into an equivalent problem of parametric optimization to be solved by SNOPT (Sparse Nonlinear Optimization) that is an appropriate solver for nonlinear optimization with sparse matrices. It is worth mentioning that sparsity is an intrinsic characteristic of the Direct Transcription problem caused by model discretization in time.

Every model has a number of uncertainties and they can be classified into parametric and nonparametric. The non-parametric uncertainties refer, for example, the unmodeled dynamics like those of actuators and sensors. The uncertain parameters are due, for example, to changes in model parameters such as the location of the poles and zeroes of the system. In hydraulic Francis turbines control problem, the parametric uncertainties can be an issue in the face of variation reported in the technical literature Fang *et al.* (2008), Cheng *et al.* (2002), and Khodabakhshian Golbon (2005), Kou *et al.* (2010), Qian, Yi and Liu (2010), Qian, Yi and Liu (2011) and Zhang and Zhang (2010). The method of optimization-based controller design is also suitable to incorporate robustness issues, i.e. taking into consideration the modeling errors. To adapt the optimal control problem to use the parametric uncertainties information it was considered the following range of parameter values, whereas the others are supposed fixed and equal to the nominal.

$$1 \le T_w \le 4 \tag{5}$$

$$3 \le T_M \le 10 \tag{6}$$

To capture the dynamic changes due to this parametric variation, it must be incorporated into the optimal control problem. As the range is wide, we use the range in the Table 1, which defines five pairs of values to try to cover a representative space for the parameters:

Table 1. Range of Parameters.

Experiment	$T_w(s)$	$T_M(s)$
1	1	3
2	1	10
3	2.5	6.5
4	4	3
5	4	10

The optimal control problems is intended to minimize the cost function J (7), subject to the set of constraints (8) to (16). It incorporates each of the five parametric variations. It is assumed here that the five possible parameters pairs are equally important, resulting in an objective function with the same penalties.

$$\min_{p} J = \int_{0}^{t} \sum_{j=1}^{5} \left(\omega_r - \omega_j\right)^2 dt \tag{7}$$

Subject to the following dynamic constraints for $j = \{1, 2, .., 5\}$:

$$\dot{z}_j = k_i(\omega_r - \omega_j) \tag{8}$$

$$\dot{g}_j = \frac{1}{T_G}\left(u_j - g_j\right) \tag{9}$$

$$\dot{x}_j = \frac{1}{T_{w_j}}\left(g_j - P_{m_j}\right) \tag{10}$$

$$\dot{\omega}_j = \frac{1}{T_{M_j}}\left[\left(P_{m_j} - P_e\right) - K_D\omega_j\right] \tag{11}$$

$$\dot{v}_j = \frac{1}{T_d}\left[\left(\omega_r - \omega_j\right) - v_j\right] \tag{12}$$

also subject to the following algebraic constraints:

$$u_j = k_p\left(\omega_r - \omega_j\right) + z_j + \frac{k_d}{T_d}\left[\left(\omega_r - \omega_j\right) - v_j\right] \tag{13}$$

$$P_{m_j} = 2\left(x_j - g_j\right) \tag{14}$$

and also subject to the following operational constraints:

$$g_{min} \le \left(g_j\right) \le g_{max} \tag{15}$$

$$\dot{g}_{min} \le \left(\dot{g}_j\right) \le \dot{g}_{max} \tag{16}$$

The block diagram of Figure 3 illustrates how to represent the range of control system parameters.

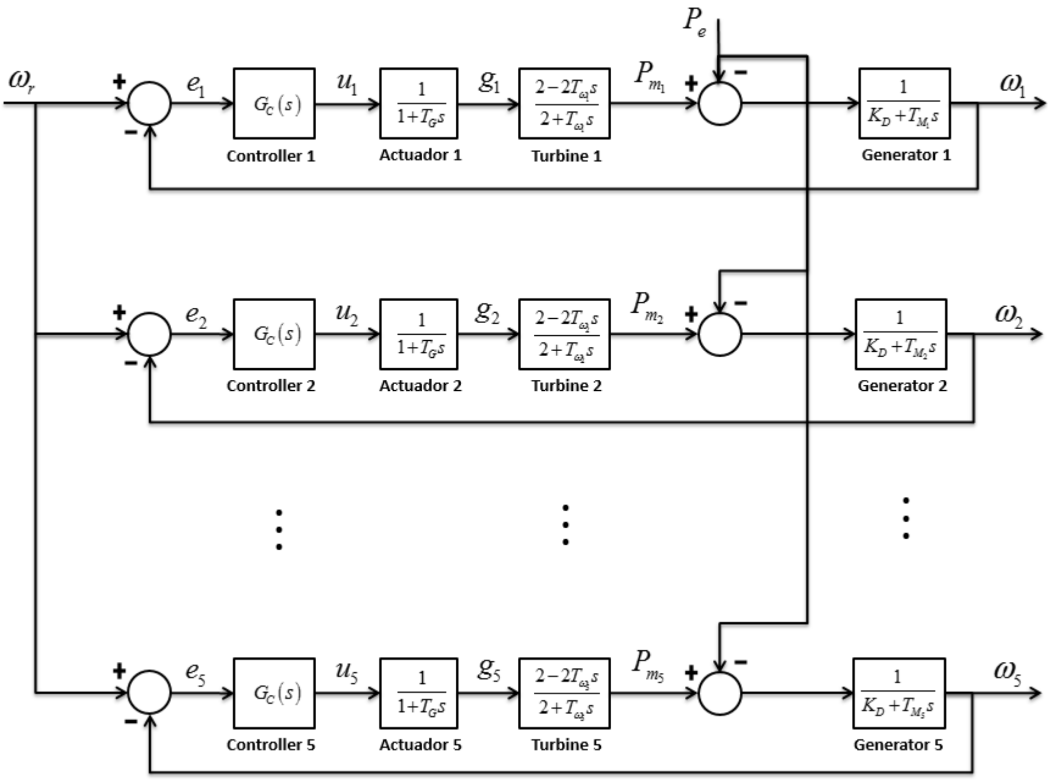

Figure 3. Block diagram of the experiment.

4. EXPERIMENTAL RESULTS

Using the sets defined in Table 1, the objective function (7), and subject to dynamic constraints (8) to (12), algebraic restrictions (13) and (14), and operating restrictions (15) and (16), and based in the block diagram of Figure 3, the robust optimization problem was solved and it was found the following tuning parameters:

$$k_p = 0.78$$

$$k_i = 0.13$$

$$k_d = 0.024$$

One might measure the effectiveness of the robust design by means of the Bode plots in open loop (see Figure 4). Bode plots (a) and (b) are for the five systems considered for the same robust PID. Each plot is associated to the respective block diagram number. The Bode plots (c) and (d) are also related to the same five systems, but controlled by a PID with the original parameters:

$$k_p = 1.44$$

$$k_i = 0.226$$

$$k_d = -0.109$$

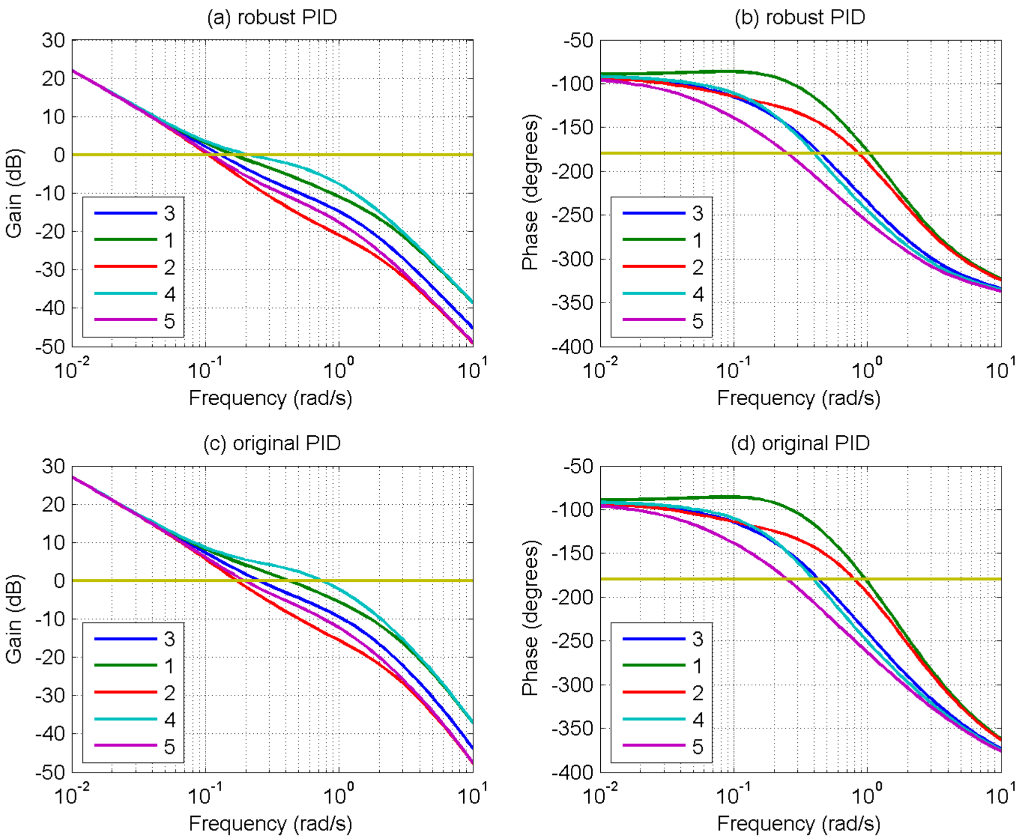

Figure 4. Bode diagrams - robust x original.

Bode plots reveals that the robust PID differs from the standard controller since it provides a smaller bandwidth than the original controller while keeping similar phase characteristics. A lower bandwidth gives increased relative stability margins and decreases the amplification of high frequency noise and modeling errors that normally are also high frequency. Thus, the optimization problem found a solution that in robust indeed. Table 2 summarizes the stability margins found for each of the five schemes when controlled by the robust PID and by the original PID. In all cases the stability margins are higher than the original system. Note that for T_W = 4 and T_M = 10, the original system is unstable, but with robust PID we have a good phase margin. Recalling that phase margin may be considered a more appropriate metric to evaluate the robustness of the system than the gain margin since phase margin represent tolerance to unmodeled dynamics, while the gain margin is a tolerance to errors in the steady state that are more easily avoided in the modeling process.

Table 2. Stability margins

			Robust PID	Original PID	Robust PID	Original PID
Experiment	T_w(s)	T_M(s)	Gain margin (dB)		Gain phase (degrees)	
1	1	3	2.9	1.5	58.1	31.9
2	1	10	3.6	1.8	90.8	59.5
3	2.5	6.5	9.8	4.9	64.7	57.9
4	4	3	1.3	0.7	41.6	-48.6
5	4	10	2.3	1.2	37.4	13

Figure 5 show the plots of the time responses obtained with the robust PID controller applied to each of the five considered systems, and again each tag is related to the corresponded block diagram. Notice that, as expected, all constraints were met.

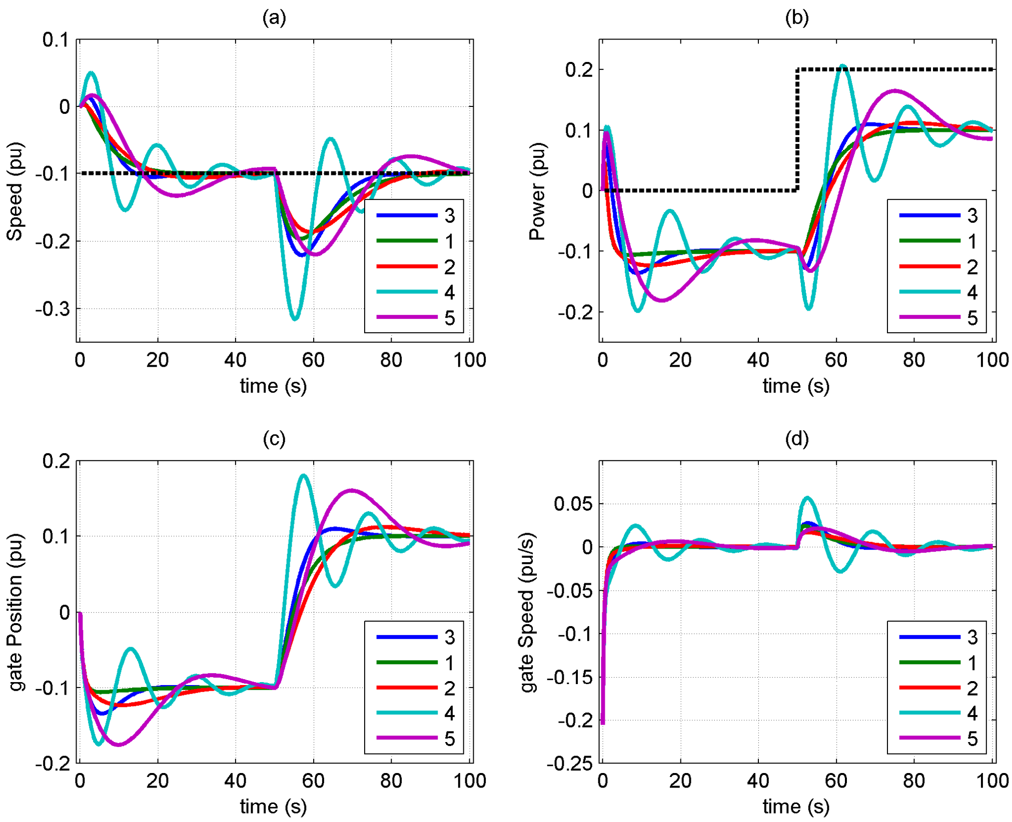

Figure 5. Time response with the robust PID.

5. CONCLUSIONS

The mathematical model of a hydroelectric plant has great complexity due to the unique characteristics of each installation and due to the difficult in fully describe all phenomena. Nevertheless, it is possible to describe the main features involved by means of a linear model if the physical and operational constraints are also include. However, the model parameters are normally uncertain. Based on this, the design of the control system should explicitly take into account the modeling errors.

This work dealt with the robust controller design problem of a hydroelectric plant through numerical optimization. The Shooting method is one candidate to solve this problem, but it is sensitive to initial estimates of the problem. To work around this difficulty, we analyzed the performance of the Direct Transcription method in face of constraints and modeling errors.

Since the PID control is the most common in this type of application, this structure was used in the optimization problem. Note that both, other kind of controllers as more elaborate models could have been considered in the proposed methodology.

The optimal robust control design problem was formulated by a finite set of models spanning the range of values of the uncertain parameters. The range of values were chosen so that a manageable number of equations were incorporated into the optimization problem. The cost function represents the cumulative squared error and the controller is required to keep all the constraint satisfied in face of parametric uncertainties.

Since the plant model is linear, it was possible to also quantifying the robustness achieved by means of stability margins. The result show that the optimal controller exhibit higher phase margins even when this feature is not included in the cost function or as constraints of the optimization problem.

As a continuation of this work, we propose to investigate the validity of the method to more complex models.

6. ACKNOWLEDGEMENTS

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