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# Identification of nonlinear finite element model through discrete-time Volterra series

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***Abstract.*** *In many mechanical systems, the dynamical behaviour is needed to design and to control the integrity of these structures. The numerical modelling is a very powerful tool to understand the dynamical behaviour. Nevertheless, the identification of the numerical parameters is not easy. This work deals with the identification of nonlinear finite element model using discrete-time Volterra series. The main idea of the Volterra series is to describe the discrete-time output of a nonlinear system using multidimensional convolutions between the Volterra kernels and the excitations. A metric based on the residue between the experimental and the numerical Volterra kernels is used to identify the parameters of the numerical model. First, the identification of the linear parameters is performed using a metric based only on the first order Volterra kernels. Then the nonlinear parameters are identified through a metric based on the higher-order kernels. To put in light the applicability of this technique, this work focus on the identification of the some parameters in a nonlinear finite element model of a buckled beam that can be preloaded by compression mechanism. This work shows that essentially the output signal from the numerical model after the updating and the experimental one are similar.*

***Keywords:*** *Volterra series, model updating, nonlinear finite element model, nonlinear identification, sensitivity.*

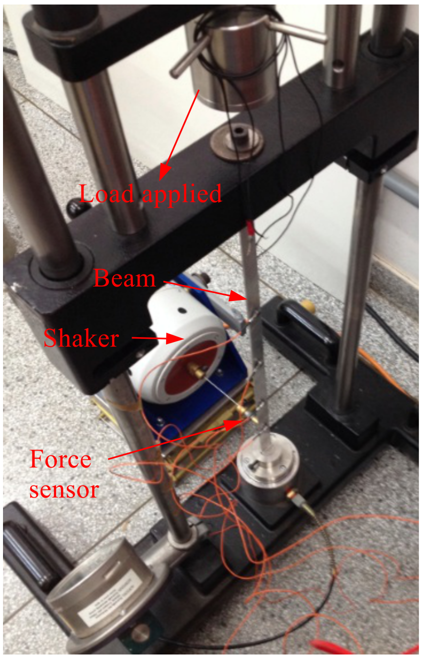
## 1. INTRODUCTION

The study of the dynamical behaviour take an important place in the design of many mechanical systems. In addition, the knowledge of the dynamical behaviour of a structure is important to control their integrity and to predict their service life. The nonlinear finite element method is a very powerful tool to study the nonlinear vibrations. Nevertheless, it is not easy to identify the parameters of the model (e.g. the mechanical properties, the boundary conditions). In the case of the linear structures, the methods for updating the numerical models are well known and are still used for industrial applications (e.g. Friswell and Mottershead (1995); Link (1998)). However, nonlinear effects due to large displacements, gaps, jumps, discontinuities, etc. are very common in the practical systems (e.g. Worden and Tomlinson (2001); Kerschen *et al.* (2006)). Despite of the important number of investigation Adams (2002); Lenaerts *et al.* (2003); Isasa *et al.* (2011); Noël and Kerschen (2013); da Silva *et al.* (2009, 2010); da Silva (2011); Shiki *et al.* (2014), the nonlinear model updating techniques are not mature as the classical linear ones. The Volterra series can be used to define metrics for the nonlinear model updating (see da Silva *et al.* (2010); da Silva (2011); Chaterjee (2010); Hansen *et al.* (2014)). With the Volterra series, the output of a nonlinear system is the sum of a linear contribution and nonlinear ones (see e.g. Rugh (1981); Wiener (1958); Schetzen (1980)). The linear contribution is the convolution between the first Volterra kernel and the excitations. The nonlinear contributions are multidimensional convolutions between the higher-order Volterra kernels and the excitations. The Volterra kernels are computed thanks to an orthogonal basis based on the Kautz filter (see da Silva *et al.* (2010); Shiki *et al.* (2014); Hansen *et al.* (2014)).

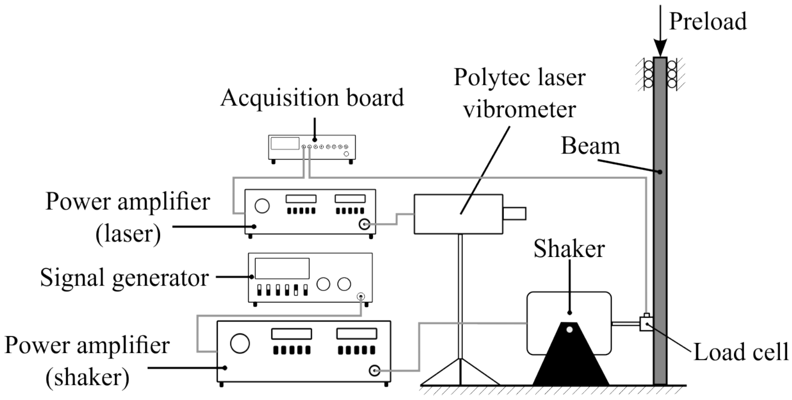
This paper is organised in four sections. First, the experimental setup is described. Then, the nonlinear finite element model is presented. The third section deals with the model updating. Finally, some remarks are summarised.

## 2. EXPERIMENTAL SETUP

The aim of this experimental setup is to study the vibration of a preloaded buckled beam. The test structure is composed of a vertical aluminium beam with dimensions of $460 \times 18 \times 2$ *mm* (see figure 1). The bottom of the beam is clamped and a preload is applied on the top of the beam. The preload force is smaller than the buckling force. The vibration of the beam are generated by an electrodynamic shaker attached at 65 *mm* from the bottom of the beam. A load cell is placed between the shaker and the beam to record the exciting force by the meaning of an acquisition board. The dynamical behaviour of the beam is measured using the velocity recorded by a laser vibrometer in the middle of the beam. The sampling frequency is equal to 1024 *Hz* and 4096 samples are recorded at each experimental test.



(a) General view



(b) Scheme

Figure 1. Experimental setup

## 3. NONLINEAR FINITE ELEMENT MODEL

The model of the preloaded buckled beam uses the classical updated Lagrangian formulation of the 3D nonlinear finite element method. In each time step the displacement field is computed according to the momentum equation (see Wriggers (2008); Belytschko *et al.* (2014)):

$$\rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2} = \boldsymbol{\nabla}.\boldsymbol{\sigma} + \rho \boldsymbol{b} \tag{1}$$

where $\boldsymbol{u}$ is the displacement, $\rho$ is the mass density, $\boldsymbol{\sigma}$ is the Cauchy stress tensor and $\boldsymbol{b}$ is the specific body forces. The mesh used in this model is composed of linear hexahedral elements. The displacement fields is computed at each node of the elements. The deviatoric part of the Cauchy stress is evaluated using a $2 \times 2 \times 2$ Gauss quadrature scheme. To overcome the locking phenomenon, the pressure is considered constant over the element and computed only at a central quadrature point. This is a so called selective Reduced Integration. After integration of the weak form of the equation (1) over the mesh, the system of equations becomes:

$$\boldsymbol{\mathcal{M}}\ddot{\boldsymbol{u}} + \underbrace{\boldsymbol{\mathcal{C}}\dot{\boldsymbol{u}} + \boldsymbol{\mathcal{K}}\boldsymbol{u}}_{\boldsymbol{\mathcal{F}_{int}}(\boldsymbol{u},\dot{\boldsymbol{u}})} = \boldsymbol{F_{ext}} \tag{2}$$

where $\boldsymbol{\mathcal{M}}$ is the mass matrix and $\boldsymbol{\mathcal{F}_{int}}$ and $\boldsymbol{F_{ext}}$ are the internal and external force vectors. $\boldsymbol{\mathcal{K}}$ is the stiffness matrix. The damping matrix, $\boldsymbol{\mathcal{C}}$, is considered to be proportional to the stiffness matrix (*i.e.* $\boldsymbol{\mathcal{C}} = d \times \boldsymbol{\mathcal{K}}$) The dynamical behaviour is computed thanks to the generalised-$\alpha$ method proposed by Chung and Hulbert (1993). With this time integration scheme, the unknowns are computed at the time $t + \Delta t$ due to the value at the time $t$. Note that the evolution of the Cauchy stress tensor, $\boldsymbol{\sigma}$, is computed thanks to the Jaumann's objective time derivative. The equilibrium equation is rewritten pondering inertia forces by the parameter $\alpha_M$, and internal and external forces by the parameter $\alpha_F$:

$$(1-\alpha_M)\boldsymbol{\mathcal{M}}\ddot{\boldsymbol{u}}^{t+\Delta t} + \alpha_M \boldsymbol{\mathcal{M}}\ddot{\boldsymbol{u}}^{t} + (1-\alpha_F)\boldsymbol{\mathcal{F}_{int}}^{t+\Delta t} + \alpha_F \boldsymbol{\mathcal{F}_{int}}^{t} = (1-\alpha_F)\boldsymbol{F_{ext}}^{t+\Delta t} + \alpha_F \boldsymbol{F_{ext}}^{t} \tag{3}$$

Moreover, the relations between the displacements, the velocities and the accelerations are:

$$\boldsymbol{u}^{t+\Delta t} = \boldsymbol{u}^t + \Delta t\,\dot{\boldsymbol{u}}^t + \Delta t^2 \left( \left( \frac{1}{2} - \beta \right) \ddot{\boldsymbol{u}}^t + \beta\,\ddot{\boldsymbol{u}}^{t+\Delta t} \right) \quad (4)$$

$$\dot{\boldsymbol{u}}^{t+\Delta t} = \dot{\boldsymbol{u}}^t + \Delta t \left( (1-\gamma)\,\ddot{\boldsymbol{u}}^t + \gamma\,\ddot{\boldsymbol{u}}^{t+\Delta t} \right) \quad (5)$$

Second order accuracy, unconditional stability and optimal numeric dissipation of high frequencies is obtained for:

$$\alpha_M = \frac{2\rho_\infty - 1}{\rho_\infty + 1} \quad (6)$$

$$\alpha_F = \frac{\rho_\infty}{\rho_\infty + 1} \quad (7)$$

$$\beta = \frac{1}{(\rho_\infty + 1)^2} \quad (8)$$

$$\gamma = \frac{1}{2}\frac{3-\rho_\infty}{\rho_\infty + 1} \quad (9)$$

$$(10)$$

where $\rho_\infty$ is approximately the percentage of numerical damping for the highest frequency of the structure.

## 4. MODEL UPDATING

Firstly, this structure is tested in a low level of amplitude force generated by the shaker in order to observe the linear response of the beam. On the other hand, a larger amplitude force is used to study the nonlinear behaviour of the structure. In addition, two kinds of input signal are used with the experimental setup. The first one is a chirp signal sweeping up the frequency range from 20 to 50 $Hz$ during the first half of the test (2 $s$). During the second half of the test the shaker is turned off and the beam vibrates freely until it reaches the rest position. The second input signal is a 35 $Hz$ sinusoidal signal. The aim of these signal is to excite essentially the first mode of the structure (experimentally identified around 37 $Hz$).

Figure 2 presents the scheme of the numerical model. Like the experimental results, the solution of the numerical model is computed to the time of 4 $s$. In absence of any indication, the value of the numerical parameters are the following ones. The time step is 0.001 $s$, the percentage of numerical damping for the highest frequency, $\rho_\infty$, is 0.01. The mesh is composed of 9600 hexahedral elements. The length, the width and the depth are respectively meshed with 300, 8 and 4 elements. This numerical model uses the classical mechanical parameters of elastic behaviour of the aluminium (Young's modulus: 58 $GPa$, Poisson's ratio: 0.33, density: 2700 $kg\ m^{-3}$). The damping coefficient $d$ is equal to 0.00018 $s$. Unfortunately, an important unknown of this numerical model is the boundary condition on the top of the beam. Since in the used experimental setup it is not possible to identify the value of the preload force, it is assumed that the boundary conditions on the top of the beam are modelled with an constant force, $F$, and a spring (with a stiffness noted $k_s$)(see figure 2).


Figure 2. Scheme of the numerical model

### 4.1 Volterra model

To identify the value of these two parameters a metric based on the Volterra kernels is used. The Volterra series are based on the fact that the output of a system can be given by the sum of linear and nonlinear relations with the input. The output is described by:

$$y(k) = y_1(k) + y_2(k) + y_3(k) + \cdots \quad (11)$$

where $y_1(k)$ is the linear term and $y_\eta(k)$ with $\eta > 1$ are the nonlinear terms. Each term of the equation 11 can be written using the discrete-time Volterra kernels:

$$y_\eta(k) = \sum_{n_1=0}^{N_1} \cdots \sum_{n_\eta=0}^{N_\eta} \boldsymbol{\mathcal{H}}_\eta(n_1, \cdots, n_\eta) \prod_{i=1}^{\eta} u(k - n_i) \tag{12}$$

where $N_j$ with $1 \leq j \leq \eta$ are the size of the Volterra kernel. In addition, the Volterra kernel are expended in orthonormal Kautz function basis (equation 13) to overcome the ill-posed and convergence problems. For more information about the Kautz function used in this example see Hansen *et al.* (2014).

$$\boldsymbol{\mathcal{H}}_\eta(n_1, \cdots, n_\eta) \approx \sum_{i_1=0}^{J_1} \cdots \sum_{i_\eta=0}^{J_\eta} \boldsymbol{\mathcal{B}}_\eta(i_1, \cdots, i_\eta) \prod_{j=1}^{\eta} \psi_{i_j}(n_j) \tag{13}$$

Where $\psi_{i_j}$ are the Kautz functions and $\boldsymbol{\mathcal{B}}_\eta$ are the projection of the Volterra kernels over this basis. Obviously, the number of terms of the Volterra kernels, $J_j$, is smaller than the number of terms with the direct computation ($N_j$ in equation 12).

### 4.2 Linear response

The experimental data acquired during the test with the low level of amplitude force is used to identify the value of the parameters – $F$ and $k_s$. This level of amplitude force correspond to 0.01 $V$ applied in the signal generator. Figure 3 shows the value of the exciting force recorded during the test and applied in the numerical model. Due to the low level of amplitude force, the response of the beam is linear. Then, the metric based on the first Volterra kernel is used:

$$J_{lin}(F, k_s) = \frac{\left\| \boldsymbol{\mathcal{B}}_1^{ref} - \boldsymbol{\mathcal{B}}_1^{mod}(F, k_s) \right\|}{\left\| \boldsymbol{\mathcal{B}}_1^{ref} - mean(\boldsymbol{\mathcal{B}}_1^{ref}) \right\|} \tag{14}$$

Where $^{ref}$ and $^{mod}$ refer respectively to the experimental data and the numerical model.

Figure 4 presents the evolution of the metric $J_{lin}$ versus the value of the parameters. Figure 4(a) shows that for the value of the spring stiffness equal to 16 $N\ mm^{-1}$ the minimum value of $J_{lin}$ is given by the value of the preload force of 20 $N$. In addition, for the value of the preload force equal to 20 $N$ the minimum value of $J_{lin}$ correspond to the value of 16 $N\ mm^{-1}$ for the spring stiffness (see figure 4(b)).

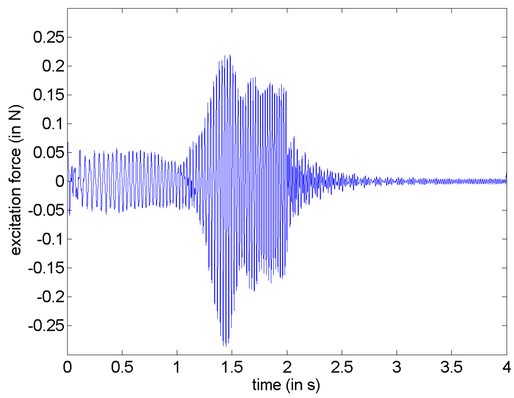
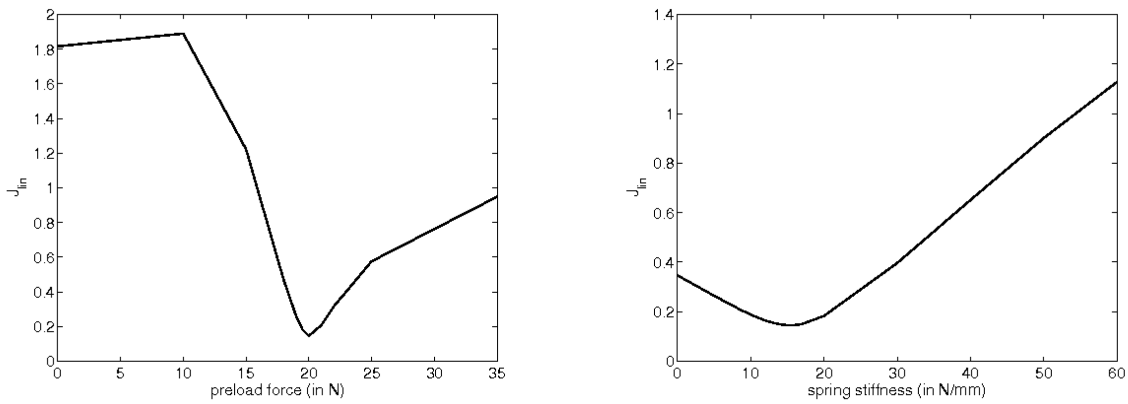



Figure 3. Value of the exciting force recorded during the test and applied in the numerical model (low input)

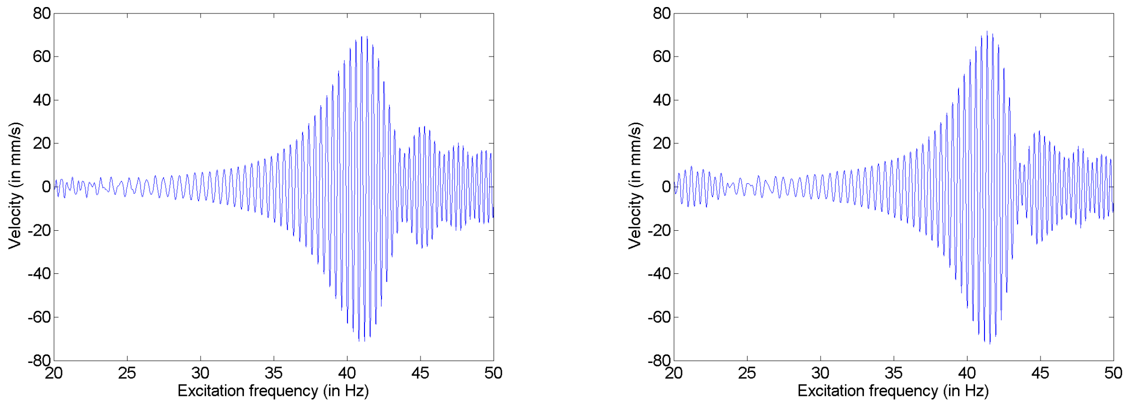
Figure 5 shows the evolution of the velocity recorded by the laser vibrometer in the middle of the beam and the velocity computed at the same position by the numerical model with the value of 20 $N$ for the preload force and 16 $N\ mm^{-1}$ for the spring stiffness. Figure 6 exposes the frequency response function (FRF) of the experimental setup and computed with the numerical model with the same value of the parameters. We can see that the output of the model and the output of the experimental setup are very similar. Then, for the next simulations, the preload force and the spring stiffness are respectively equal to 20 $N$ and 16 $N\ mm^{-1}$.




(a) Versus the value of the preload force ($k_s = 16\ N\ mm^{-1}$)

(b) Versus the value of spring stiffness ($F = 20\ N$)

Figure 4. Value of the metric based on the first Volterra kernel, $J_{lin}$, for a low level chirp input.




(a) Experimental data

(b) Response of the numerical model ($F = 20\ N$ and $k_s = 16\ N\ mm^{-1}$)

Figure 5. Response to chirp excitation of the experimental setup and computed thanks to the numerical model

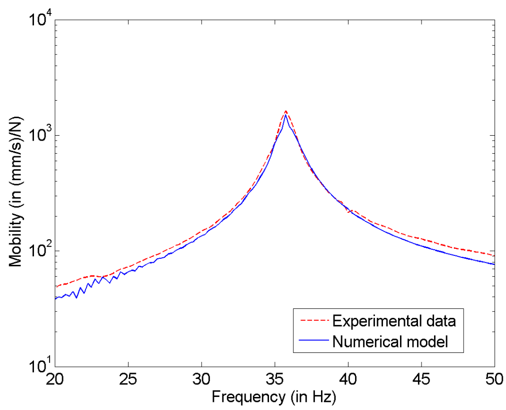
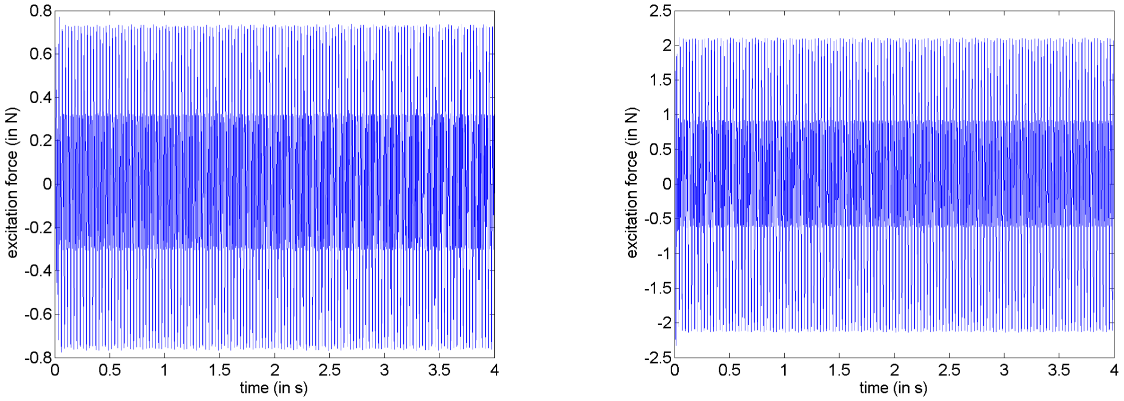



Figure 6. FRF of the experimental setup and computed with the numerical model ($F = 20\ N$ and $k_s = 16\ N\ mm^{-1}$)

### 4.3 Nonlinear response

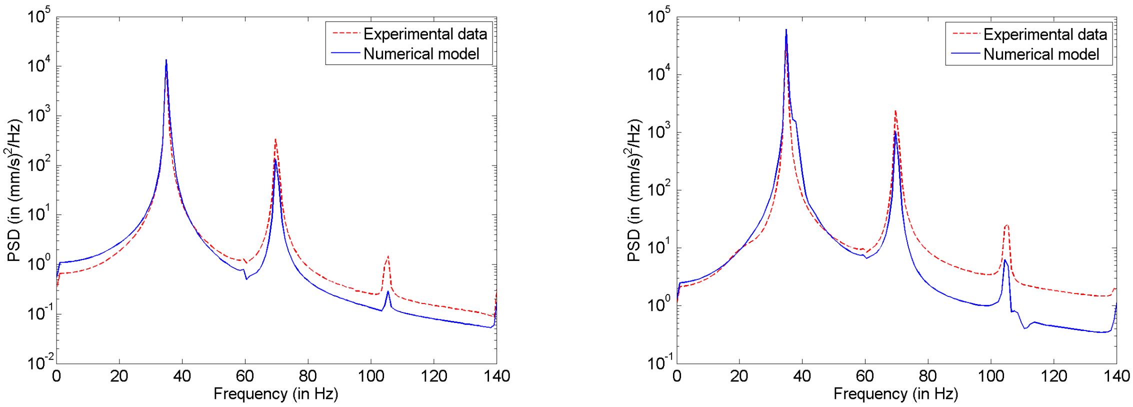
The system is exited by higher level signals to put in light its nonlinear behaviour. A medium and a high input level of 35 $Hz$ sinusoidal signal are used. The medium input level signal corresponds to 0.05 $V$ applied in the signal generator and the high one to 0.1 $V$. Figure 7 presents the evolution of the value of the exciting force for both signals.




(a) medium input (b) hight input

Figure 7. Value of the exciting force recorded during the test and applied in the numerical model

The comparison between the power spectral density (PSD) of the numerical model and the experimental data are presented in figure 8(a) for the medium input and in figure 8(b) for the high input. These figures show that the model is able to represents all harmonics in this range of frequencies (between 0 and 140 $Hz$). Thus, these comparison proves that the model updating based on the first Volterra kernel can be used for non linear systems. However, for the high input the damping of higher frequencies is more important for the numerical model. Therefore, to increase the reliability of the solution the numerical model have to be updated. A metric based on the higher-order Volterra kernels or a hybrid one is a good way to update this kind of highly-nonlinear system.




(a) medium input (b) hight input

Figure 8. PSD of the experimental setup and computed with the numerical model

## 5. FINAL REMARKS

This paper proves that the metric based on the Volterra kernel can be used to identify the nonlinear finite element model. This model updating technique is validated thanks to an experimental setup of a buckled beam preloaded by compression mechanism. Furthermore, one of the advantage of the Volterra model is that the system response is split on a linear part and a nonlinear one. Then, it is possible to identify the linear parameter with a linear response of the system. In a second step, the nonlinear parameters can be updated thanks to more complex response of the system. The future study will be focused on the model updating using a metric based on the higher-order Volterra kernels or a hybrid one applied to highly-nonlinear system.

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